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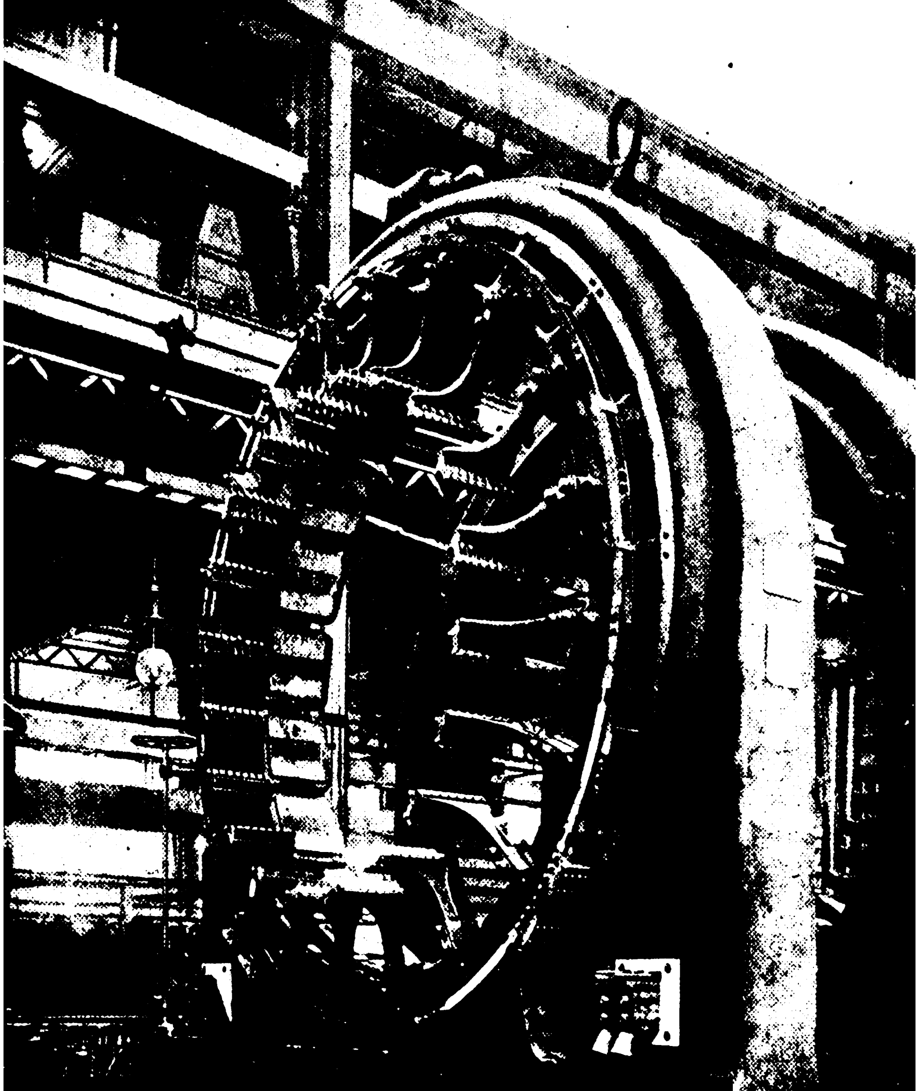

Continuous current dynamos and motors and their control

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CONTINUOUS CURRENT

DYNAMOS AND MOTORS

AND THEIR CONTROL.

BEING A SERIES OF ARTICLES REPRINTED FROM
THE PRACTICAL ENGINEER, AND

COMPLETED BY

W. R. KELSEY, B.Sc., A.I.E.E., F.Ph.S.,
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PREFACE.

THE first portion of this volume, so far as the end of the treatment of windings for closed-coil armatures, appeared originally as a series of articles in *The Practical Engineer*, from the pen of another writer. I can thus claim no credit for the merits of this part, my own attempt to set forth the subject as instructively as possible being confined to the later pages.

I have endeavoured to give a fairly complete account of the ways in which the principles previously dealt with are applied in the design and construction of the various forms of generators and motors, and to consider this in conjunction with the mechanical points involved.

Perhaps the features in which the present volume differs chiefly from similar works on the subject is in the rather fuller treatment of electrical traction, so far as tramway motors and their gear are concerned, and in the discussion of the flux-speed-torque curves for motors excited by the different standard methods. The former is, I think, justified by the ever-increasing importance and extension of electric tramway systems, while the latter strike the mind more vividly than many figures or worded statements of the facts they indicate.

I would invite the reader's attention to the words of Mr. Heaviside which terminate the introduction, and impress upon him the absolute necessity for taking actual figures and making for himself calculations and curves, as only in this way can a proper understanding of electrical design be possibly obtained.

In conclusion, I must express my thanks and indebtedness to the various firms who have kindly placed information or illustrations at my disposal, to the various authors whose works I have extracted from or cited—in every case, I hope, with due acknowledgment, and to my late assistant, Mr. H. Nutton, for much help in preparing the diagrams.

W. R. KELSEY.

Municipal Technical Institute,
Taunton, January, 1903.

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ERRATA.

Page 182, line 13, for "brush" read "Brush."

Page 194, line 9 from bottom, for "are not uncommon" read "are the usual practice."

Page 195, line 5 from bottom, for "rim" read "ring."

Page 206, line 7 from bottom, for "energy loss one-eighth," read "energy loss one-fourth of C^2R."

Page 215, line 8 from bottom, for "n = number of parallel circuits in the winding" read "n = number of pairs of parallel circuits in the winding."

Page 222, line 16, read "$\frac{284 \times 30 \times \cdot 66}{\cdot 33 \times 10^6} = \cdot 017$ square inch."

Page 251, second equation, for "12^6" in denominator read "10^6."

Page 252, line 7 from bottom, for "seven" read "thirteen."

Page 256, last two lines, for figs. 113, 114, and 119 read figs. 119, 120, and 121.

Page 334. In fig. 170 the right hand contact of the no-load release should be shown connected to the wire above it.

Page 350, line 2, for "which are here shown. fig. 180, free," read "as shown in fig. 180."

Page 381. For uniformity with figs. 193, 195, fig. 194 should be inverted.

Page 421, line 5, for "economical coefficient" read "economic coefficient."

Page 431, line 17, for "N" read "N′."

INTRODUCTION.

In recent years electrical energy has been largely employed in effecting many industrial operations. Its application to the production of light has now become quite familiar; electricity works, affording a public supply for this purpose, being now in successful operation in very many towns in England.

Many workshops and factories are also lighted electrically, usually by plant erected and worked within the works; with better results, as regards a purer atmosphere, than can be obtained by other systems of artificial illumination, and in most cases at a slightly reduced cost.

Great attention has also been given to the electric transmission of power. At first transmission over long distances formed a very attractive problem to engineers, and numerous examples of such transmissions are now successfully at work.

More recently the distribution of power to the various tools, &c., employed in workshops has been undertaken. In place of one large engine driving numerous lines of shafting by belts or ropes, or of several engines, erected in different parts of the works, driving by belts or ropes lines of shafting or geared to large machines; one or more large engines are employed directly driving dynamos, from the terminals of which conductors lead to the various points where the tools are to be driven. In the case of large tools, a separate motor is usually employed for each machine; but with small tools, this is not at present considered quite feasible, and the shafting is broken up into short lengths, each section being driven by a separate motor. The results so far attained are decidedly encouraging. The cost of fuel and labour is in all cases less than has been observed with mechanical transmission.

The measurement of electric power is an extremely simple operation, practically amounting to the reading of two gauges—one giving directly the current in amperes, the other the electrical pressure in volts,—the product of these two quantities gives the electrical power in watts; or even to the simple inspection of a single instrument which reads directly the power in watts. This simplicity of measurement has led to one remarkable fact being established. It has been observed in an ordinary works that usually more power is required when all the tools are running, but doing no cutting, than is required for the regular work of the shop. All the tools are very seldom fully employed in cutting operations—some are idle from time to time, from one cause or another.

This method of distribution of power will undoubtedly be adopted in all newly-equipped workshops; and as competition becomes keener, ultimately such a system will prevail in all workshops.

The manufacture of dynamos and motors is daily becoming more the work of mechanical engineers—for example, the cores of the armatures were in early machines carried on wooden bosses. Then much more mechanical methods of support were introduced, metallic spiders directly driving the core, the conductors being driven and prevented from flying off from centrifugal force by binding bands of wire. Next, insulated metallic driving horns were made to project amongst the conductors, positively driving them at numerous points, the binding wires being retained to prevent the flying off of the conductors. The most recent practice, which at the present time bids fair to become universal, is the embedding of the conductors in channels in the iron core, all the conductors being positively driven. The binding wires also disappear ; the conductors are fixed in position, and prevented from flying out by long taper dovetailed hardwood wedges driven in on the top of them.

The field magnets also have undergone a similar process of evolution. The many strange forms, which at one time seemed to be looked on as essential to their design, have almost disappeared. The modern field magnet is made as simple and massive as possible, and almost without exception the exciting coils are now so arranged that they can be wound by machinery on separate formers with metallic end-plates, and afterwards placed in position on the cores and mechanically fixed there.

Probably in years to come we shall see dynamos and motors designed and made in engineering workshops quite as freely as steam engines are to-day.

Under these circumstances it behoves the practical engineer, who is anxious to keep abreast with the times, to bestir himself. Not only will he have to deal with steam engines, shafting, and machinery, but he will also be expected to manufacture and put in the dynamos and motors as well. In order that he may do this in an intelligent manner, it is necessary that he should be, at any rate, familiar with the elements of dynamo and motor design, and the first portion of this book is written with the object of presenting this, for continuous-current dynamos and motors, in as simple and practical a manner as possible consistent with the necessary degree of accuracy. It is due to the labours of the brothers Hopkinson, Kapp, Swinburne, Esson, Ewing, Crompton, amongst others, who have freely published the results of their work and experience, that it is possible thus early in the history of the practical application of the dynamo machine, to deal fully with the design of dynamos in a scientific manner. Constant reference to their writings is inevitable.

The designing of dynamos is a compromise of conflicting conditions, and practical experience is required before it is possible to arrive at the best result under given conditions. This will be clearly kept in view, and an attempt made to define the limits found best in practice and to show the methods pursued, in order that a successful result may be attained.

The practical construction of dynamos and motors of various types will also be dealt with concurrently with their design; and finally their working and practical application to lighting, transmission of power, &c., will receive attention.

A sentence in Mr. Oliver Heaviside's book on "Electro-magnetic Theory"* is well worthy the earnest consideration of all practical engineers:—"The best plan is to sit down and work; all that books can do is to show the way." The author hopes that this little book may be the means of directing some practical engineers to that work which is essential to a clear understanding of the applications of electrical energy.

---

* "Electro-magnetic Theory," vol. i., Electrician Series.

DYNAMOS AND MOTORS.

PART I.

CHAPTER I.

THE DESIGN, CONSTRUCTION, AND WORKING OF CONTINUOUS-CURRENT DYNAMOS AND MOTORS.

Definition of Dynamo and Motor.—When a conductor of electricity is moved in a magnetic field of force, experiment proves that, between any two points in the conductor, a difference of electrical pressure is maintained so long as the motion of the conductor is continued in the magnetic field. If this conductor is made part of a closed circuit, a current of electricity flows through this circuit during the motion of the conductor.

This principle was discovered by Faraday in 1831, and electrical energy so produced is said to be generated by magneto-electric induction, or more frequently by **induction.**

The dynamo machine is the practical application of this principle of magneto-electric induction to the production of electrical energy, the dynamo being simply a machine by which mechanical energy is converted into electrical energy, usually by the rotation of conductors in a magnetic field.

If electrical energy is applied to a closed conductor, part of which is situated in a magnetic field, the conductor tends to move in the field. An electro-motor then is a machine for applying this reciprocal principle of a dynamo, it converts electrical energy into mechanical energy.

To take a familiar analogy, two cisterns, one placed at the earth's surface, the other at a higher level, say 25 ft. above, the lower one. The cistern at the earth's surface is filled with water, and the suction pipe of a common lift pump is inserted into it. The delivery pipe of the pump communicates with the upper cistern. On working the pump, a difference of pressure is created between the pressure on the surface of the water in the suction pipe and the atmospheric pressure on the surface of the water in the lower cistern ; and in virtue of this difference of pressure, a stream of water flows from the lower cistern to the upper one so long as this difference of pressure is maintained, *i.e.*, so long as the pump is kept in motion. The dynamo is analogous to the water pump, it generates a difference of electrical pressure, and it is in virtue of this difference of electrical pressure that when, the circuit of a dynamo is completed by a conductor of electricity (equivalent to the pipe of the pump), a current of electricity flows through the conductor.

Further, the mechanical energy required to work the pump and raise the water from a lower to a higher level imparts to the water in the upper cistern potential energy, and by leading from the upper to the lower cistern a pipe, a stream of water could be made to flow through this pipe and used to drive, say a water wheel, so producing mechanical energy again. The electro-motor is equivalent to the water-wheel; a difference of electrical pressure is maintained between its terminals, and in virtue of this difference of pressure a current of electricity flows through the motor, producing rotation of the conductors conveying this current in a magnetic field and transforming electrical into mechanical energy.

Obviously, if it is desired to keep the water-wheel in continuous rotation, the pump must be worked continuously, requiring the expenditure of mechanical energy. So also in the case of a dynamo, if electrical energy is required to supply lamps, drive motors, &c., the dynamo must be kept in continuous motion by the application of mechanical energy.

In order then that electrical energy may be obtained in a closed conductor, it is requisite that a difference of pressure must be created between two points in the conductor, and this difference of pressure must be maintained to keep the current flowing under this pressure. This is the function of the dynamo; it generates a difference of pressure, and maintains it when a current flows under this pressure.

More mechanical energy would be expended in working the water pump and keeping in continuous rotation the water wheel, than would be returned as mechanical energy by the wheel, from loss by friction, &c., in the pump; friction, &c., of the water in the pipe; and friction, &c., of the water wheel. This is also true of the dynamo; more mechanical energy is required to be supplied to the dynamo than appears as electrical energy at its terminals, from loss by air friction, friction of bearings, and electrical losses in its conductors, &c. Further, if this energy is conveyed by conductors to a distant point, a further loss takes place in the conductors, and if the energy is there used to drive a motor, still more energy is used in overcoming air friction, bearing friction, and electrical losses.

Notwithstanding these losses the dynamo is a machine of remarkable efficiency; with a well designed machine of medium size, 90 per cent of the mechanical energy required to drive it may be obtained at its terminals as electrical energy at full load. It further possesses a distinct characteristic in that it maintains a high efficiency over a large variation of output. At one-fourth the full load it would return probably 75 per cent of the energy supplied to it; and on open circuit, the energy required would be that absorbed by air and bearing friction, with, say, 4 to 5 per cent for exciting its field and magnetic loss in the armature core. Very large machines usually give an efficiency of from 95 per cent to 96 per cent at full load.

Magnetic Fields of Force.—To design successfully any type of machinery, a clear idea of the principles underlying the method of working of that type is necessary. Further, it is essential to precisely specify any given machine, that the system of weights and measures applicable to that type should be well understood; hence it will be necessary to deal with these principles briefly and to describe the units of measurement employed.

It is not possible to determine by simple inspection whether a bar of steel is magnetised; no visible change takes place during the process of magnetisation. But if two bars are taken, one magnetised, the other not, and both plunged in iron filings, evidence of a marked difference in the properties of the two bars is apparent. The magnetised bar attracts the iron filings strongly; more than this, the filings are unequally distributed along its length—scarcely any are attached to its middle portion—nearly all are concentrated in two regions near the ends of the bar. A bar of steel possessing these properties is called a **permanent magnet**, and the regions near the ends of the bar are termed the **poles** or **polar regions** of the magnet.

The space surrounding such a magnet can be mapped out in a more complete manner by a method due to Faraday. Take a sheet of cardboard, or a plate of glass, by preference having one surface coated with paraffin wax, or with gum which has been allowed to set. Sprinkle fine iron filings uniformly over the prepared surface of the card (by a fine muslin bag) and place gently in position over a bar magnet. With care no disturbance of the filings will take place, the friction of the filings on the card being sufficient to prevent movement. Tapping the card gently, so as to throw up the filings from the surface of the card a short distance in the air, the filings while in the air are free to move under any force acting on them, and in falling take up definite positions on the card due to that force, and actually in this case the filings are found to have arranged themselves in a series of separate curves (fig. 1). Such a space is referred to as a magnetic field of force, and the curves are called magnetic lines of force.

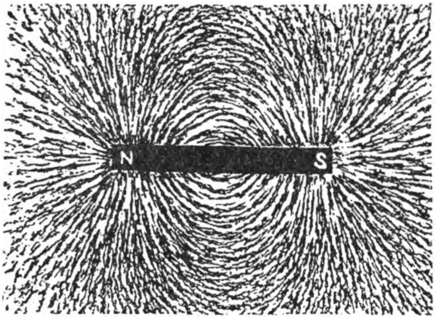


FIG. 1.—Magnetic Field of Force of a Bar Magnet (Longitudinal Plane).

Applying heat to the surface of the card, and melting the paraffix wax; or, if gum is used, by allowing steam to play on its surface, the curves can be permanently fixed in position.

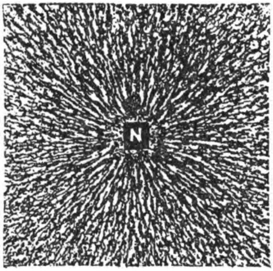


FIG. 2.—Magnetic Field of Force of Bar Magnet (End on).

Examining a card made in this manner, it is apparent that the curves are denser near the polar regions by the crowding together of the lines of force, and that proceeding further away from these regions the lines are not so dense, some extending beyond the surface of the card, appearing as far as they can be traced as lines growing wider apart and fainter, but all, if followed, would form distinct curves, no line ever crossing any other line.

Having thus obtained a representation of the lines of force in one plane, in a similar manner the fields of force of all other planes could be plotted, and for all longitudinal planes would be of a precisely similar character. For planes, however, at right angles to the longitudinal planes, the curves would appear different. Placing the magnet vertically, with a card above it, and proceeding as before, the result would be a number of radial lines; the plotting now giving a sectional view of the curves previously mapped out (fig. 2).

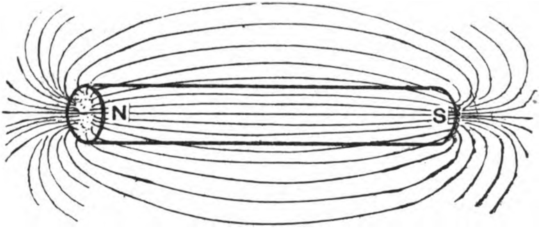


FIG. 3.—Magnetic Field of Force Encircling Cylindrical Bar Magnet.

With a cylindrical bar magnet the lines emerge equally all round the circumference on any given plane at right angles to the length of the magnet, at one end, curl round, and re-enter the other end, completely enveloping the bar with lines of force (fig. 3).

So far the fields plotted by iron filings show only the existence, in the space surrounding a magnet, of lines of force in the form of curves. Can any other information be obtained from them?

If two steel bars are magnetised (two knitting needles are convenient), and one is suspended by a silk thread at the centre of its length, so that it can vibrate in a horizontal plane, it will oscillate about a certain position, and, after a few swings, come to rest pointing in a certain direction; if no other magnets are present, magnetic north and south. Mark the end of this needle which points towards the north, N (north-seeking end), and the end which points towards the south, S (south-seeking end), and remove it from the silk thread. Suspend the second needle, allow it to come to rest, and mark also its north and south-seeking ends. Now bring the N end of the first needle near the N end of the suspended needle, and observe the effect produced on the suspended needle—the two ends repel one another. Next take the S end of the first needle near the N end of the suspended needle, and the two ends now attract one another; or, stated generally, like poles repel, and unlike poles attract, one another. What then would be the effect of placing a suspended needle in different positions near a bar magnet?

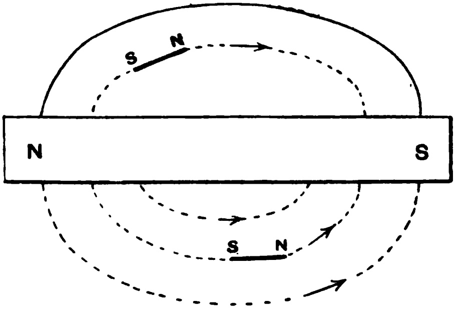


FIG. 4.—Magnetic Field of Bar Magnet Plotted by Means of Test Needle, showing Direction of Lines of Force.

Take a sheet of white paper and place a bar magnet on it near the centre of the sheet. Bring near say the N end of the magnet a suspended magnetic needle short in length as compared to the length of the magnet. The S end of the small needle will be strongly attracted by the N end of the magnet. Make a mark with a pencil directly under the N end of the small needle. Now move the suspended needle so that its S end is directly over the pencil mark and again mark the new position of its N end. Proceeding thus, trace out the positions of the N end of the needle until the suspended needle arrives at the S end of the bar magnet. Joining the pencil marks by straight lines and drawing a curve through the middle of the length of these short lines, one curve is traced out exactly as was plotted by the iron filings. The needle in fact sets itself tangentially to the line of force. Commencing at other positions near the N end, other curves can be drawn to the S end.

These curves then not only show the existence of the lines of force, they also show the direction of the force, *i.e.*, if a single N pole was placed near the N pole of the magnet it would be repelled along such a curve until it arrived at the S pole of the magnet, where it would be attracted and come to rest.

A "Free" Pole.—It is impossible, however, to make a magnet having only one pole, every N pole has its accompanying S pole. If a long magnet is broken in two exactly at the middle of its length, two new poles equal in strength but opposite in sign appear. The system of measurement employed in determining these quantities demands the existence of a single magnetic pole, or, as it is termed, a "free" magnetic pole. This cannot be realised in practice, but an approximation can be made to a "free" pole by taking a bar of considerable length as compared to its diameter, so that the N pole is as far removed as possible from the vicinity of the S pole. If one of the magnetised knitting needles is suspended by a silk thread so that it hangs vertically with its N pole downwards, and so held is brought near the N pole of the permanent magnet, the knitting needle will be urged from the N polar region along a line of force towards the S polar region of the magnet. That this really occurs is perhaps made more apparent if the knitting needle is thrust into a cork so that its N pole just projects above the surface of the cork, and thus equipped, placed in a vessel of earthenware containing water, on the surface of which it will float. Over this vessel the bar magnet is suspended horizontally by silk threads, so that it is fixed at about the same level as the pole of the floating needle. On bringing the floating needle near the N polar region of the magnet and releasing it, it will float away from the N polar region of the magnet in a curved path along a line of force towards the S polar region, where it will be attracted and come to rest, and similarly for any other positions near the N polar region, but each time starting from a new position and floating along a new path. As regards the direction of the lines of force, it only remains then to point out that it is agreed that the direction in which a "free" N pole would move along a line of force is taken as the **positive direction.** This is usually indicated, when a field is plotted, by marking the lines coming from the N polar region of a magnet with arrow heads directed towards the S polar region, as in fig. 4. The lines are supposed to emerge from the N polar region, curl round, and re-enter the S polar region.

The number of lines of force shown by the curves plotted with iron filings is limited, yet it is clear that they really are infinite in number, as will be evident if the floating needle is started from an infinite number of points near the N polar region, an infinite number of lines would be traced out by it in floating round to the S polar region. The mutual action on one another of the filings, magnetised by induction, is such that they tend to congregate in lines with comparatively bare spaces between them.

C.G.S. System of Weights and Measures.—It is evident, however, that the lines thus shown are denser near the polar regions than at a distance from them, and the magnetic field is said to be **strong** where the lines are dense, and **weak** where they are further apart. These are only comparative terms, and to define the strength of a field more exactly it will be necessary to explain more in detail the system of measurement usually used in determining these quantities.

This system is known as the C.G.S. (centimetre-gramme-second) system, from the centimetre being taken as the unit of length, the gramme as the unit of mass, and the second as the unit of time. All the other units used are multiples of these fundamental units; the system may, in fact, be termed a system of units, simply related to one another.

Unit of Force.—A force is measured on this system by the velocity imparted to a mass of one gramme after acting upon it for one second, and the **unit of force** is defined as the force which, after acting for one second, imparts a velocity of one centimetre per second to a mass of one gramme. This unit of force is called **one dyne**, so a force of 1,000 dynes would impart a velocity, at the end of one second, of 1,000 centimetres per second to a mass of one gramme.

Unit Pole.—In the application of this system to the measurement of magnetic forces, **unit pole** is defined as a magnetic pole of such strength that, if placed in air at unit distance from another pole of equal strength, unit force will act between them. Thus a "free" N pole of unit strength, placed at a distance of 1 centimetre in air from a "free" S pole of unit strength, will be attracted with a force of one dyne. It has been determined experimentally that a mass of one gramme, falling under the action of gravity at Paris, has imparted to it a velocity at the end of one second of nearly 981 centimetres per second—*i.e.*, expressed in units of force, the attraction of the earth at Paris on a mass of one gramme is 981 dynes. If, then, the attraction or repulsion in air of a unit pole placed in a magnetic field, as measured by a balance, is 10 grammes, this weight multiplied by 981 = 9,810 dynes, the force exerted on the unit pole; and this number, 9,810, is a measure of the strength of the field at that point.

Unit Field.—As previously pointed out, the lines of force plotted out by iron filings show the direction of the force at any point. It is also possible to use them as a measure of the strength of the field. Suppose that a piece of card, 1 square centimetre in area, is placed in a magnetic field so that it cuts the lines of force at right angles to their direction at any point. Near the polar regions, many more lines would be cut by the surface of the card than if it was placed where the lines are not so dense. Hence the number of lines, which are cut in a plane perpendicular to their direction by this piece of card, would also be a measure of the strength of the field at different points over that area.

This method is the one usually employed in the practical design of dynamo machines, and on the C.G.S. system, unit field (*i.e.*, a field having one line per square centimetre) represents a field acting on a "free" pole of unit strength, with a force of one dyne. Thus, a field whose strength is stated in C.G.S. measure as 5,000 lines, is a field in which the number of C.G.S. lines per square centimetre in a plane perpendicular to the direction of the lines is 5,000; and, therefore, a "free" pole of unit strength placed in this field, would be acted on by a force of 5,000 dynes.

By these conventions, then, the strength and direction of magnetic fields can be completely expressed.

Units of Resistance, Current, and Electrical Pressure. In the analogy of the water pump and dynamo, it was pointed out in the case of the pump, that it maintained a difference of pressure between the water in the pump and that on the surface of the water in the lower cistern, in virtue of which a stream of water flowed from the lower to the upper cistern; also that the action of the dynamo was analogous to that of the pump—it maintained a difference of electrical pressure between two points in a closed conductor, in virtue of which a certain current flowed in the conductor.

It will next, therefore, be necessary to define the units employed for the measurement of these quantities.

When a conductor is made part of a closed circuit, a difference of electrical pressure is found to exist between any two points of the conductor when a current flows in the conductor. If the current is varied, the difference of electrical pressure varies also, and it has been proved experimentally that the ratio of $\frac{\text{electrical pressure}}{\text{current}}$ is a constant quantity. This ratio is called the **resistance** of the conductor; it is a property of all conductors, which is independent both of the difference of electrical pressure applied and of the current resulting from the application of this difference of pressure; so long as the temperature of the conductor remains the same, and no change occurs in its molecular state. The constancy of this ratio was first discovered by Dr. Ohm, and the law stating this relation is known as Ohm's law, and stated as follows:

The current varies directly as the electrical pressure and inversely as the resistance of the circuit.

With the C.G.S. system, Ohm's law would be written:—

$\frac{\text{One C.G.S. unit of electrical pressure}}{\text{one C.G.S. unit of current}}$ = one C.G.S. unit of resistance.

Now, the C.G.S. unit of resistance has been found so inconveniently small, when applied to the measurement of resistances which occur in the practical applications of conductors to the heavier branches of electrical engineering, that a unit (usually known as the practical unit of resistance) 10^9 (*i.e.*, 1,000,000,000) times as large has been adopted; and this is defined as "the resistance offered by a column of mercury—at the temperature of melting ice—14·4521 grammes in mass, of a constant cross sectional area, and of a length of 106·3 centimetres."

The real standard in England, legalised by an Order in Council dealing with electrical units and standards, dated 23rd August, 1894, is the resistance between the copper terminals of a certain coil of wire in the Board of Trade Standardising Laboratory to the passage of an unvarying electrical current, when the coil of wire is in all parts at a temperature of 15·4 deg. Cen.

To this unit the name of **one ohm** is given. To give some idea of this unit, it is almost exactly the resistance offered by a wire of pure copper 10 ft. in length and 0·01 in. diameter.

The C.G.S. unit of current, on the other hand, would in practice be far too large, and a unit 10^{-1} (*i.e.*, $\frac{1}{10}$) as large has been adopted. This is defined as "the unvarying electric current which, when passed through a solution of nitrate of silver in water, prepared in accordance with an appended specification, deposits silver at the rate of 0·001118 of a gramme per second."

The legalised standard is the current which is passing in and through the coils of wire forming part of the Board of Trade standard instrument, when on reversing the current in the fixed coils, the change in the forces acting upon the suspended coils in its sighted position is exactly balanced by the force exerted by gravity in Westminster upon a certain weight of iridio-platinum.

To this unit the name of **one ampere** is given. The current in an ordinary 16 candle power incandescent lamp is about $\frac{6}{10}$ of an ampere.

In order that the relation stated by Ohm's law, when expressed in these practical units, may still be directly comparable with it as expressed in C.G.S. units, the unit of electrical pressure is made 10^8 (*i.e.*, 100,000,000) times as great as the C.G.S. unit. This is defined as "the electrical pressure that, if steadily applied to a conductor whose resistance is 1 ohm, will produce a current of 1 ampere, and which is represented by ·6974 $\left(\frac{1000}{1434}\right)$ of the electrical pressure at a temperature of 15 deg. Cen. between the poles of the voltaic cell known as Clark's cell, set up in accordance with an appended specification."

The legalised standard is one-hundredth part of the pressure which, when applied between the terminals of the Board of Trade standard instrument, causes the rotation of the suspended portion from coincidence of the reflection of a fixed mark in the suspended mirror with the cross-wire of an eyepiece, before and after the application of the pressure, to that of coincidence of the reflection of a second fixed mark with the cross-wire of the eyepiece during the application of the pressure.

This unit is called **one volt**.

The electrical pressure at the terminals of the voltaic cell (Leclanche) usually used for working electric bells is about $1\frac{4}{10}$ volt.

Ohm's law with these units, would be written:

$$\frac{\text{Electrical pressure in volts}}{\text{current in amperes}} = \text{resistance in ohms};$$

or if E = electrical pressure in volts,
C = current in amperes,
R = resistance in ohms,

$$R = \frac{E}{C};$$

and by transposition, $$C = \frac{E}{R},$$

and $$E = CR.$$

Unit of Power.—To specify the power or rate of doing work in a circuit, or in any part of a circuit, two factors must be measured—the amperes of current in the circuit, and the difference of electrical pressure in volts between the two given points in the circuit. The product of these two quantities expresses in volt-amperes the power developed in that circuit.

In a circuit, then, with a difference of electrical pressure of 1 volt at its terminals when a current of 1 ampere flows through it, 1 volt-ampere of power is developed, and 1 volt-ampere is taken as the unit of power, and is called **one watt.**

746 watts = 1 horse power.

Unit of Energy.—Electrical energy or work done is in practice usually measured in **watt-hours**—*i.e.*, the unit is the energy supplied in an hour by a circuit delivering power at the rate of 1 watt. In the commercial supply of electrical energy from electricity works, the Board of Trade has stipulated that the energy used shall be charged for in units of 1,000 watt-hours; this is sometimes called 1 B.T. unit, or simply 1 unit.

Certain prefixes are used to denote very large or very small multiples of these units: thus, mega denotes a million; 10 megohms is a resistance of 10,000,000 ohms; micro is one-millionth; 1 microhm is ·000001 ohm. Kilo equals one thousand, so 5 kilowatts equal 5,000 watts; 1 kilowatt equals about 1⅓ horse power. One B.T. unit equals 1 kilowatt-hour, equals 1,000 watts for an hour.

To recapitulate briefly the units used in practical work, so far defined:

Practical unit of	Name.
Strength of magnetic field..................	Line
Resistance	Ohm
Current....................................	Ampere
Electrical pressure	Volt
Power	Watt
Power	Kilowatt
Energy	Watt-hour
Energy	B.T. unit = kilowatt-hour

Other units employed will also need to be defined, but these will perhaps be more easily understood when the effects to be measured have been explained.

CHAPTER II.

Field of Force of Wire Conveying a Current.—It was pointed out in the case of a bar of steel that it was not possible to tell by inspection whether it was magnetised or unmagnetised. The same remarks apply to the case of a wire conveying an electric current; the wire is not visibly different from other wires. But in the year 1820 the Danish physicist, Oersted, made an important discovery respecting the space surrounding such a wire. He found, on bringing a small pivoted magnetic needle near a wire conveying an electric current, that the needle was affected, and took up a definite position relative to the wire.

As a simple illustration of this, take the case of a long straight wire, held horizontally over the small suspended needle, so that the wire is parallel to the direction in which the needle sets itself in the earth's field. On sending a strong current through the wire, the needle is violently deflected, and, after a number of oscillations, comes to rest very nearly at right angles to its former position. On interrupting the current the needle returns to its former position; it again points north and south.

Around a wire, then, in this condition, a field of magnetic force evidently exists; it affects a compass needle just as if a magnet had been brought near it. Will such a wire attract iron filings? The magnetic effect of a single wire is small, but if a No. 15 S.W.G. copper wire, carrying a current of from 40 to 50 amperes, is plunged in iron filings, the filings will be magnetised by induction, and will arrange themselves around the wire. On withdrawing the wire, the whole of its surface will be found to be enveloped in a complete layer of filings.

The field of force around a straight wire conveying a current can be mapped out more completely by means of iron filings, just as the field surrounding a bar magnet was mapped out.

Pass the wire vertically through a hole in the centre of a horizontal sheet of prepared cardboard or glass (the wires leading to the source of electric current should be kept at some distance from the wire passing through the card). Then cover the surface of the card with fine iron filings. On sending the current (40 to 50 amperes) through the wire, and gently tapping the card to throw the filings from its surface, they will be found, in falling, to arrange themselves in certain curves, which on examination will be found to be concentric circles surrounding the wire, fig. 5. In the immediate vicinity of the wire, the magnetic force is so great that the magnetised filings attract one another, and are swept up into a little annular heap closely surrounding it. Proceeding further away, the circles become less dense; the strength of the field is greater near the wire than at a distance from it.

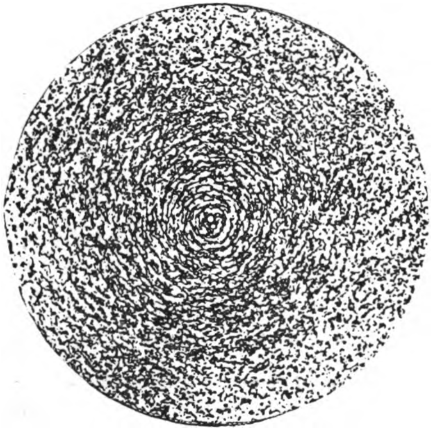
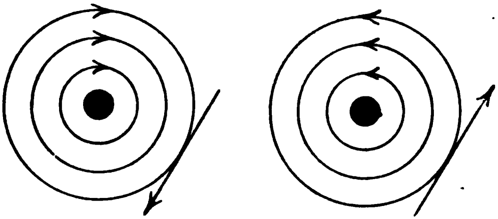


Fig. 5.—Magnetic Field of Force surrounding a Straight Wire Conveying a Current in a Plane Perpendicular to the Wire.

The circles thus obtained show the distribution of the lines of force of a straight wire for a plane at right angles to the length of the wire. By moving the card along the wire, the distribution for other planes parallel to this plane will be found to be exactly the same. The wire is completely surrounded throughout its length by a magnetic field of this character.

A short magnetic needle, held just above the surface of the card near the wire, would set itself tangentially to these circles at any point, and, as in the case of a bar magnet, the N end of the needle will always point in the positive direction of the lines—*i.e.*, the direction in which a "free" north pole would move round the wire. To an eye placed vertically above the wire, and with the current flowing downwards through the wire, a free north pole would move in a right-handed direction around the wire; whereas, if the current flowed upwards through the wire, a free north pole, to the eye placed vertically above the wire, would move in a left-handed direction, fig, 6.



Current flowing downwards. Current flowing upwards.

FIG. 6.—Direction of Lines of Force surrounding a Straight Wire Conveying a Current.

Field of Force of Circular Current.—The wires, or conductors, used in practice are not often quite comparable to the case of a single straight wire; usually the conductors take the form of loops, and it is important that a clear idea should be obtained of the field of force of such loops.

Bend the wire so as to form it into a circle, and arrange the two ends so that the current can be sent around the circle in a right-handed direction, to an observer facing the circle, as in fig. 7.

With the wire thus arranged, it is evident, if the field obtained with a straight wire is maintained, that all the lines will pass through the circle, emerging from one face, curling round, and re-entering the other. On examining by means of a small needle the field produced, this will be found to be the case; the needle will not only take up positions outside the circle, but at any point within the area enclosed by the wire as well.

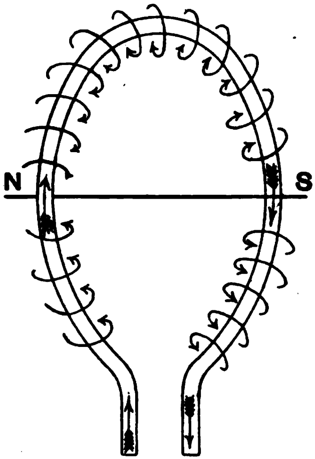


Fig. 7.—Magnetic Field of Force of Circular Current.

If care is taken with the plotting, the lines of force, however, will no longer be found to be circles, but ovals, closely crowded together within the ring, and gradually growing wider apart as they emerge and curl round. This is due to their mutual action on one another, and to their close packing together in passing through the ring.

If the direction of the current passing round the circle is reversed, the direction of the lines as shown by the testing needle will be found to be reversed also.

Such a circle of wire with a current passing round it, is, as pointed out by Arago, equivalent to a thin disc of steel magnetised so that one face is N. polar and the other S. polar.

The circle, if suspended, will set itself in the earth's field with one face pointing north and the other south. It will be attracted or repelled when the pole of a magnet is brought near it, and two such circles will behave to one another just as if they were two magnets—like faces will repel and unlike faces attract one another. If a very large current is sent round a ring of this description, its field can be plotted by means of iron filings.

Field of Force of a Spiral Current.—The magnetic effect of a circular current can, however, be increased in a simple manner, by placing a number of such circles side by side in a straight line, or, what is practically equivalent, winding a long length of wire into a close spiral, and passing the current through. On testing such a solenoid, as it is termed, with a suspended needle, one end will be found to be N. polar and the other S. polar. Plotting the space in any longitudinal plane surrounding the solenoid, the lines will be found to emerge from the N. end, curl round, and re-enter the S. end, practically just as in the case of a cylindrical bar magnet of similar size, fig. 8.

Further, if a card covered with iron filings is inserted in the interior of the solenoid, and a current passed through it, the filings will be found to arrange themselves in nearly parallel lines. Thus, in this case, the complete paths of the lines in air can be traced, whereas in the case of a bar magnet the lines can only be traced from their point of emergence from the magnet to their point of re-entry.

Suspending the solenoid, and sending a current through it, it will be found to set itself north and south in the earth's field. If a magnet pole is presented to it, the solenoid will be attracted or repelled, and two such solenoids will attract or repel one another precisely as if they were two bar magnets.

To an observer facing the solenoid, and with the current flowing in a right-handed direction, the end near the observer will be S. polar, and the end furthest from him will be N. polar. If the direction of flow is left-handed, the end near him will be N. polar and the remote end S. polar.

It is perhaps scarcely necessary to remark that these magnetic effects are only perceptible when a current passes, or is said to pass, through a straight wire, a circular ring of wire, or a solenoid, &c. On interrupting the current all these phenomena vanish. The lines of force thus created must be regarded as interlinked with the current—they co-exist with it, when it flows in a straight, circular, spiral, or other path.

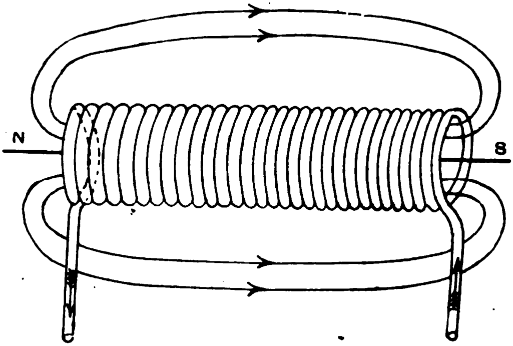


FIG. 8.—Magnetic Field of Force of Spiral Current.

Magneto-electric Induction.—With this explanation of the magnetic fields of force accompanying currents flowing in various paths, it will next be necessary to examine some facts first discovered by Faraday, and called by him the induction of currents, or currents produced by magneto-electric induction.

For this purpose an instrument, known as a galvanometer, capable of detecting the presence of a current in any circuit attached to its terminals, is required. This usually consists of a coil of insulated copper wire, with a very small steel magnet, suspended by a single silk or quartz thread at the centre of the coil, so that, when the plane of the coil is placed parallel to the direction of the earth's field of force, the axis of the magnet is parallel to the face of the coil. To enable small deflections of the magnet from its position of rest to be readily observed, some form of pointer is attached to it; and for work requiring a sensitive instrument, this most frequently takes the form of a small silvered glass mirror attached to the magnet. A beam of light is arranged to fall on the mirror, from the surface of which it is reflected back on to the scale of the galvanometer, placed at some considerable distance from the instrument (usually about 3 ft.). In this way the galvanometer is provided with a pointer equivalent in length to twice the distance between it and the scale, but possessing this important advantage, that it adds a very small weight (a few grains only) to the small suspended magnet.

When a current passes through the coil, the little magnet tends to set itself at right angles to its former position, as in the case of the simple experiment with a single wire and suspended magnet, already described (page 20), due to Oersted. Owing, however, to the small weight of the magnet and mirror, an instrument of this character is readily made which is extremely sensitive to very small currents, and which will show by a movement to and fro of the spot of light on the scale the presence of a current of very short duration.

Induction by a Moving Coil—Provided, then, with such an instrument, take a flat coil of cotton-covered copper wire, and attach the ends of the coil to the terminals of the galvanometer. Place the coil of wire vertically on a table, with the plane of the coil perpendicular to the horizontal component of the direction of the earth's field of force. Proceed now to turn the coil on a vertical axis through a right angle in a right-handed direction. The spot of light on the galvanometer scale will swing, say, to the right of the observer, will oscillate a few times to and fro, and will finally return to its position of rest in the earth's field. During the motion of the coil a current of short duration, as evidenced by the deflection of the magnet, has passed through the coil of the instrument. Still in a right-handed direction, turn the coil through a second right angle, and again the spot will swing to the right, oscillate a few times, and come to rest in its original position. Again in a right-handed direction, turn the coil through a third right angle; the spot of light will now swing to the left, oscillate, and come to rest. Turning the coil through a fourth right angle in the same direction (one complete revolution), the spot will again swing to the left, oscillate, and come to rest.

Commencing, then, with the plane of the coil at right angles to the direction of the lines of force of the earth's field, for half a revolution the currents produced, as indicated by the swings of the galvanometer needle, are in the same direction for each right angle through which the coil is rotated; for the other half revolution the currents produced are also in the same direction for each right angle of rotation, but opposite in direction to those produced for the first half revolution of the coil.

How, then, is the production of these currents explained? Evidently the presence of the coil in the earth's field of force does not account for their generation, for so long as the coil is not rotated no deflection of the galvanometer needle is noted; it is only during the motion of the coil relative to the lines of force that a transient current is produced in the rotating coil. It will therefore be necessary to examine the field of force in which the coil is rotated more closely.

The field of force of the earth is too weak to allow of its being plotted by iron filings, but it can be plotted by means of a suspended magnet, as explained, for a bar magnet (page 11, fig. 4). Proceeding to plot the field in this manner, a series of parallel lines would be traced out in any longitudinal plane, and if the plotting magnet is started from a number of equidistant points in any plane, a number of equidistant parallel lines would be traced out—*i.e.*, the field is everywhere of the same strength. A plan of such a field is represented in fig. 9, in which also the coil of wire is shown in full lines, with its plane perpendicular to the direction of the lines of force.

As depicted, the number of lines of force—or, as it is called, the **flux of force**—passing through or threading the coil is a maximum. When, however, the coil is rotated through a right angle, as shown in dotted lines in fig. 9, the number of lines of force threading the coil is evidently reduced, for whereas in the first position five lines are shown as threading it, after rotation one line only does so; the flux of force threading it is now a minimum. For the first right angle, then, of rotation, the coil moves from a position relative to the field of force in which the flux is a maximum to that in which the flux is a minimum. Continuing the rotation through the second right angle, the flux is increased

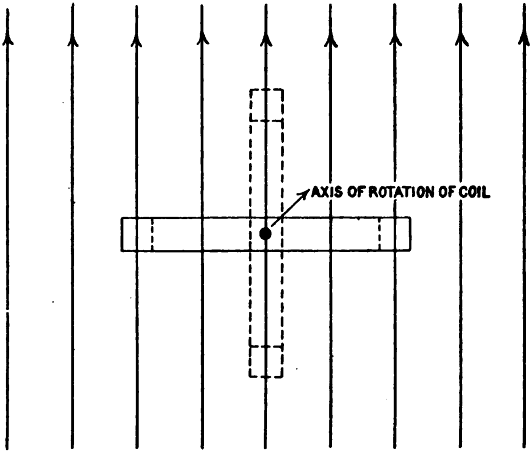


FIG. 9.—Coil of Wire Rotated in Magnetic Field of Force.

from a minimum to a maximum; but now the lines thread the coil in the opposite direction. For the third right angle the flux again decreases from a maximum to a minimum, and for the fourth right angle the flux increases from a minimum to a maximum again. It is this increase and decrease in the flux of force which threads the coil that generates the transient currents shown by the galvanometer, and such currents are said to be produced by induction.

Induction by a Moving Magnet.—With the ends of the coil still attached to the terminals of the galvanometer, thrust into the coil the N. polar region of a bar magnet; the galvanometer needle is violently deflected, say to the right, swings to and fro, and finally returns to its position of rest. Withdrawing the magnet, a violent deflection in the opposite direction, to the left, is produced.

So long, however, as the magnet remains stationary, even when placed in the interior of the coil, its presence does not affect the position of the needle of the galvanometer; it is only during the relative motion of the magnet with reference to the coil that transient currents are produced.

An examination of the field of a bar magnet plotted in fig. 1, page 9, will suffice to explain the generation of current in this case. With the polar region of the magnet thrust into the coil, the flux of force from the magnet threading the coil is a maximum; with the magnet withdrawn from the coil, the flux is a minimum. Again, then, a variation from maximum to minimum takes place when the magnet is withdrawn, and from minimum to maximum when the magnet is introduced.

Turning the magnet, end for end, and thrusting the S. polar region into the coil, the deflection produced is to the left, in the same direction as on the withdrawal of the N. polar region; on withdrawing the S. polar region, the deflection is to the right, as on the introduction of the N. polar region into the coil.

Induction by the Approach or Recession of a Coil conveying a Current.—Next lay the coil attached to the galvanometer terminals on its face, and place a similar coil of cotton-covered copper wire on the top of it. To the ends of the second coil attach a source of electrical energy, say a voltaic battery, or a few secondary cells. The coil attached to the galvanometer is usually spoken of as the secondary, and the coil attached to the battery the primary coil.

Now lift up the primary coil conveying the current; the galvanometer needle is deflected by a transient current, say to the right, and after a few swings returns to its position of rest. On bringing the primary coil again near the secondary coil, a current is produced deflecting for an instant the needle to the left. Again, when the two coils are together, the flux of force threading the secondary coil is a maximum; when the primary coil is lifted up, the flux threading the secondary is a minimum; and it is only during the relative motion of the two coils to one another that the transient currents are produced.

With this arrangement a further experiment can be carried out. Introduce in the circuit of the primary coil a switch, so that the current can be made or broken at pleasure.

Placing the two coils together, with the switch open, the flux threading the secondary is a minimum. On closing the switch, the flux becomes a maximum, and this change in the flux from minimum to maximum is indicated by a movement of the galvanometer needle to the left, in the same direction as on bringing the primary coil when conveying the current near the secondary coil; on breaking the circuit of the primary coil—*i.e.*, making the flux threading the secondary coil a minimum—a deflection to the right is produced, in the same direction as on moving the primary coil when conveying the current away from the secondary coil.

In this case no relative motion of the coils takes place, but what does occur is that the flux of force threading the secondary coil varies, and it is this increase or decrease in the flux of magnetic force which generates a current in a closed circuit.

It is not sufficient that the coil should move in a field of force; for instance, if the coil in fig. 9 is moved parallel to the direction of the field, as indicated by the arrow heads, no current will be produced, because the flux threading the coil remains the same; when, however, the coil is rotated, the flux varies; and, as proved by Faraday, it is this rate of change in the flux, which generates a difference of electrical pressure between any two points in a conductor, proportional to the rate of change of the flux, and hence in a closed circuit produces a current in the conductor.

Direction of Induced Current.—So far no attention has been given to the magnetic effects of the currents induced in the coil beyond the fact that, as shown by the movement of the galvanometer needle, the currents generated alternate in direction.

It will now therefore be as well to determine the direction of the induced currents in the coil. Take, as a simple case, that of the coil and bar magnet. When the N. polar region of the magnet was plunged into the coil, the galvanometer needle was deflected to the right. Introduce now into the circuit of the coil a battery, so that a steady current flows through the coil in such a direction that the galvanometer needle is permanently deflected to the right, and examine the field produced by the coil by means of a small compass needle. The face of the coil into which the N. polar region of the magnet is thrust will be found to be N. polar. The approach of the N. polar region of the magnet is accompanied by the creation on the face of the coil, into which the magnet is introduced, of a region similar in sign to that of the magnet pole introduced. The deflection on withdrawing the magnet is to the left, and the battery would now have to be connected in the opposite direction to deflect the galvanometer needle permanently to the left. Again examining the field produced, the region on the face of the coil, from which the magnet is withdrawn, will now be found to be S. polar, opposite in sign to the pole of the receding magnet. With the magnet reversed, the face of the coil, on approach of the

S. polar region, will be found to be S. polar, and on withdrawal N. polar.

Lenz's Law.—These results are expressed in a law formulated by Lenz, which may be stated as follows: **The induced current in any conductor is always in such a direction that the magnetic force produced by the current opposes the motion which induces the current.**

Many rules have been given for determining the direction of the induced electrical pressure (and consequently, in a closed conductor the direction of the current as well) in a conductor when it is moved in a magnetic field, so as to vary the flux threading the conductor. One proposed by Dr. Fleming is very simple. Hold the first and middle fingers and thumb of the right hand in a position as nearly as possible at right angles to one another in three directions; let the forefinger point in the direction of the lines of force; let the thumb point in the direction in which the conductor is moving, at right angles to the direction of the lines of force; then the middle finger represents the direction of the induced electrical pressure. If the conductor moves in a circular path, the thumb must point in a direction tangential to this path. By Lenz's law, the motion of a conductor in a magnetic field, due to a current passing through it, is opposite in direction to that required to induce a current in the conductor when it is mechanically put in motion. Hence, in applying this rule to motors to determine the direction of rotation of conductors conveying currents in a magnetic field, the fingers and thumb of the left hand must be used, whereas for dynamos the right hand must be employed.

The Dynamo Machine.—Having now explained the production of electrical pressure by induction, the method of applying this principle in the dynamo to the generation of electrical energy can be described. Take as an example the two-pole dynamo, shown in fig. 10.

It consists of two essential parts, one of which is capable of motion relative to the other. The fixed portion in this machine includes the bed-plate or frame *a a*, provided with the necessary bearings *b b* for the rotating part. This frame carries the two vertical iron limbs of a large magnet, called the field magnet, round which are wound the exciting coils E E (covered by a protecting plate), consisting of a large number of turns of copper wire. The polar regions P P of the magnet are extended, and bored out concentrically to the bearings, so that the revolving part A A can be introduced and rotate freely between the pole pieces. The rotating part A A, known as the armature, consists of a shaft with a pulley, and a cylindrical iron core keyed to the middle portion of the shaft. On this core a number of longitudinal turns of copper wire is wound in sections, and the ends of these sections are connected, so as to form a continuous winding, to a number of insulated copper bars

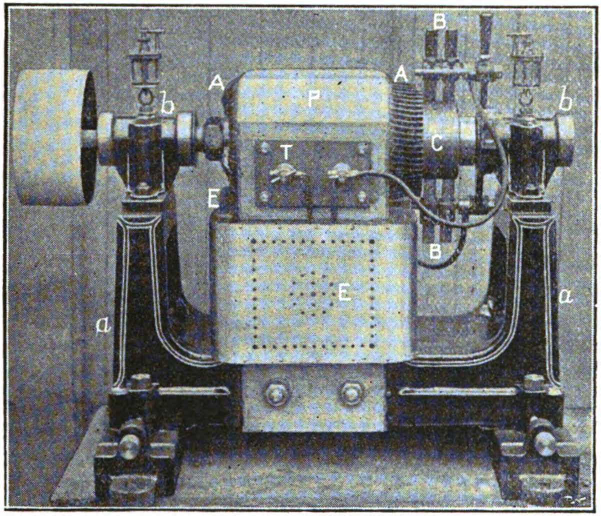


Fig. 10.—Two-pole Continuous-current Dynamo.

built up into a cylinder C, called the commutator, on the surface of which two sets of stationary metallic brushes B B press, diametrically opposite to each other, and make contact with the copper bars as they revolve. From these brushes wires lead to the two terminals T of the machine (the second terminal is on the opposite side of the machine). The effect of the presence of the iron in the magnet limbs and armature core will be discussed later, but the working of the machine may be described as follows: A current is sent round the exciting coils E E on the limbs of the magnet, and creates a strong magnetic field of force between the pole pieces P P. When the armature is set in rotation the longitudinal conductors are caused to move for a portion of a revolution practically parallel to, and for the remainder practically at right angles to, the direction of the lines of force, so that the flux of force embraced by any of the conductors varies from a maximum in one direction to a minimum; then to a maximum in the opposite direction, and again to a minimum in this direction; then to a maximum again in the first direction, as at the commencement of the cycle of revolution.

This variation in the flux generates a difference of electrical pressure at the terminals of the machine, and if the external circuit between these terminals is closed by a conductor, a current flows round the circuit in virtue of this difference of pressure. Electrical energy is now expended in the closed circuit, and mechanical energy must be applied to the armature pulley to keep the armature in rotation and maintain the difference of electrical pressure, in virtue of which the current flows.

The dynamo thus directly transforms mechanical into electrical energy.

CHAPTER III.

Lines of Induction.—In describing the fields of force produced by permanent magnets, the paths of the lines of force considered have been entirely traced in the space surrounding the magnet. They have been regarded as emerging from a N polar region, curling round and re-entering a S polar region. The fields of force of an electric current flowing in various paths have also been depicted, and attention was drawn to the fact that the complete paths of the lines in air in the form of closed curves could be traced, whereas for the magnet they could only be traced from their point of emergence from the magnet to their point of re-entry.

As the magnetic force produced by a current has been shown to be similar in its effects to that produced by a permanent magnet, it is reasonable to conjecture that the curves in the case of a magnet are closed also; and if this is so, the only available path seems to be that offered by the substance of the magnet itself.

The method of exploring the field in the space surrounding a magnet cannot, however, be applied to an examination of the substance of the magnet, and thus it is not possible in this manner to decide whether the lines are continued through the substance of the magnet, but experiment provides strong evidence that such is the case.

If a narrow transverse slot is cut through the magnet, and a "free" unit pole is introduced into the slot, a certain force will be found to act on the "free" unit pole, and this force will have two components, one due to the magnet itself and the other due to the free magnetism which exists on the faces of the slot. The actual force exerted on the "free" unit pole in the slot is called the **magnetic induction** in that space.

At any position in the space surrounding a magnet, the magnetic force has the same meaning and the same direction as the magnetic induction, and with the units of measurement employed, the magnetic force is numerically equal to the magnetic induction, and may be regarded as the **magnetic induction through the air**; but in the substance of the magnet, the magnetic force is compounded of that due to the magnet itself as well as that due to the free magnetism developed at the polar regions. This second component acts in a direction opposite to that due to the magnet, and tends to demagnetise it. In the case of a long straight bar magnet, the magnetic force due to the polar regions acts from the N polar region to the S polar region, both in the space surrounding the magnet and in the magnet itself; whereas the magnetic induction is from the N polar region to the S polar region in the space surrounding the magnet, and from the S polar region to the N polar region in the substance of the magnet. If the long bar magnet is formed into a circular ring, an endless ring without a joint, such a ring will possess no polar regions, although strongly magnetised, and the magnetic force within the substance of the ring will then be that due to the magnet alone.

The lines of force in the space surrounding a magnet may thus be regarded as the continuation of the lines of induction in the substance of the magnet, and the complete path of any line from its point of emergence to its point of re-entry, and thence through the substance of the magnet, is a closed curve, and is called a **line of induction.**

At any point where these lines of induction emerge from a magnet into the air a N polar region is developed, and wherever they re-enter a magnet from air a S polar region appears. It will therefore not be necessary to distinguish further between lines of induction in the substance of a magnet and lines of force in air, which, as pointed out, may be regarded as lines of induction through the air. In any dynamo machine the difference of electrical pressure produced is due to the rate at which these lines cut or thread the loops of a conductor, and in future the production of a difference of electrical pressure will be referred to as due to

the cutting or threading of lines of induction. What has been previously said as to the measurement of the strength of the field by means of the number of lines of force threading unit area, can also be extended to mean the density of the lines of induction per square centimetre in air.

Magnetic Force Produced by an Electric Current.—The direction only of the magnetic force produced by electric currents flowing in various paths has hitherto been discussed, without any reference to the magnitude of the force. To completely specify any given force it is requisite that its direction and magnitude should be known, and it will therefore now be as well to deal with the magnitude of the force created by a current in any given case.

In all practical applications of electric circuits the conductors are closed and a current flows in them. Around such a circuit a decrease of electrical pressure occurs, and in virtue of the rate of decrease of electrical pressure, *i.e.*, the difference of pressure from point to point, a current flows in the circuit. To maintain the flow of current, electrical energy must be supplied from a source outside the circuit, this energy being continually converted into heat in the circuit. Such an electric circuit, with a current flowing in it, has created around it a field of magnetic force, and any path chosen around this field of force, interlinked with the electric circuit, may be regarded as a magnetic circuit. The conditions in a magnetic circuit are closely analagous to those of an electric circuit. Magnetic pressure is regarded as existing around such a circuit, and the rate of decrease of magnetic pressure is called the **magnetic force**.

No conversion of energy into heat takes place, however, in a magnetic circuit, and the two circuits, electric and magnetic, are further dissimilar, in this respect, that while it is possible to define exactly the path of an electric circuit, the wire forming the circuit can be perfectly insulated, this cannot be accomplished in the case of a magnetic circuit; no magnetic insulator is known. A magnetic circuit may be compared to an electric circuit composed of two conductors, one possessing very great conducting power, surrounded by the second with very low conducting power; in fact, a very leaky electric circuit with no well defined path for the current.

To return now to the measurement of the magnitude of the magnetic force. Unit pole was defined (page 14) as a magnetic pole of such strength that, if placed in air at unit distance from another pole of equal strength, unit force will act between them; and unit field—*i.e.*, a field having one line per square centimetre—represents a field acting on a unit pole with a force of one dyne (unit force).

The unit pole is supposed to be of very small dimensions, merely a mathematical point, which, of course, cannot be attained with a material pole. It can, however, for this purpose be assumed to be a sphere of very small radius. If, then, such a spherical pole is placed at the centre of a sphere having a radius of 1 centimetre, every portion of the surface of this sphere will be at unit distance (1 centimetre) from the centre of the unit pole, and a unit pole placed anywhere on this surface will be acted on by unit force (one dyne).

The surface of a sphere, however, is $4 \pi r^2$ where $\pi = 3{\cdot}1416$ and $r =$ the radius of the sphere, and for a sphere with a radius of 1 centimetre, the surface is therefore 4π square centimetres. The lines of induction are supposed to emerge uniformly from the unit pole radially over its surface, and by the definition of unit field—*i.e.*, a field having 1 line per square centimetre, the number of lines which will thread the surface of the sphere of 1 centimetre radius is 1 line per square centimetre of surface, or a total for the entire surface of the sphere of 4π lines, and therefore 4π lines must emerge from the unit pole. If the unit pole is placed at the centre of a sphere having a radius of 2 centimetres, the surface of this sphere is $4 \pi r^2 = 16 \pi$ square centimetres, and as only 4π lines emerge from the unit pole, 1 line only will cut this sphere for each 4 square centimetres of surface.

The density will now be only one quarter of a line per square centimetre, and therefore a unit pole placed at a distance of 2 centimetres from a similar pole, will only be acted on with a force of one quarter of a dyne, or, as taught by experiment, the force of attraction or repulsion between two poles varies inversely as the square of the distance between the poles.

The C.G.S. unit of current is defined as the current, which, flowing in a small wire bent into a circle of unit radius (1 centimetre), acts on a unit pole placed at its centre with unit force (1 dyne) per unit of length of its circumference; or conversely, action and reaction being equal and opposite, each centimetre in length of the wire is acted on with a force of 1 dyne.

The difference of pressure produced in a circuit is proportional to the number of lines cutting the circuit per second, and C.G.S. unit difference of pressure is generated when a conductor 1 centimetre long is moved at a velocity of 1 centimetre per second across a field having 1 C.G.S. line per square centimetre of area.

The C.G.S. unit of energy or work done is defined as a force of 1 dyne acting through a distance of 1 centimetre, and a conductor therefore 1 centimetre long and conveying C.G.S. unit current moved across a field with 1 C.G.S. line per square centimetre of area for a distance of 1 centimetre has unit work done on it.

Now suppose a loop of wire is taken, and a difference of electrical pressure is applied to the terminals of the loop, so that C.G.S. unit current passes around the loop. This loop conveying a current will act on a magnetic pole, and the pole will tend to move in a closed path interlinked with the loop—*i.e.*, from whatever position the pole is allowed to move it will pass through the loop and return to the position again from which it started. By the convention previously explained, if the magnetic pole employed is a unit pole 4π lines of induction will emerge from it, and therefore 4π lines will have threaded the loop when the pole has made one complete circuit of its closed path interlinked with the loop.

If the pole is compelled to move uniformly, so that it exactly makes one complete circuit of its path interlinked with the loop in one second, 4 π lines will thread the loop per second, and will therefore generate a difference of electrical pressure equal to 4 π C.G.S. units in the loop during one second. The direction of this difference of pressure will be such as to oppose that applied to the terminals of the loop. If, however, the current in the loop is maintained constant in value at 1 C.G.S. unit by increasing the difference of pressure at the terminals of the loop, more energy will be supplied to the loop, and the additional work done by this addition of pressure to the circuit will be 4 π units. If π lines cut the loop per second, the difference of pressure generated will be equal to π, but now the pole will take four seconds to complete its circuit, and the difference of pressure will be maintained four times as long, and therefore the work done, the current being maintained constant, is again 4 π units; or, with the current maintained constant, the work done is proportional to the total number of lines threading the loop without any reference to the rate of the threading. A unit magnetic pole then, in moving along any path interlinked with a loop conveying a current, has work done on it equal to 4 π times the current.

If the loop consists of two turns, the work done for C.G.S. unit current in each loop is 4 π units for each loop, and so on for any number of loops; or, stated generally, the work done on a unit magnetic pole, in moving along a closed path interlinked with a number of loops conveying a current, is equal to 4 π times the current multiplied by the number of loops, the current of course being measured in C.G.S. units.

In practice, however, the current is not measured in C.G.S. units, but in practical units, and, as pointed out on page 17, the practical unit of current, the ampere, is $\frac{1}{10}$ of a C.G.S. unit. When therefore, the current is measured in amperes, the work done on a unit magnetic pole moving in a closed path, interlinked with a number of loops conveying a current, is $\frac{4}{10}\pi$ times the current in amperes, multiplied by the number of loops; or, if W = the work done on a unit magnetic pole in moving once around a closed path, interlinked with a coil of wire of t turns, with C amperes flowing through the coil, then—

$$W = 0{\cdot}4\,\pi\,C\,t.$$

The product $C\,t$ (ampere-turns) is called the exciting power of the coil. It is quite immaterial how this product is attained, whether by a large current and a small number of turns, or by a small current and a large number of turns; for instance, 5,000 turns and 1 ampere would give the same exciting power as 100 amperes and 50 turns, 5,000 ampere-turns in each case.

The work done on a unit magnetic pole can also be obtained in another manner. The rate of decrease of magnetic pressure around any magnetic circuit, usually called the **magnetic force**, and symbolised by H, is the force in dynes which acts on a unit magnetic pole in a magnetic field; and the number expressing this force in dynes, multiplied by the length of the magnetic circuit in centimetres, also gives the work done in dyne-centimetres in carrying a unit magnetic pole once round a magnetic circuit, whose length is expressed in centimetres; or, if

H = the magnetic force in dynes,
l = the length of the magnetic circuit in centimetres,
W = the work done,

then

$$W = \mathsf{H}\,l.$$

Two equations have now been obtained for the work done in carrying a unit magnetic pole once round a magnetic circuit interlinked with an electric circuit—

$$(a)\quad W = 0{\cdot}4\,\pi\,C\,t.$$
$$(b)\quad W = \mathsf{H}\,l.$$

The first (*a*) gives the work in terms of the exciting power of the current (ampere-turns); the second (*b*) in terms of the magnetic force, and the length of the magnetic circuit.

It is a great convenience in practice, however, to express or indirectly measure the magnetic force H directly in ampere-turns per centimetre of length of the magnetic circuit, and thus, by adding up the turns per unit length, obtain the total excitation required in ampere-turns.

These two equations may therefore be combined, and written—

$$\mathsf{H}\,l = 0{\cdot}4\,\pi\,C\,t,$$

or

$$\mathsf{H} = \frac{0{\cdot}4\,\pi\,C\,t}{l} = 1{\cdot}257\,\frac{C\,t}{l} \quad . \ . \ . \ . \quad (1)$$

which gives at once the ampere-turns for any magnetic circuit of length l centimetres to produce a given magnetic force H at a point in that circuit.

Magnetic Force within a Solenoid.—By means of this equation the magnetic force H within a solenoid, such as is illustrated in fig. 8, page 25, may be calculated when a current flows through the turns or loops of wire forming the solenoid.

Let the solenoid consist of t turns with C amperes flowing in them, and let its length from end to end be l centimetres; then—

$$\mathsf{H} = 1{\cdot}257\,\frac{C\,t}{l};$$

or the magnetic force H within the solenoid is equal to 1·257 times the current in amperes multiplied by the number of turns per centimetre of length, and from this it is evident that the magnetic force H, for any given current, is independent of the size of the loop or the substance of which the wire is made.

In the case of a solenoid with current flowing through it, polar regions are developed at its ends, and the magnetic force due to these polar regions is opposite in direction to that produced by the current. Hence the magnetic force H is less, by the force due to the polar regions, than that calculated, for any positions near the ends of the solenoid. For a position exactly at the centre of the solenoid, the value of H is practically equal to that calculated, and the longer the solenoid is made the greater will be the length over which H is practically equal to the calculated value.

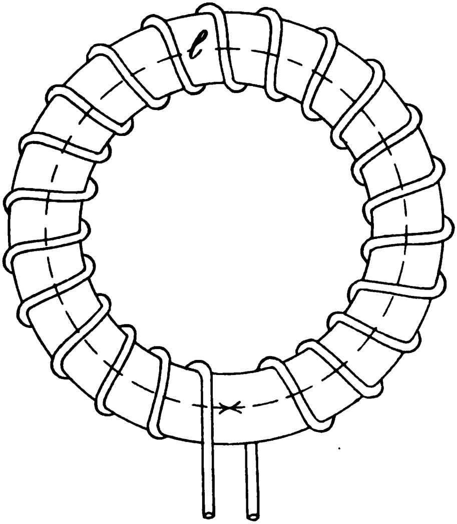


FIG. 11.—Non-magnetic Ring with Magnetising Coil wound round it.

The polar regions can, however, be entirely eliminated by substituting for the straight solenoid a solenoid bent round into a circle, as in fig. 11, made, say, by winding a number of turns of wire round a non-magnetic ring, and sending a current through the turns of wire. The lines of induction now form closed circles within the wire coiled on the ring, and a strong field will exist within the turns of wire. The length of the solenoid is, in this case, the circumference at the mean diameter of the ring, and therefore, assuming no external magnetic forces in the vicinity of the ring :—

$$H = 1{\cdot}257 \frac{Ct.}{l}$$

When H = the magnetic force in dynes,
t = the total number of turns of wire,
C = the current in amperes flowing through t turns,
l = the mean circumference of the ring in centimetres.

Solenoid with an Iron Core.—So far the magnetic circuits described have been regarded as only enclosing air or non-magnetic materials. If a bar of soft iron is inserted into the solenoid, fig. 8, page 25, and a current passed through the turns of wire, the iron is situated in a magnetic field and will be magnetised by induction.

Examining the bar by means of a test needle, polar regions will be found to be developed at the ends of the bar, N at one end and S at the other. The attractive power of the solenoid will, however, be found to be very much increased, and the magnetism developed will almost entirely vanish when the current through the coils of the solenoid is interrupted, *i.e.*, the magnetism developed is interlinked with the exciting current flowing round the coils of wire; it can be made into a magnet at pleasure by passing a current through the coils of wire. Such a solenoid, with an iron core passing through it, is called an **electro-magnet.**

The field in the space surrounding such an excited electro-magnet is practically the same as that produced by a solenoid without an iron core, but for the same current through the coils the field will be found to be much stronger.

Around such a magnetic circuit, lines of induction are regarded as existing, issuing from the N polar region of the electro-magnet, curling round in the air, re-entering the

S polar region, and continuing their path through the iron core in closed curves.

If a closed ring of iron, similar to the non-magnetic ring, fig. 11, page 43, with a number of turns of insulated wire wound on it, is substituted for the straight bar and solenoid, the lines of induction will form closed curves within the substance of the ring; but again, with the same number of turns and for the same current passed through them, the total number of lines of induction will be found to be largely increased as compared to a ring of non-magnetic material.

Permeability.—This increase in the number of lines of induction passing through an iron core is spoken of as due to the greater **permeability** of iron to these lines.

All known substances are permeable to lines of induction, and may practically be divided into two groups, widely differing in permeability; non-magnetic substances and magnetic substances. As examples of the first and larger group, the metal bismuth is slightly less permeable than air; some of the salts of iron are slightly more permeable; brass, copper, and all non-magnetic metals are practically equal to air; but the difference in permeability among the various substances included in this group amounts only to a very small percentage. The second group consists practically of only three metals—iron, cobalt, and nickel. These metals, unless subjected to very great magnetic forces, have permeabilities very much greater than air. Especially is this so in the case of very soft wrought iron, the permeability of which, for a good sample, with an induction of from 4,000 to 5,000 lines per square centimetre, may amount to over 3,000, and for an induction of about 20,000 lines equals about 100.

The function of the field magnet of a dynamo machine is to produce a strong magnetic field in which the conductors attached to the armature are rotated, and owing to this enormous permeability this part of a dynamo is constructed with iron cores, excited by a current flowing round coils of copper wire of many turns surrounding these cores—are, in fact, very powerful electro-magnets.

The permeability of a substance is thus a numerical coefficient by which the magnetic force H has to be multiplied to give the density of induction through a substance forming a magnetic circuit; or, if

H = the magnetic force in dynes
B = the density of magnetic induction in C.G.S. lines per square centimetre (usually referred to simply as the induction density).
μ = the permeability,

then $$\mu H = B \quad \ldots\ldots\ldots \quad (2)$$

and $$\mu = \frac{B}{H},$$

i.e., the ratio of the induction B to the magnetic force H equals the permeability.

The permeability of air is used as the standard, and its value taken as 1, and therefore the density of magnetic induction per square centimetre through a magnetic circuit of air is numerically equal to the magnetic force, or

$$B = 1 \times H = H.$$

The total induction Z, through a cross-section of any magnetic circuit whose area is a square centimetres, is the induction density B per unit area multiplied by the area, or

$$Z = Ba \quad \ldots\ldots\ldots \quad (3)$$

When iron is introduced into a magnetic circuit the relation between the induction B and the magnetic force H cannot be so simply expressed; for this reason, that the permeability μ of iron varies with B, it is not a constant quantity but a very variable one.

The method adopted in practice, in order to deal accurately with this variation of μ for different values of B, will therefore next be described in discussing the magnetisation of iron.

CHAPTER IV.

Magnetisation of Iron.—For a magnetic circuit consisting only of air or non-magnetic materials, it has been shown that the induction density B is numerically equal to the magnetic force H; μ, the permeability for such materials, being a constant quantity, and its value for air being taken as 1. It is also evident from equation (1), page 42, that when the current flowing around loops of wire, enclosing only air or non-magnetic material, is varied, the magnetic force H, produced by a given number of turns, varies also, and is directly proportional to the current flowing in the turns. As the induction density B is numerically equal to H for such a magnetic circuit, B also varies for a given number of turns directly as the current flowing in the turns.

If, now, a ring of iron, wound round with turns of insulated wire, is substituted for the non-magnetic ring, such a simple relation between B and H no longer holds; and the methods adopted in practice for determining the values of B for magnetic substances corresponding to different values of H will now be briefly referred to.

Two methods are employed in practice for experimentally determining the induction density B produced, in a magnetic material, by the application of magnetic force.

In one method, called the **magnetometric method**, the magnetism developed in the specimen is measured by the effect produced on a magnetic needle, called a magnetometer, suspended near it. This method is only applicable to specimens equivalent to very long rods—*i.e.*, whose length is great as compared to their diameter, owing to the effect of the polar regions developed near the end of the rod; and, according to Ewing, to render this effect negligible, the ratio, length : diameter must be about 400 : 1 to secure accurate results.

This method, therefore, can only be applied to wires of comparatively small diameter. For a rod ½ in. diameter, the length required would be over 17 ft., which would be unwieldly, and would not enable specimens, say from field magnets forgings, to be tested without further forging, and consequent alteration of their magnetic character.

The second method is, however, much more frequently employed in practice, as by its aid not only can the induction produced in specimens equivalent to long rods be determined, but also that developed in the case of a closed ring as well. A short description of the principle of this method will therefore be given.

Take the case of a closed iron ring. Around the ring an insulated wire is wound over its whole circumference, as in the case of the non-magnetic ring, fig. 11, page 43, so that by passing a current through the turns of wire the ring will be subjected to magnetic force and magnetised by induction. A second layer of insulated wire is wound sometimes over and sometimes beneath the first layer, but perfectly insulated from it, and frequently this layer does not extend over the whole circumference of the ring, but only over a short portion of it, although it also can extend over the whole length of the circumference without detriment.

The first layer of wire is usually spoken of as the primary coil, and the second layer as the secondary coil.

When a current is made to flow in the primary coil, a transient current is produced (supposing its circuit closed) in the secondary coil, by induction; further, any variation in the current flowing in the primary coil is accompanied by a transient current in the secondary coil, and the whole quantity of this transient current (the time integral of the current) in the secondary coil is proportional to the change of induction in the iron ring produced by the variation of the current in the primary coil.

If, now, the whole quantity of the transient currents produced can be measured, the changes of induction accompanying a definite change in the current flowing in the primary coil can be determined; thus commencing with a current in the primary coil and suddenly increasing or reducing the strength of this current by definite steps, the change of induction in the iron ring can be determined for each step. Instead of increasing or reducing the current by successive steps in the primary coil to obtain successive values of the induction in the iron ring, the change of induction may be produced by suddenly making or breaking a current in the primary coil, or by quickly reversing the current flowing in the primary coil.

To measure the transient current, a galvanometer is employed with a magnet having a large moment of inertia, this being attained in practice by making the suspended magnet of the galvanometer much heavier than for an ordinary galvanometer, or by attaching a weight to the magnet of an ordinary reflecting galvanometer, so that the whole of the transient current has passed through the coils of the instrument before the needle moves perceptibly from its position of rest. An impulse is, however, given to the needle by the transient current passed through its coils, and the needle makes a swing proportional to the transient current, and hence proportional to the change of induction threading the secondary coil. Such an instrument is called a ballistic galvanometer, and Lord Kelvin proposed the name **Ballistic Method** for this method of measuring the change of induction, by which it is now generally known.

To obtain quantitative results, it is necessary to know the value of a division of the galvanometer scale, or the swing corresponding to a known change of induction in the secondary coil.

Three methods can be employed to effect this calibration :

1. By an earth coil—*i.e.*, a coil of wire, whose mean area in square centimetres is known, and also the number of turns of wire on the coil. This coil is turned through 180 deg. in a field of magnetic force, usually the earth's field, and the change of induction threading the coil can then be calculated from the mean area of the coil, and the value of the component of the earth's magnetic field of force. This

method cannot be employed in workshops where large masses of iron are moved about, as the value of either the horizontal or vertical component of the earth's field is not a constant quantity under such conditions.

2. A known quantity of electricity may be discharged through the galvanometer from a condenser of known capacity charged to a certain difference of pressure. This method is not very often applied in practical work.

3. A long straight rod of wood or a brass tube is provided, whose length is 40 to 50 times its diameter. At the centre of the length of this rod a small groove is turned, and the mean diameter of the groove carefully measured. A large number of turns of fine insulated wire is then wound in the groove, the number of turns being counted during the winding, and the ends of the wire brought out for attachment to the terminals of the galvanometer. A long solenoid of insulated wire is then wound, extending from end to end of the wooden rod, but entirely insulated from the coil in the groove.

When a known current is passed through the turns of the solenoid, the magnetic force H at its centre can be calculated by means of equation (1), page 42, and since H for a non-magnetic material is numerically equal to B the induction density, the total induction Z threading the secondary coil, when the current in the solenoid is interrupted, is the mean area of the secondary coil in square centimetres (which can be obtained from the mean diameter of the groove), multiplied by B the induction density, or,

$$\mathsf{Z} = \mathsf{B}\, a,$$

where Z equals the total induction.

This total induction Z threads all the turns of the secondary coil, and if t_s is the number of turns of the secondary coil, the (integral) difference of pressure generated at the terminals of this coil will be

$$\mathrm{E} = \mathsf{Z}\, t_s$$

in C.G.S. units, and since 10^8 C.G.S. units equal 1 volt, E in volts will be

$$Z\, t_s\, 10^{-8}.$$

Now, by Ohm's law, the current through the secondary coil due to this difference of pressure when attached to the galvanometer terminals is

$$C = \frac{E}{R},$$

and, substituting in this equation the value of E above,

$$C = \frac{Z\, t_s\, 10^{-8}}{R},$$

where C = the current in the secondary coil in amperes;
Z = total induction threading secondary coil;
t_s = number of turns of wire on secondary coil;
and R = resistance of galvanometer circuit (galvanometer, secondary coil, &c.) in ohms.

The current C in passing through the coils of the galvanometer will give a certain swing to the galvanometer needle, the magnitude of the swing depending on the constant of the galvanometer employed.

Let δ = the throw of the galvanometer in divisions of the galvanometer scale, when a transient current C passes through the instrument; then

$$C = \frac{\delta}{g},$$

where g is the constant of the galvanometer.

Substituting this value for C in the equation above,

$$\frac{\delta}{g} = \frac{Z\, t_s\, 10^{-8}}{R}$$

and, therefore,
$$g = \frac{\delta\, R}{Z\, t_s\, 10^{-8}}.$$

To obtain the value of this constant g of the galvanometer, the long solenoid is connected in circuit with a battery, usually of secondary cells, a delicate ammeter and a switch, which interrupts the circuit very quickly, being interposed between the solenoid and the battery. The terminals of the galvanometer are connected to the ends of the secondary coil wound in the groove at the centre of the long solenoid. The switch is now closed, the current through the turns of the solenoid noted on the ammeter, and the spot of light on the galvanometer scale brought to rest. The switch is now turned so as to break the primary circuit quickly, a current is induced in the secondary coil, and the needle of the galvanometer makes a throw, the deflection of the spot of light on the galvanometer scale being noted.

From the dimensions and number of turns of solenoid and secondary coil, the current flowing in the solenoid, the resistance of the galvanometer circuit, and the observed deflection, g can be calculated.

Thus described, the process of standardisation appears complicated. In reality it is quite easy to carry out, and a numerical example will perhaps best exemplify this.

Say the diameter of the solenoid is 3 centimetres, and that the ratio of length to diameter is about 42 : 1 ; then—

Length of solenoid l = 125·7 centimetres ;

Current in amperes flowing through solenoid C = 10 amperes ;

Number of turns of wire on solenoid t = 2000 ; then, by equation (1), page 42,

$$\mathsf{H} = 1{\cdot}257\,\frac{C\,t}{l}$$

$$\mathsf{H} = \frac{1{\cdot}257 \times 10 \times 2000}{125{\cdot}7} = 200,$$

and for a non-magnetic substance $\mathsf{H} = \mathsf{B}$;

$$\therefore \mathsf{B} = 200.$$

Let the mean area a of the secondary coil be 5 square centimetres ; then

$$\mathsf{Z} = \mathsf{B} \,.\, a = 200 \times 5 = 1000.$$

Let the number of turns t_s in the secondary coil be 1,000, the resistance R of secondary coil, galvanometer, &c., be

200 ohms, and the resulting deflection δ by breaking 10 amperes in the solenoid, 20 divisions.

Now,

$$g = \frac{\delta \text{ R}}{\mathsf{Z}\ t_s\ 10^{-8}};$$

therefore

$$g = \frac{20 \times 200}{1000 \times 1000 \times 10^{-8}} = 400000,$$

which is the required constant. One observation, therefore, gives the value of this constant, and this value can be checked as often as thought necessary.

Such a solenoid and secondary coil can also be used to calibrate the scale of the galvanometer, to determine whether the deflections of the galvanometer on the scale are strictly proportional throughout its length.

For a number of different values of the strength of the current in the solenoid a number of different deflections will be obtained, on breaking the circuit of the solenoid, on the galvanometer scale. If the scale is strictly proportional, the calculated value of the constant g should be the same whatever throw the galvanometer needle makes, so long as the resistance R remains the same.

Having thus determined g, the total induction Z through any coil attached to the galvanometer terminals can be obtained from the throw of the galvanometer needle, the resistance of the secondary circuit, the mean area of the coil, and its number of turns of wire.

By transposition from the preceding equation,

$$\mathsf{Z} = \frac{\delta \text{ R}}{g\ t_s\ 10^{-8}};$$

but

$$\mathsf{Z} = \mathsf{B}\, a;$$

therefore

$$\mathsf{B}\, a = \frac{\delta \text{ R}}{g\ t_s\ 10^{-8}},$$

and

$$\mathsf{B} = \frac{\delta \text{ R}}{g\ a\ t_s\ 10^{-8}},$$

which gives the induction density B per square centimetre through the coil.

This result is, however, only true if the resistance of the new coil is equal to that of the secondary coil of the solenoid, by which the galvanometer was standardised. As the resistance of the galvanometer circuit decreases, the throw of the galvanometer, for a transient current constant in value through the secondary coil, diminishes, due to the damping effects of the currents generated in the coils of the galvanometer by the movement of its own magnetic needle. If, however, the resistance of the galvanometer circuit is kept high, this error is not likely to exceed 1 per cent, and, if greater accuracy is required, the damping can be allowed for, as explained by Ewing; also in a very practical paper by Vignoles, read before the City and Guilds of London Institute, Old Students' Association.*

Having determined the constant g, as above, the galvanometer can also be applied to measure the total induction Z produced by the application of various values of H, in rings, &c., made of other than non-magnetic materials, and from the cross-sectional area of the rings the induction density B, corresponding to any given value of H, can be ascertained.

The rings employed for this purpose are usually from 4 in. to 6 in. external diameter. The length of the ring is frequently made greater than the radial depth, so as to give a rectangular cross-section with an area of from 1 to 3 square centimetres, the proportions being as about in fig. 12.

With a sample from a field magnet forging of wrought iron, it is better to turn the ring from the solid block than to weld it up from a bar, in order that its magnetic qualities may not be altered; if from a field magnet casting, the ring should be either cast at the same time as the magnet casting, or, what is perhaps preferable, turned from a solid block poured from the same metal as the casting. After machining, the sample should be thoroughly annealed by heating to a red heat, and then allowed to cool very slowly. In the case of armature iron, punchings of the required dimensions, and from all the sheets which are to be used in making the armature core punchings, are, after careful annealing, placed side by side to form a ring.

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* Ewing's "Magnetism in Iron and other Metals." Vignoles' "Some Researches in Electro-magnetic Induction." *Electrician*, May 15 and 22, 1891.

The cross-sectional area and the mean circumference of each sample is then carefully determined; each ring is then insulated with tape, and a primary coil wound on it completely enveloping its circumference, the number of turns of primary winding being counted. The wound ring is then covered with another layer of tape, its cross-section again measured, and the secondary coil (usually of fine wire, to keep up the resistance of the galvanometer circuit) wound on, the number of turns in this coil being also noted.

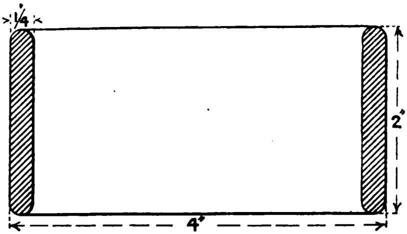

FIG. 12.—Ring of Magnetic Material for Ballistic Method.

When the secondary coil is wound, as in this example, outside the primary coil, a certain number of lines will thread the secondary coil which do not pass through the iron ring, and as the object is to determine the induction density through the iron, this in exact work should be allowed for. Thus, if

a = the cross-sectional area of the iron ring,
a^1 = the cross-sectional area outside the primary coil,

the number of lines threading the secondary coil which do not pass through the iron will be $(a^1 - a)$ H, and this number should be deducted from Z, the total induction before dividing by a the area of the iron ring, to obtain the induction density B.

The secondary coil of a ring so prepared, with a variable resistance in series with it, is then coupled to the terminals of the galvanometer. To the terminals of the primary coil a few secondary cells are attached, this circuit also including an ammeter, switch, and variable resistance, as in fig. 13, the coils, for clearness, in the figure being shown as extending over only a portion of the circumference, but the primary coil is always wound so as to envelop the whole circumference of the ring: the secondary, in some cases, does so as well; in others it only extends over a portion of the circumference.

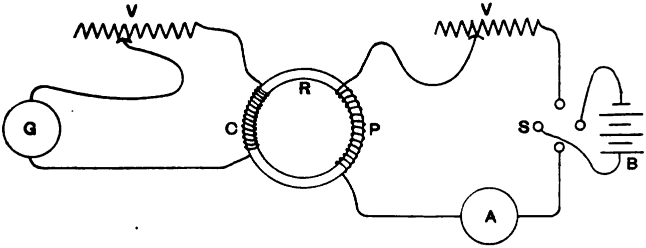

FIG. 13.—Arrangement of Apparatus for Ballistic Method.

A. Ammeter. B. Battery. C. Secondary Coil. G. Galvanometer. P. Primary Coil. R. Iron Ring. S. Switch. V. Variable resistance.

One or other of the methods of procedure previously described (page 49) is now determined on—say the method of increasing or decreasing the current in the primary coil by definite steps.

The galvanometer needle having been brought to rest, a small current is sent through the primary coil, the resulting throw on the galvanometer scale noted, and the current passing through the primary coil read on the ammeter. Then by a series of definite steps the variable resistance in the primary circuit is suddenly reduced, the resulting galvanometer throws taken, as well as the ammeter reading after each successive decrease in the resistance, until the whole of the resistance is cut out; the iron ring at this stage of the experiment being subjected to the greatest magnetising force which the given primary coil and battery can produce.

The current in the primary coil is next reduced in value by definite steps until all the resistance has been again introduced, when, finally, the switch is quickly opened, and the current in the primary coil interrupted. The throws of the galvanometer needle and the ammeter readings are taken for each step with a decreasing current, just as with an increasing current.

The battery terminals are then reversed, and a similar series of observations, first with an increasing and then with a decreasing current, as before, are taken.

Now, by means of equation (1), page 42, the magnetic force H, to which the ring has been subjected at each step, can be calculated from the number of turns of wire in the primary coil, the mean circumference of the ring, and the current; and the total induction Z, corresponding to definite values of H, can also be determined from the sum of the throws of the galvanometer needle for any given value of H, the resistance of the galvanometer circuit, &c., by the equation on page 53; and since $\mathsf{Z} = \mathsf{B}\,a$, B can be calculated for each value of H by dividing Z by a, the cross-section of the ring in square centimetres.

The results thus obtained give the necessary information for determining the value of H required to produce a definite induction density B in the given sample of iron, and could then be arranged in tabular form for facility of reference. As previously pointed out, the ratio $\frac{\mathsf{B}}{\mathsf{H}}$ varies very greatly for magnetic substances, and it is better to plot these results for B and H graphically, as the resulting curves show in a striking manner the behaviour of iron when subjected to magnetic force.

Such a curve for soft wrought iron is shown in fig. 14. The value of H is plotted along the horizontal axis, H being regarded as + (positive) when the battery terminals are coupled one way to the primary coil, and – (negative) when the battery terminals are reversed. The resulting induction density B is plotted along the vertical axis above or below the horizontal axis to correspond with the + or – direction assumed for H.

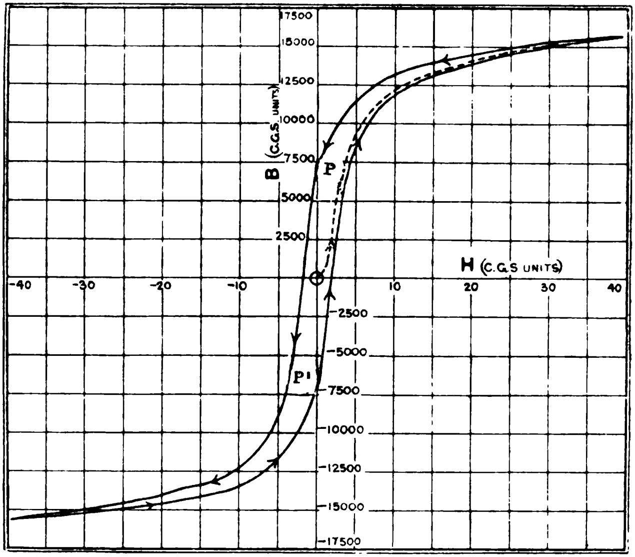

FIG. 14.—Cyclical Curve of Magnetisation for Soft Wrought Iron.

Commencing with the ring thoroughly annealed, B rises slowly at first with an increasing value of H, as shown at the commencement of the dotted line; next it ascends almost vertically, then bends over and finally continues to increase, until subjected to the full magnetic force of the primary turns, but very slowly during the latter portion as compared to the increase for the earlier portion of the curve. The current was then decreased in successive steps, and the resulting curve is shown in the full line above the dotted line, which will be seen to occupy a different position to that of the ascending curve. When the battery circuit is broken, the curve does not descend to the origin O, from which it started, but cuts the vertical axis at a point P, and O P represents the number of lines of induction still passing through the ring, although not subjected to any magnetising force from the primary coil, and is called the **residual induction.** At this stage of the experiment the battery terminals were reversed, and a certain force $-$ H had to be applied to reduce B to zero, and the force represented by the length along the horizontal axis from the point where the curve cuts it to the origin O is called the **coercive force** of the iron. Following the curve downwards, the current was again increased to its full value, and a curve similar to the ascending curve, but reversed in position, is traced out until the maximum $-$ value is reached. The current was then decreased, and the curve continues until it arrives at a point P^1, at which it again cuts the vertical axis, when the current in the primary coil is interrupted. The battery is now again reversed, and a force $+$ H is required to reduce $-$ B to zero. The application of $+$ H is then continued in steps, and the complete curve, shown in full lines, traced out. After being subjected to a few complete cycles of this character, such a curve will always be re-traced, no matter how many times the complete cycle of magnetisation is repeated. The direction of the cycle of magnetisation is represented on the curve by the arrow heads.

Such a curve is typical of the variation of B with H for wrought iron. The values, however, of B, for different specimens will be found to vary widely with the chemical composition of the iron; and H may also be raised to far higher values than represented; but whether the value of $+$ H to $-$ H for any complete cycle is large or small, a complete curve of this character is traced out when the various values of B and H are plotted. It will be found that μ, the ratio of $\frac{B}{H}$, is very large during the early portion of the cycle, and that as the curve bends over μ diminishes in value very rapidly. The iron at this stage is spoken of in practice as being **saturated**, although, so far as is known, with an increasing value of H, the curve still continues to ascend, and never becomes parallel to the horizontal axis.

The magnetic metals employed in dynamo construction are wrought-iron forgings, and castings of cast iron and mild steel (practically nearly pure ingot iron) for the field magnets; and for the armature cores, thin punchings of very soft wrought iron and mild steel. The metal in the form of thin sheets is usually slightly better magnetically than for the larger forgings, but in practice it will be found quite accurate enough to assume that the curve for thin stampings is the same as for good wrought-iron field-magnet forgings.

Cast iron varies very much in quality magnetically with its chemical composition, and only the softest qualities should be used for field-magnet castings. The curves in fig. 15 give the average curve for B and H, obtained from a number of specimens of both the ascending and descending values for very soft wrought iron, and for cast iron.

Mild steel castings are being used at the present time to a considerable degree in dynamo construction, and a curve for mild steel is added, taken from a number of curves determined by different experimenters from samples supplied by Messrs. Allen and Co., Sheffield. Although the induction density B for small values of H is less than for wrought iron, yet for high values B is frequently greater than for wrought iron.

Ampere-turns for Magnetic Substances.—It has been previously pointed out (page 42) that it is a great convenience in practice to express or indirectly measure H directly in ampere-turns per centimetre of length of the

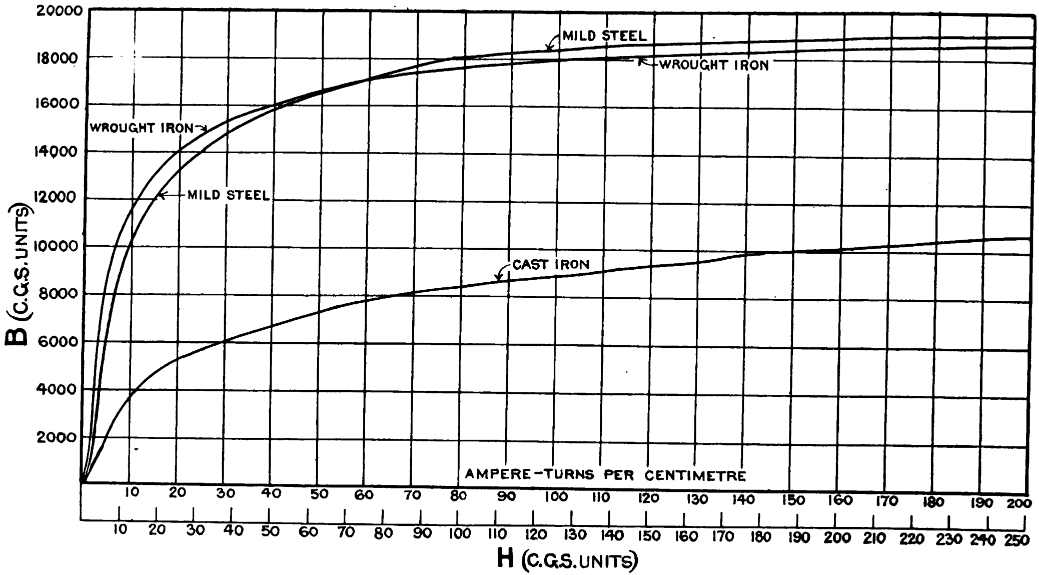

FIG. 15.—B-H Curves for Wrought Iron, Mild Steel, and Cast Iron.

magnetic circuit. By transposition from equation (1), page 42,

$$C\,t = \frac{H\,l}{1{\cdot}257}\,;$$

and, putting $l = 1$ centimetre,

$$C\,t = \frac{H}{1{\cdot}257}\,;$$

or, dividing H by 1·257, the exciting power in ampere-turns ($C\,t$) required per centimetre of length of the magnetic circuit is obtained.

The ampere-turns per centimetre of length, corresponding to the different values of H, have been added at the bottom of fig. 15, and it is thus possible to read directly from the curves the ampere-turns required per centimetre of length to produce a given induction density B for these magnetic substances.

By this method the difficulty caused by the wide variation of μ for magnetic substances can be completely overcome.

It may be as well to repeat here that the curves in fig. 15 can only be applied with success to iron, &c., of good quality, and thoroughly annealed, the permeability varying greatly with the chemical composition of the iron and its degree of hardness from mechanical strain.

Ampere-turns for Non-magnetic Substances.—In the case of non-magnetic substances it is not necessary to plot a curve for B and H, as for such substances $H = B$; the ampere-turns required per centimetre of length can be more readily obtained by calculation.

Since $$H = B,$$

and $$C\,t = \frac{H}{1{\cdot}257}\,;$$

$$\therefore\ C\,t = \frac{B}{1{\cdot}257}\,;$$

or, dividing the required induction density B by 1·257 gives the ampere-turns required per centimetre of length of the non-magnetic substance.

This may be still further simplified. As $\frac{1}{1 \cdot 257}$ = nearly 0·8, the above can be written without very great error,

$$C\,t = 0 \cdot 8\,B$$

for non-magnetic substances.

Hysteresis.—The curves in fig. 14, page 58, show the induction density B produced by the application of magnetic force to a certain ring of soft wrought iron. The iron ring was subjected first to increasing values of H, by increasing the current through the primary coil in definite steps, until the ring was subjected to the greatest value of H which the given primary coil and battery could produce. At this stage of the experiment the current through the primary coil was reduced by definite steps, and the ring subjected to a series of decreasing values of H, until the circuit of the primary coil was opened and the ring subjected to no magnetic force from the primary coil.

The curve for B, with descending values of H, does not coincide with the curve for ascending values—*i.e.*, for a given decrease and increase in the value of H the rate of decrease of B is less for the decreasing value of H than the rate of increase of B with an increasing value of H; and further, a certain residual induction was still found to be passing through the ring when no current was passing through the primary coil; the iron, once subjected to magnetic force and magnetised by induction, tending to remain in its acquired magnetic state.

It is thus not possible to precisely specify the induction density for a given magnetic force, even for a known sample of iron, as it is seen to depend on whether the magnetic force is ascending or descending in value, and on the previous processes of magnetisation to which the iron has been subjected. Fortunately in practice this does not cause any great difficulty, as the iron used for dynamo machines is subjected to very high magnetic forces, and the value of B for the ascending and descending curves does not vary very greatly for such high values of H. As regards the previous magnetic history of the iron, if a given cycle is repeated a number of times, and the measurements then made, the ascending and descending curves will still be distinct, yet the complete curves traced for each succeeding cycle will be found to coincide with one another; B will have arrived at a steady state for any portion of this cycle. The object of the curves in fig. 15, page 61, is to give the average value (mean of ascending and descending curves) for good iron, &c., in a steady state magnetically.

If, now, instead of continuing the complete cycle by reversing the battery terminals, the battery is again applied in the same direction as for the first portion of the cycle, and a current sent through the primary coil so as to again subject the ring to a series of steps of increasing values of H, the induction through the ring will not be further reduced below the residual value, but will begin to increase again. This increase will take place slowly at first, owing to the tendency of the iron to remain in its acquired magnetic state; will then proceed more rapidly during a further stage, and finally, when the full value of H is again applied, the induction through the ring will be found to have acquired its former value.

The curves thus plotted for increasing and decreasing values of H will again be found to follow different paths, and will form a closed loop of the same general character as that obtained for a complete cycle. This is also the case whether the circuit of the primary coil is broken or not; for instance, if during a portion of a cycle with increasing values of H the value of H is decreased by inserting resistance again into the circuit of the primary coil, the resulting curve for B, with decreasing values of H, will not coincide with that for increasing values. On again increasing H, a small complete loop will be formed with the descending curve, and the new ascending curve will slowly attain its original value. For any cyclical change, then, in the magnetic force H applied to the iron ring, the resulting induction density B through the ring is represented by two curves, which, however, do not coincide, owing to the reluctance of the iron to a change in its acquired magnetic state, but which form a closed loop; and this tendency of the induction through a magnetic substance to lag behind the magnetic force which produces it, has been called by Ewing the **magnetic hysteresis** of the substance.

The practical importance of these closed loops is the fact, first pointed out by Warburg, and later independently by Ewing, that the area enclosed by the B—H curve $\div 4\pi$ (B and H being expressed in C. G. S. units) represents the work done per cubic centimetre of the substance in C. G. S. units of work (ergs) in carrying a magnetic substance through a complete cycle of magnetisation. For a complete cycle, during the ascending portion of the curve, in carrying the substance from the value $-$ H to $+$ H, a certain amount of work is done; during the descending portion of the curve $+$ H to $-$ H, some of this work is returned; and the substance has returned to its initial magnetic state. The work done, however, from $-$ H to $+$ H is more than the work returned from $+$ H to $-$ H; and the difference in the energy expended and recovered during a complete cycle is the energy dissipated by the magnetic hysteresis of the substance. This dissipated energy appears as heat, and a magnetic substance which exhibits hysteresis has its temperature raised when it is subjected to cyclical variations of magnetic force. For a complete cycle $-$ H to $+$ H, the area of the enclosed loop is greater than for a small variation in the value of H, and hence the energy dissipated is also greater.

This dissipation by hysteresis occurs whether the cycles are performed slowly or quickly, and for a given induction appears to be a constant quantity, at any rate up to 400 cycles per second.

Modern views regarding these observed facts of induced magnetism, are based on what is known as the molecular theory. The original conception, due to Weber, is that the molecules of magnetic substances—iron, nickel, and cobalt—are magnets with polar regions, and the process of magnetisation consists in turning them from pointing in many directions to that in which similar polar regions all point in the same direction. If, however, the molecules were perfectly free to turn, the slightest application of magnetic force would, as pointed out by Maxwell, suffice to bring all the molecules into complete parallelism, and produce magnetic saturation; but such is not the case. Two reasons were given by Maxwell to account for this failure in the theory to fit in with observed facts: the first, adopted by Weber, that each molecule is acted on by a force, tending to preserve it in its original direction; and the second, that an equivalent effect to this is produced by the mutual action on each other of the entire system of molecules. The latter view has received remarkable confirmation from some experiments carried out by Ewing, described in a paper read before the Royal Society, 19th June, 1890, from which the following is extracted:—

"The experiments consist in the making of models to represent the supposed molecular structure of the magnetic substance. A number of small magnets, strongly magnetised, made either from short bars of round steel or from lozenge-shaped punchings from a thin steel plate, each pivoted like a compass needle upon a sharp vertical support and balanced to swing horizontally, represent the magnetic molecules. It is, of course, only possible to provide freedom of movement in one plane, whereas the axes of the actual molecular magnets have two degrees of rotational freedom; still a group of magnets swinging in one plane gives a sufficiently good general idea of the nature of the equilibrium which is brought about by inter-molecular forces, and the manner in which that equilibrium is disturbed when an external magnetic force is applied. A number of these little pivoted magnets is then arranged on a horizontal board at regular distances apart to represent the intended specimen, and two circular coils of insulated wire are provided, one at each end of the board, and with their planes at right angles to the surface of the board, so that when a current is passed through the coils the pivoted magnets are subjected to an external magnetic force. It is important to remark that the magnetic force which these coils can produce is too weak to have any material effect on the magnetism of the small magnets; it can alter their alignment only. A liquid rheostat with a sliding terminal is used to secure continuity in varying the magnetic force.

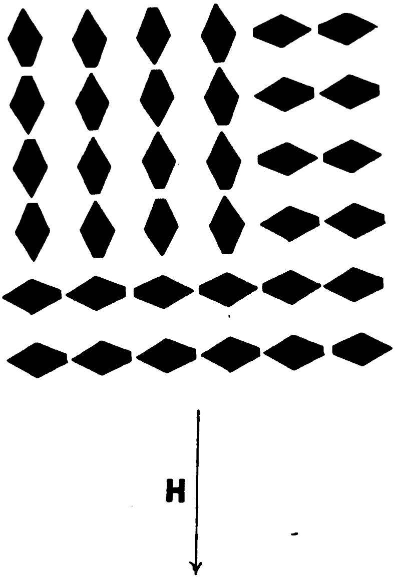

FIG. 16A.—Arrangement of small magnets to illustrate the process of magnetisation (unmagnetised).

The pivoted magnets are first disturbed, say by moving a permanent magnet near them, and after a number of oscillations will arrange themselves in certain groups, each group possessing stability for small displacements; and if again violently disturbed it is quite possible that a new arrangement of groups will result, but again each group will be stable for small displacements. Take as an illustration 6 rows of 6 magnets, each equi-distantly arranged, as in fig. 16A. A weak magnetic force H is applied in the direction marked by the arrow by sending a small current through the two coils, and all the magnets will deflect slightly from their position of rest, but none become unstable, and if H is removed all the magnets return to their former positions, this stage corresponding to the initial portion of the dotted curve in fig. 14, page 58. Next increase the value of H until some of the groups become unstable; these will now oscillate quickly about a new position,

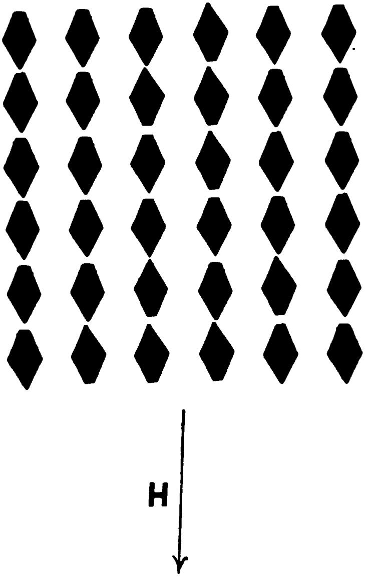

FIG. 16B.—Arrangement of small magnets to illustrate the process of magnetisation (magnetised).

and arrange themselves in a different order. Again increasing H, more groups become unstable and break up from their original positions, until finally all the magnets have passed through an unstable condition and re-arranged themselves in new order in the general direction of the force H, fig. 16B. This corresponds to the steep portion of the curve, fig. 14, when the induction is increasing rapidly Let H be now further increased, and the magnets will be still further pulled into line with H, fig. 16C, and with a nearer approach to saturation, corresponding to the flat portion of fig 14. If now H is removed, the magnets will be found to nearly all retain their new positions, and this

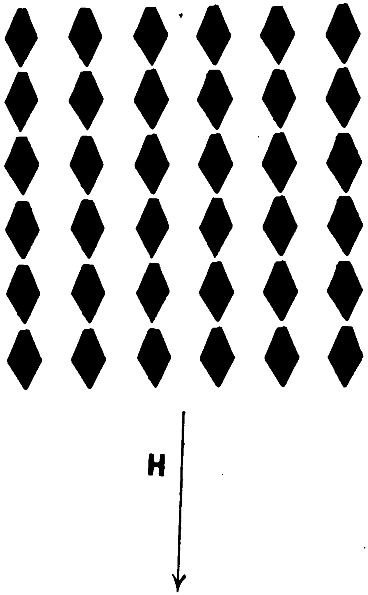

FIG. 16C.—Arrangement of small magnets to illustrate the process of magnetisation (strongly magnetised).

corresponds to the residual induction O P, fig. 14. Now, suppose H to be reversed and slowly applied in the opposite direction. At first the effect produced is slight; presently, as H is increased, and within a narrow range, instability begins, and at last all the little magnets are reversed in direction corresponding to the steep part of the descending curve of fig. 14. Further increase of H again results in a closer approximation to parallelism of the little magnets to the applied magnetic force."

Such a model, therefore, satisfactorily explains the hysteresis of iron when subjected to a cycle of magnetic force. When some of the molecules of iron become unstable, they oscillate rapidly about a new position of equilibrium, and communicate their oscillations to the neighbouring molecules, until the motion is damped out by the production of electric currents in the surrounding mass of the iron, the energy of these currents being expended in the substance of the iron; converted, in overcoming the resistance of the iron, into heat and raising its temperature.

In continuous-current dynamos and motors, the iron cores of the armatures are subjected to a cyclical change of this character, and a certain amount of energy is converted into heat by magnetic hysteresis. As the rise of temperature, which a given machine attains in work, is one of the factors limiting its output, it is necessary, in designing an armature, that the amount of energy wasted from this cause should be capable of estimation. Ewing* gives a curve connecting the energy dissipated in ergs per cubic centimetre per cycle with the maximum induction density B for complete cycles of magnetisation for soft wrought iron. This curve is reproduced in fig. 17. For any given maximum induction density the dissipation in joules per cubic centimetre per cycle is obtained by dividing the reading in ergs from the table corresponding to the maximum induction density by 10^7 (10^7 ergs = 1 joule), and this value multiplied by the

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* Ewing, "Magnetism in Iron and other Metals."

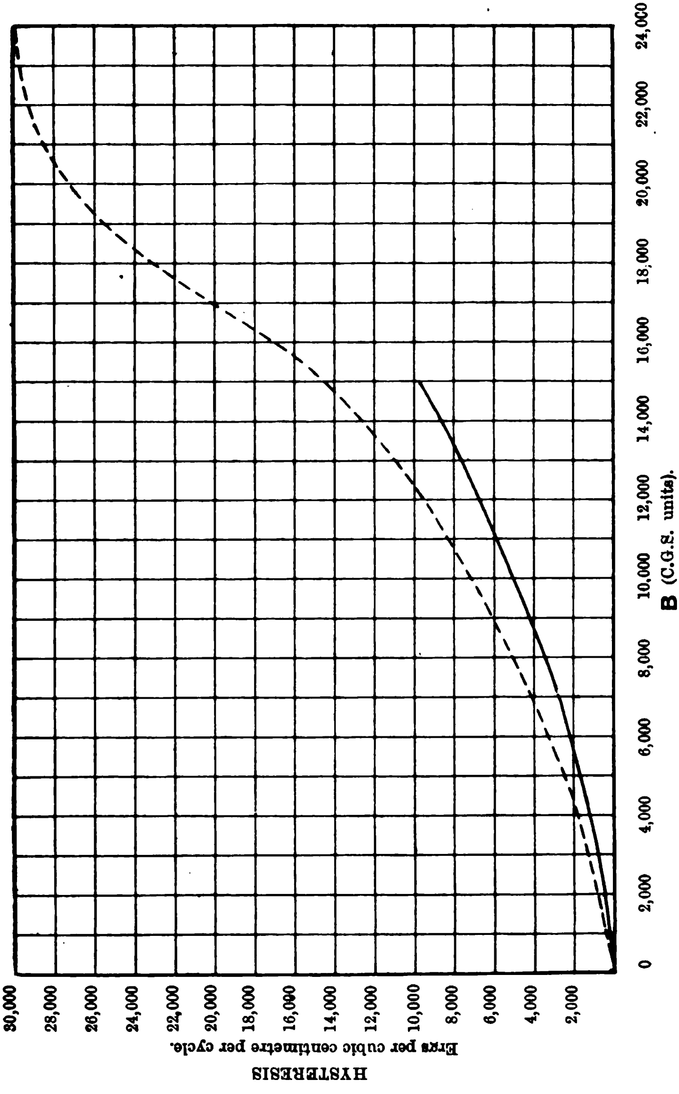

FIG. 17.—Hysteresis curves for soft wrought iron.

volume in cubic centimetres of the iron gives the hysteresis in joules per cycle. This product multiplied by the number of complete cycles per second, gives the work done per second, or the heat produced in the iron in joules per second. Now, 1 joule equals 1 watt per second, and if

x = joules dissipated per cubic centimetre per cycle
$= \frac{\text{ergs}}{10^7}$;
v = the volume of iron in cubic centimetres;
f = the number of complete cycles per second,

the power developed or rate of doing work from hysteresis loss is equal to $x\,v\,f$ watts.

The iron, however, in the armature of a continuous current dynamo is not quite in the same magnetic condition as iron which is being subjected to cycles alternating in value from $-$ H to $+$ H. In the cyclical variation from $-$ H to $+$ H the iron is stationary and the magnetic field is reversed periodically, and varies in strength from a maximum negative value, through zero, to a positive maximum, then to zero, and again to a maximum negative value, &c.; while the iron core of a continuous-current armature rotates in a strong stationary magnetic field; the iron is turned round in a field whose direction is fixed.

The model, then, representing this case would consist of the little magnets pivoted on a disc, the disc being capable of rotation. When the disc, thus equipped, is rotated with no outside magnetic field, no re-arrangement of the magnets will take place; the magnets are quite stable, and always remain in the same position relative to the rotating disc.

If, now, as the disc rotates, magnetic force with a fixed direction is applied, the magnets, when a certain force has been attained, will become unstable, their previous arrangement will be broken up, and a re-arrangement will result. When the field is made sufficiently strong the magnets will be found to always point in the same direction—*i.e.*, along the magnetic field; but now their positions, relative to the rotating disc, will vary from instant to instant. The direction of the little magnets is fixed by the direction of the stationary field, and the pivots rotate within the magnets.

The hysteresis loss under these conditions should therefore be less than for the cyclical variation. There is an entire absence, when the field is strong, of the breaking up, with violent vibration, accompanying an alternating cycle, before a new condition of stability is reached.

Swinburne, in discussing Ewing's conclusions, first pointed out this necessary corollary of a strong field constant in direction, and concluded that iron under such conditions would have no hysteresis if the field could be made sufficiently strong. Baily,* by direct experiment, has proved that this is the case. He has found that the hysteresis for iron in a strong stationary magnetic field does not increase continuously as the magnetic force is increased, but that the curve attains a maximum, and then bends over and descends again towards the axis. Baily† has also published details of experiments, in which for complete cycles, alternating in value from − H to + H for very high magnetic forces, the curve obtained shows a marked falling off in the rate of increase of hysteresis as the saturation point is approached; the hysteresis at this point becoming practically constant in value. The maximum value of B in Ewing's curve, fig. 17, is 15,000 lines, and the dotted curve to 24,000 lines has been added from Baily's results.

The probable result of determining the hysteresis in continuous-current dynamo armatures for high induction densities from this curve will be that the waste is rather over-estimated; but this error will be on the right side, and cannot differ by a very large percentage from the real loss.

Hysteresis Tester and Permeability Bridge.—The curves for hysteresis loss and induction density can, in either case, only give the values for these quantities for a given sample of iron, or the average for a large number of samples.

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* British Association, 1894. † British Association, 1895.

As previously pointed out, the induction density for a given magnetic force depends very greatly on the chemical composition of the iron. This is also the case with the hysteresis: values will be obtained differing widely even from samples from the same sheet of iron, since it is not possible to secure absolute uniformity of chemical composition in commercial iron; the impurities are irregularly distributed throughout the substance of the sheet. Obviously, then, it would lead to greater accuracy in design if samples from the actual sheets or forgings to be employed in a given machine could be tested, and attempts have been made to invent instruments to determine rapidly, and with sufficient accuracy, the hysteresis and the induction density for a given magnetic force.

The two latest instruments, of a very practical character, for enabling these measurements to be quickly performed are due to Professor Ewing, who has kindly furnished the photographs from which the accompanying blocks have been prepared.

The hysteresis tester was described in a paper read before the Institution of Electrical Engineers, 25th April, 1895,* from which the following account is extracted:—

The basis of the instrument is the mechanical measurement of the work done in causing reversals of magnetism to take place in the iron under examination. The iron to be tested is cut or stamped in the form of strips, which are 3 in. long and $\frac{5}{8}$ in. wide, and filed accurately to length in a hardened steel clamp. The number of these pieces taken to form a sample depends on the thickness of the sheet, and usually does not exceed seven.

The bundle of pieces forming the sample is placed in a carrier *a*, fig. 18, and is covered with a vulcanite washer and secured by two clamps *b*, *b*. The carrier is made to rotate by means of the friction pulley *c* and driving wheel *d*. This causes the sample to revolve between the poles of the permanent magnet *e*, with the effect that its magnetism is

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* Journal of the Proceedings of the Institution of Electrical Engineers. Part 118, vol. xxiv.

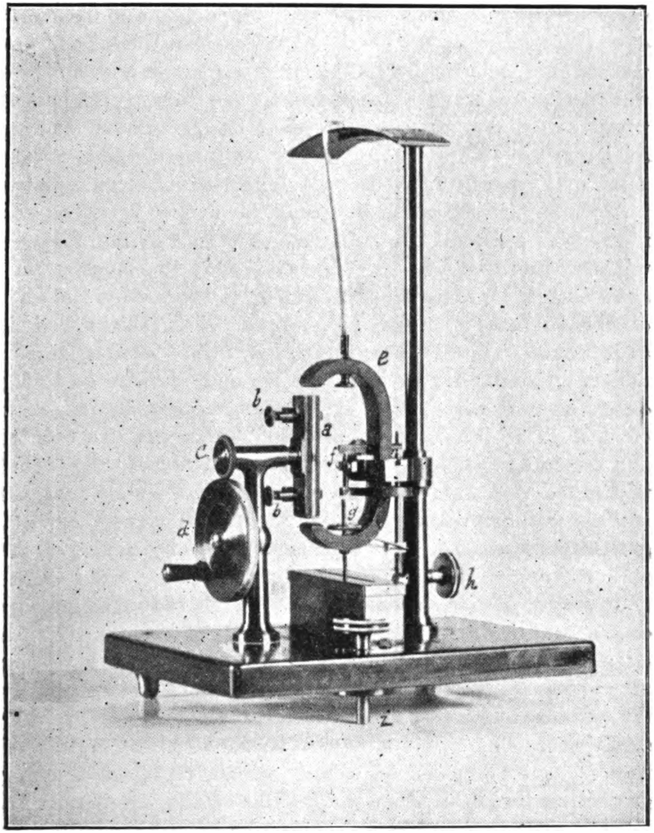

Fig. 18.—Ewing's Hysteresis Tester.

periodically reversed. The work done in reversing the magnetism in consequence of hysteresis causes a mechanical moment to be exerted by the revolving sample upon the magnet; and the magnet, being supported upon a knife edge at *f* in line with the axis of the carrier, tends to follow the sample, and is deflected through an angle which serves to measure the work expended. Since a definite amount of work is done per reversal, whatever the frequency (so long as that is not so high as to make a sensible addition to the work by inducing currents), the deflection of the magnet is independent of the speed at which the carrier revolves, and no special care has to be taken to turn the handle at a uniform rate. If the rate is very slow, the magnet will show each individual impulse which it receives as the ends of the sample pass its poles, but when the speed is sufficiently raised these impulses blend into a steady deflection, and the speed may be further augmented to the extent of doubling it, or more, without making the deflection change. The deflection is observed by means of a pointer and scale above the magnet, and the swinging of the magnet is checked by a dash-pot below, consisting of a vane or spade moving in a box filled with oil. The stability is adjusted to give any required degree of sensitiveness, by means of the weight *g*, which travels as a nut upon a screw fixed to the magnet, and serves to raise or lower the centre of gravity of the oscillating system. The magnet swings about a knife edge working in an agate trough, and a lifting arrangement like that of a balance is provided, operated by the handle *h*, to save the knife edge from unnecessary wear or injury. The pointer is set to zero in the middle of the scale by means of a nut which runs on a screw projecting sideways from the middle of the magnet, and a more delicate adjustment of the zero may be effected by means of the levelling screw *i*.

Two standard samples of iron are provided with each instrument, along with a table stating their hysteresis, as found by reference to ballistic experiments, in ergs per cubic centimetre per cycle, and also in watts per pound at an assigned frequency.

To determine the hysteresis, the necessary strips of iron are cut and filed accurately to length in the gauge. The instrument is then levelled, and the pointer adjusted to zero of the scale. One of the standard samples is then inserted in the carrier *a*, care being taken to keep the ends of all the strips flush, and to push the bundle up against a stop at one end of the carrier. The magnet is now lowered on to its knife edge, and the handle turned just quick enough to give a steady deflection, and the scale reading observed. The direction of rotation of the handle is now reversed, and the

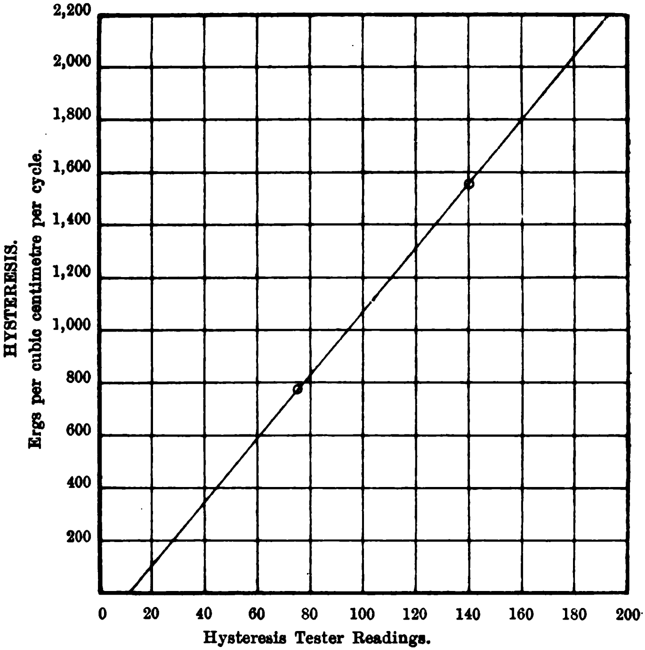

FIG. 19.—Determination of Hysteresis from Hysteresis Tester Readings.

steady scale reading taken on the opposite side of the zero. (This eliminates any inaccuracy in the initial adjustment of the zero.) The sum of the two readings is then taken as the total deflection. The deflections given by the second sample are then determined in the same manner, thus standardising the instrument as the hysteresis of the standard samples has been determined ballistically.

Each bundle of prepared sample strips is then inserted in turn in the carrier, and the scale readings taken for right and left handed rotation of the handle, just as for the standard samples.

To determine the hysteresis in ergs per cubic centimetre per cycle, the scale readings of the tester for the two standard samples are plotted on squared paper in relation to their known hysteresis, and a straight line drawn passing through these points. Thus, suppose the hysteresis of the two standard samples is stated as 780 and 1,560 ergs per cubic centimetre per cycle, and that the tester reading for the first sample is 76 scale divisions, and for the second sample 140 scale divisions. These values are plotted as in fig. 19, in relation to their known hysteresis, and a straight line drawn passing through these two points. The hysteresis of the prepared samples can then be read off in ergs per cubic centimetre per cycle by observing where the scale reading for each sample cuts this inclined line. Say the tester reading, for a sample, is 100 scale divisions; then, by inspection of fig. 19, the hysteresis corresponding to this scale reading is 1,070 ergs per cubic centimetre per cycle.

The induction density B attained in the specimens is about 4,000 lines, and for higher or lower induction densities the hysteresis in ergs has to be multiplied by a factor to determine the loss at the given induction density. The following table of factors is given in the paper:—

Induction density B.	Relative amount of hysteresis.
2,000 lines.	0·33
2,500 ,,	0·47
3,000 ,,	0·63
4,000 ,,	1·00
5,000 ,,	1·41
6,000 ,,	1·89
7,000 ,,	2·41
8,000 ,,	3·00

These values for B well cover the ground for the induction densities employed in alternating-current transformers, but B in continuous-current armatures is usually much higher, and the following table is therefore added to give the approximate factors by which the hysteresis, as determined by the tester, should be multiplied to obtain the hysteresis per cubic centimetre per cycle, and hence to determine the rate of doing work from hysteresis in watts.

Induction density B.	Relative amount of hysteresis.
10,000 lines.	3·9
12,000 ,,	5·0
14,000 ,,	6·6
16,000 ,,	8·5
18,000 ,,	12·0
20,000 ,,	13·0

The magnetic quality of the iron employed has progressively improved in recent years; nevertheless bad batches do come to hand occasionally; and although the tester, as usually made, cannot give directly the hysteresis at the high values of B usual in continuous-current armatures, still it enables very satisfactory tests to be made quickly and simply of the quality of the iron used. Sheet iron or mild steel which shows a greater hysteresis than about 1,200 ergs per cubic centimetre per cycle for B = 4,000 lines should be rejected.

The permeability bridge was exhibited at the May soirée of the Royal Society,* and was also described in a paper read before the Institution of Civil Engineers on the 19th May. †

The method employed is to compare magnetically the sample of iron with a standard sample whose B—H curve has been previously determined. In the instrument shown in fig. 20, the standard and sample are in the form of solid cylindrical bars, the latter, say, from a field magnet forging; but samples may be formed of sheet iron strips as well, prepared from armature stampings. These two bars are placed horizontally and parallel to each other, and their ends are joined together by two short wrought-iron yokes. The ends of the bars at *b*, *b* are seen projecting from one of the yokes *y*, the other yoke *y* being at the opposite end of the bars. Carefully fitted to the yokes, and fastened to them by screws, two long iron arms *a*, *a* rise vertically upwards, and these are bent over so that their free ends face each other. In the short gap between the free ends of the two arms a box *c* is placed, containing a pivoted magnetic needle. This needle serves to indicate when the ends of the two arms are at the same magnetic pressure. Surrounding the bars *b*, *b* are two parallel coils of insulated wire, shown covered in fig. 20 by a brass cover. These coils, when a current is sent through them, act as magnetising coils, and magnetise the bars by induction. The two coils are connected in series, so that the same current passes through each, but the number of turns surrounding the sample bar can be varied by means of the two dial switches in front of the instrument, which have double contacts, and at each step introduce a coil equal in resistance to the turns by which the magnetising coil is reduced, thus keeping the total resistance of the instrument in circuit constant, and maintaining the current at the same value as an adjustment is made.

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* *Electrician*, May 18th, 1896. † *The Practical Engineer*, May 29th, 1896.

Now, suppose two bars are used exactly equal in permeability, then for the same induction through each

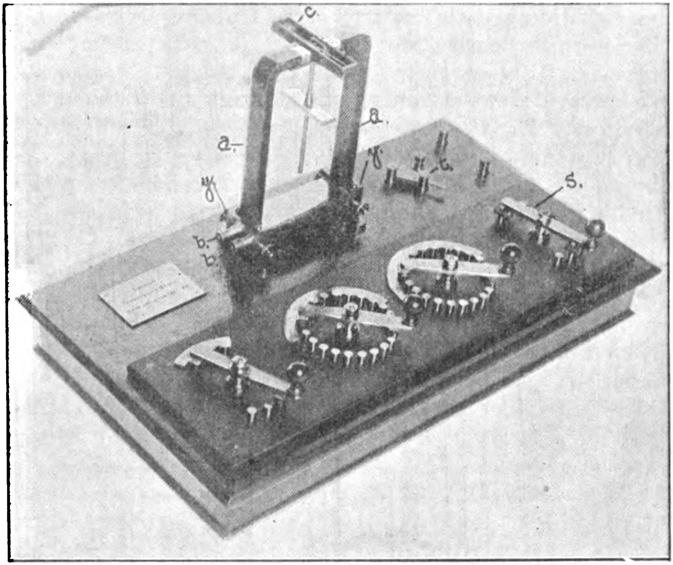


Fig. 20.—Ewing's Permeability Bridge.

bar the same number of ampere-turns must be applied by each coil. The lines of induction will then go from right to left along one bar, cross over one yoke to the other bar, return along it from left to right, cross over the other yoke, and again from right to left, &c. The magnetic circuit under these conditions will be quite complete through bars and yokes, and each yoke will be at the same magnetic pressure, as well as the arms a, a, connected to the yokes. No polar regions will be developed at the free ends of the arms, and the magnetic needle will not be deflected.

If, however, the two bars differ in permeability, more lines of induction for equal ampere-turns on each bar will pass through one bar than through the other, and some of the lines will pass up one arm, leap across the gap, and pass down through the other arm, to return again through the bar of greater permeability. Under these conditions, polar regions will be developed at the free ends of the arms, and the pivoted magnetic needle will be deflected from its zero position. To restore the needle to its zero, the number of turns in the coil on the sample is increased or diminished by means of the dial switches, until, as shown by the needle, no difference of magnetic pressure exists between the free ends of the arms, and consequently between the two yokes.

To eliminate hysteresis in the bars, yokes, and arms, a reversing switch s is provided, and as the adjustment is being made the exciting current is reversed in direction from time to time by it until a balance is obtained. A transient kick, even when the adjustment is complete, will usually be observed at the needle at each reversal, due to a difference in the time rate with which the two bars take up their magnetism.

By formula (1), page 42,

$$\mathsf{H} = 1{\cdot}257\,\frac{C\,t}{l},$$

and to make the constant of the instrument a round number the length of the magnetising coils is made 12·57 centimetres. The number of turns in the magnetising coil is 100. Hence for one ampere through this coil

$$\mathsf{H} = \frac{1{\cdot}257 \times 1 \times 100}{12{\cdot}57} = 10.$$

By placing an ordinary ammeter, then, in series with the instrument, reading from 0 to, say, 20 amperes, and coupling the instrument in circuit with a few secondary cells and a variable resistance, H can be given any value from 0 to 200. The relation of B to H has, however, been previously determined for the standard bar, and therefore, when the current is known, the value of H is also known, and B for the standard bar, corresponding to this value of H. When a balance is obtained, B is, of course, the same for both bars and the ratio of the number of turns on the sample bar (read from the dial switches) to the number of turns on the standard bar (100), multiplied by the value of H for the standard bar, gives the new value of H for the sample bar. As the value of B corresponding to H for the standard bar is known, and B is the same for both bars, one point on the B—H curve of the sample bar is determined. By varying the current, and proceeding in this manner, a number of points on the B—H curve are obtained. Thus, commencing with 1 ampere through the coils, one point is determined; then the current is increased to 2 amperes, and a second point found; then 3 amperes, and so on, giving H successively values of 10, 20, 30, &c.

By means of the dial switches the number of turns on the sample bar can be increased from 100 to 210 turns; and in cases where the sample is magnetically much worse than the standard, a two-way switch *t* is added, which reduces the number of turns on the standard bar to 50, thus giving H successive values of 5, 10, 15, &c., for 1, 2, 3, &c., amperes through the 50 turns on the standard bar, with the switch in its second position.

The magnetic needle is sufficiently sensitive to allow of an adjustment being made to at least one turn of the coil, and a directing magnet is added below the needle to adjust its sensitiveness, and bring the needle initially to zero.

Such an instrument appears specially fitted for the rapid determinations of B required in practical work. The final values are, of course, dependent on the initial measurements of the standard bar; but this once carefully determined, further comparisons would be very quickly made, and a great saving of labour effected. Ballistic tests require some skill to carry out, and the labour involved in the final reduction of the results is by no means light. Moreover, a workshop subject to vibration, and where large masses of iron are frequently moved, is not quite an ideal place for a ballistic galvanometer. The permeability bridge, employing a method in which the needle is finally adjusted to zero, overcomes some of these difficulties, and its use in practice will undoubtedly extend, and lead to greater certainty in the pre-determination of the characteristic curves of dynamos.

CHAPTER V.

Armatures.—When a conductor of unit length (1 centimetre) is moved at a constant velocity of 1 centimetre per second across a uniform field of unit strength—*i.e.*, having 1 C.G.S. line per square centimetre—1 C.G.S. line cuts or threads the conductor per second, and 1 C.G.S. unit of electrical pressure is generated between the two ends of the conductor (page 39). If the conductor is 2 centimetres long, each unit of length will cut 1 C.G.S. line, or the total number of lines cut by the conductor, 2 centimetres long, will be 2, and therefore 2 C.G.S. units of pressure will be generated between the ends of the conductor. A conductor, then, of unit length, moving with a constant velocity of v centimetres per second, will cut v times as many lines per second and generate v C.G.S. units of electrical pressure. Further, if this conductor is made b centimetres long, the conductor will cut $v\,b$ lines per second, and $v\,b$ C.G.S. units of electrical pressure will be generated between its ends when moved across a field of unit strength. Instead of a field of unit strength, let B equal the number of C.G.S. lines per square centimetre; then a conductor b centimetres long, moving with a constant velocity of v centimetres per second across a field having B C.G.S. lines per square centimetre, will cut $\mathsf{B}\,v\,b$ lines per second, and will generate a difference of $\mathsf{B}\,v\,b$ C.G.S. units of electrical pressure between the ends of the conductor. This difference of electrical pressure will be maintained constant in value so long as the conductor is moved with constant velocity v across a uniform field having B C.G.S. lines per square centimetre.

Practically, however, it is not easy to give a continuous rectilinear motion in one direction, with constant and sufficient velocity, to the conductors. Mechanically, it is preferable to group the conductors around a central axis and rotate them with uniform velocity in the magnetic field, and it is necessary to determine the difference of electrical pressure generated in this case.

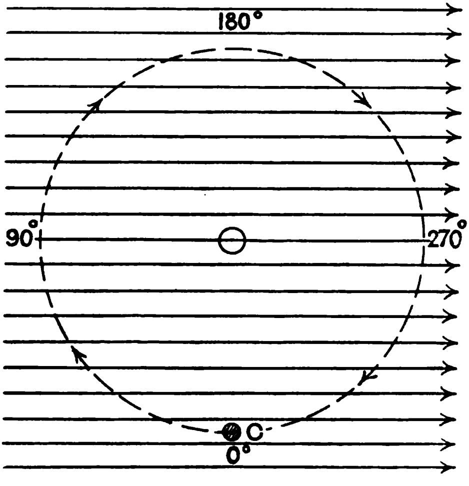


FIG. 21.—Single conductor rotating in a uniform field.

Take the simple case of a single conductor rotating at constant velocity in a uniform field. Let the magnitude and direction of the field in a vertical plane be represented by the horizontal lines in fig. 21. Around a centre O let a straight conductor, shown in section at C, be rotated with uniform velocity in a right-handed direction. For the first right angle of revolution—0 deg. to 90 deg.—the wire will move from a position parallel to, to a position at right angles to the lines of induction. For the second right angle—90 deg. to 180 deg.—the rotating conductor will move from a position at right angles to the lines, to a position parallel to them. For the third right angle—180 deg. to 270 deg.—the conductor will move from a position parallel to, to a position at right angles to the lines; and for the fourth right angle—270 deg. to 0 deg.—from a position at right angles to, to a position parallel to the lines. Starting from 0 deg., where the conductor moves practically parallel to the lines, the rate of cutting is a minimum, and the difference of pressure generated is zero. As the conductor rotates the rate of cutting increases, until at 90 deg. it has attained a maximum; it now moves at right angles to the lines, and the difference of pressure produced is also a maximum. From 90 deg. the rate of cutting decreases again until the conductor arrives at 180 deg., when it is again a minimum, and the difference of pressure has fallen to zero. From 180 deg. the rate of cutting rises from a minimum to a maximum at 270 deg., and the lines are now cut in the opposite direction; so also the difference of pressure produced: it rises from zero to a maximum, but the difference of pressure is now opposite in direction to the first right angle of revolution. For the fourth right angle—270 deg. to 0 deg.—the rate of cutting falls from a maximum to a minimum, and the difference of pressure also falls from a maximum to zero. The number of lines cut by a conductor rotating in a uniform field varies, then, from instant to instant, and the actual rate of cutting at any instant will, for a single conductor in a uniform field, be proportional to the sine of the angle through which the conductor has been rotated. Let θ be this angle; then the rate at which a conductor b centimetres long, moving with a velocity of v centimetres per second, cuts the lines at any instant in a uniform field having B lines per square centimetres, is

$$\mathsf{B} \,.\, v \,.\, b \sin \theta.$$

$\mathsf{B} \,.\, v \,.\, b \,.\, \sin \theta$ also represents in C.G.S. units the difference of electrical pressure generated at any instant, or, as it is called, the instantaneous value of the electrical pressure.

Let N be the number of revolutions per minute of the conductor; then $\frac{\mathrm{N}}{60}$ equals the number of revolutions per second.

Now, $$v = 2 \pi r \frac{\mathrm{N}}{60},$$

where r equals the radius of rotation in centimetres, and, substituting this value in the above, the instantaneous difference of pressure is

$$\mathsf{B}\, 2 \pi r \frac{\mathrm{N}}{60} b \,.\, \sin \theta.$$

But $2\, r \,.\, b$ is the projection on a plane at right angles to the lines of induction of the area traversed by the rotating conductor, and this area, multiplied by B, the density of lines per square centimetre, gives the total number of lines cut by the conductor in half a revolution. Let Z equal this number; then, by substitution in the above, the instantaneous difference of pressure generated is

$$\pi\, \mathsf{Z} \frac{\mathrm{N}}{60} \sin \theta.$$

As θ increases from 0 deg. to 90 deg., the value of the sine varies from 0 to + 1; from 90 deg. to 180 deg. the sine varies from + 1 to 0; from 180 deg. to 270 deg., from 0 to − 1; from 270 deg. to 0 deg., from − 1 to 0, or the maximum value of $\sin \theta$ is $+1$ or -1.

The instantaneous difference of electrical pressure generated in a complete revolution varies, then, from 0 at 0 deg. to

$$\pi Z \frac{N}{60} \text{ at 90 deg.;}$$

again to 0 at 180 deg.; then to

$$-\pi Z \frac{N}{60} \text{ at 270 deg.;}$$

and again to 0 at 0 deg. Such a difference of electrical pressure, which periodically varies from a maximum positive to a maximum negative value, is called an alternating difference of pressure, and its value and direction at any instant would be represented by a point on the sine curve, fig. 22.

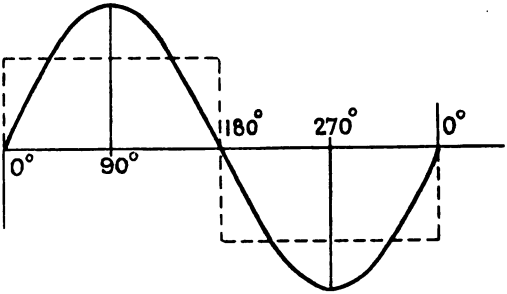


FIG. 22.—Sine curve representing an alternating difference of pressure.

In practical applications of the dynamo it is not the maximum difference of pressure generated that is required, but the mean or average difference of pressure; and since the average value of sin θ is $\frac{2}{\pi}$, the average difference of pressure produced is

$$\frac{2}{\pi} \pi Z \frac{N}{60} = 2 Z \frac{N}{60}.$$

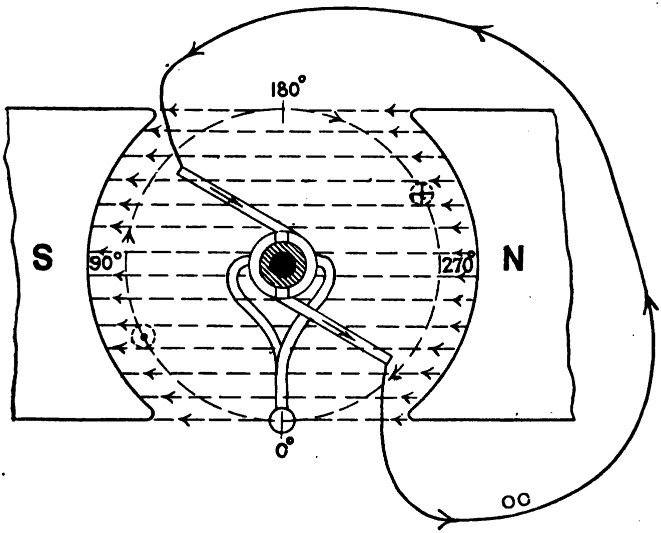
This average difference of pressure would be represented on fig. 22 by the dotted rectangles, and is simply the number of lines cut per second, without any regard to the rate at which the lines are cut from instant to instant. The rotating conductor cuts all the lines once from 0 deg. to 180 deg., and a second time from 180 deg. to 0 deg. Now, Z equals the total number of lines crossing the projected area traversed by the conductor, and therefore $2Z$ lines are cut by the single conductor per revolution, and if this conductor makes $\frac{N}{60}$ turns per second, the average difference of pressure is equal to the number of lines cut in one second, and this is—

$$2Z\frac{N}{60}$$

If, now, two insulated metallic rings were placed on the axis round which the conductor rotates, the two ends of the conductor being joined respectively to a ring by wires, and on each ring a fixed spring is made to press, so that the conductor can be freely rotated while still making metallic contact with the springs, a wire attached to the springs would complete the outside circuit, and in virtue of the difference of pressure generated by the rotation of a conductor in a magnetic field, an alternating current would flow through the closed circuit.

For many practical purposes it is, however, requisite that the difference of pressure generated, and the current resulting from this difference of pressure, should have a continuous direction as regards the outside circuit. This is effected by means of an arrangement called a commutator, attached to and rotating with the rotating conductors. For a single conductor this simply consists of one of the rings cut along a diameter into two semicircular segments, both being insulated from the shaft and from each other. Each segment is joined to one end of the rotating conductor, as shown diagramatically in fig. 23.

From 0 deg. to 180 deg. the induced difference of pressure will act, and with a closed outside circuit the current also will flow vertically upwards, to the observer's eye, the dot in the centre of the dotted conductor to the left representing the point of an arrow coming towards the observer. From 180 deg. to 0 deg. the current will flow in the opposite direction, as shown by the cross in the dotted conductor to the right, representing the tail of the arrow flying from the observer. Now, if the springs pressing on the divided ring are set opposite to each other along a diameter, it is obvious that by properly placing them relatively to the rotating conductor, one part of the divided ring will turn out of contact, and the other part come into contact with the spring, just at the instant the direction of pressure and



O C = Outside circuit.

FIG. 23.—Single conductor with split-ring commutator.

the resulting current is reversed in the rotating conductor, as shown in full lines in fig. 23. The result of this change in the contacts between the divided ring and the springs is that the current is commuted or rectified with regard to the outside circuit, and flows in one direction. The current still, however, undulates; it has not a constant value, but is now of one sign only, as shown in fig. 24, two currents being generated for each complete revolution of the conductor.

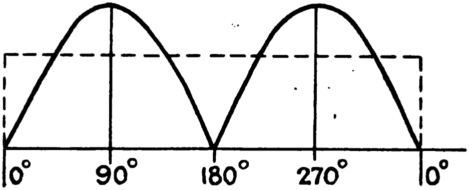


FIG. 24.—Commuted or rectified undulating difference of pressure.

The difference of electrical pressure produced by a single conductor rotating in a magnetic field cannot, however, attain a high value, since the size and density of the magnetic field employed in practice is limited, and the rate of rotation of the conductor is also fixed by mechanical considerations. To increase the difference of pressure, the number of conductors employed is increased, and these are so joined as to add together the difference of pressure produced by each—*i.e.*, are joined in series.

The simplest method of effecting this is to place a second conductor diametrically opposite the first one, and join the back ends of the two conductors together by a wire, as shown in fig. 25, the conductors now forming a single loop. The difference of pressure produced by each conductor is opposite in direction with regard to a fixed point in space (the two conductors move in the same direction, but through fields of opposite sign); but, as shown by the arrows in the figure, in the same direction with regard to the conductors forming the loop.

Such an arrangement, therefore, provides two active conductors, *a b*, *c d*, joined in series by a connecting wire *a c*, and the difference of pressure produced for the same rate of rotation will now be double that produced by a single conductor. The free ends of the loop are joined to a simple split commutator at *f* and *g*, as for the single conductor, and the currents are rectified as regards the outside circuit, and as collected by the springs B_1 and B_2 are of one sign only. In place of a single loop, a conductor may be arranged so as to form a rectangular loop of many turns, as shown in fig. 26, the free ends of the loop being again joined respectively to the two segments of the split ring commutator. The differences of pressure produced by the two active conductors forming each single loop are thus added together, and the total difference of pressure can with such an arrangement attain any required value.

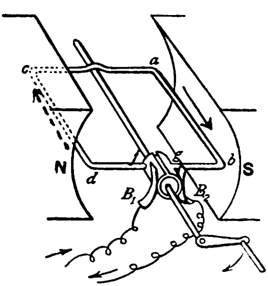


Fig. 25.—Single loop with split-ring commutator.

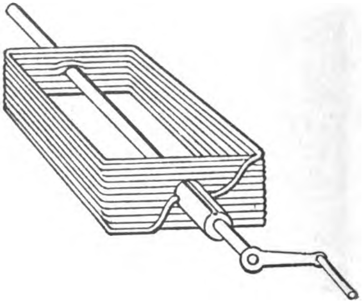


Fig 26.—Loop of many turns with split-ring commutator.

The difference of pressure produced, however, still undulates; it is not constant in value, although continuous in direction as regards the outside circuit, since the differences of pressure produced in all the active conductors are only added together, and practically in all the conductors varies from zero at a given instant, when the conductors move parallel to the lines, to a maximum value at a later period, when the conductors move at right angles to the lines; the current also with a closed circuit varying from zero to a maximum value.

To obtain a difference of pressure continuous in direction and more nearly constant in value, it is necessary to arrange the conductors on the armature, so that as one conductor leaves the position where the difference of pressure produced is a maximum, a second conductor approaches that position, and this in turn must be followed by a third, and so on. To fulfil this condition, the conductors, or more usually groups of conductors, are arranged at equal intervals around the circumference of the armature, so that when some of the conductors are generating a maximum difference of pressure, in others, at right angles to these, the difference of pressure produced is zero, and in intermediate positions the difference of pressure produced rises from zero to the maximum value. With such an arrangement of conductors, the commutator also requires modification; the number of segments employed must be increased, but the precise modification employed depends on the system of winding adopted for the armature conductors.

The methods of armature winding used in continuous-current dynamos may be divided into two types called respectively (1) closed-coil armatures, (2) open-coil armatures. In both types the difference of pressure produced, and with a closed circuit, the current in the circuit is of one sign only. With closed-coil armatures, however, the difference of pressure produced at the terminals of the machine is practically continuous in value, while with open-coil armatures the difference of pressure still undulates, but the number of undulations per revolution is increased by employing a number of groups of conductors in place of a single conductor or a single group of conductors.

Closed-coil Armatures.—The methods of arranging the conductors on armatures of this type, so as to add together the differences of pressures generated by a number of conductors, while at the same time maintaining at the terminals of the machine a difference of pressure constant in value and direction, are practically three in number, and these are known as—

(*a*) Ring armatures.
(*b*) Drum armatures.
(*c*) Disc armatures.

Ring Armatures.—The active conductors on closed-coil ring armatures are arranged symmetrically around the circumference of a ring or cylinder, and these active conductors are connected together in series by connectors which pass through the interior of the ring or cylinder. An active conductor and a connector is spoken of as a turn, and a number of turns is usually called an armature coil. To secure a constant difference of pressure at the terminals of the machine it is necessary, as previously pointed out, to arrange the turns or coils so that some of the turns are cutting the lines of induction at right angles, while others are cutting no lines—*i.e.*, are passing through a neutral region.

As a simple example, take the case of four coils, 1, 2, 3, 4, arranged equi-distantly on a ring or cylinder, figs. 27 and 28. The direction of the difference of pressure produced is simultaneously reversed in two of the coils as they pass through the neutral region, when the coils move practically parallel to the lines of induction, coils 2 and 4 occupying this position in fig. 27. When the ring has revolved through a quarter of a revolution, coils 1 and 3 will occupy this position, and the direction of the difference of pressure induced in them will be reversed, and so on, a reversal of the direction of the difference of pressure produced in two of the coils taking place at each quarter of a revolution when four coils are employed. At the instant this change in the direction of the difference of pressure generated in these coils takes place, it is necessary, to produce a difference of pressure acting in one direction only, to interchange their connection with the outside circuit; and in closed-coil armatures this is effected without removing the coils from the armature circuit. All the four coils are wound around the ring in the same direction; the end of coil 1 is joined to the beginning of coil 2, the end of coil 2 to the beginning of coil 3, the end of coil 3 to the beginning of coil 4, and the end of coil 4 to the beginning of coil 1. The winding is thus continuous in direction around the ring, and forms a circuit closed on itself. The number of commutator segments is also usually made equal to the number of coils, and will now consist of four segments insulated from each other. To the first segment the common junction of coils 1 and 2 is attached; to the second segment, the junction of

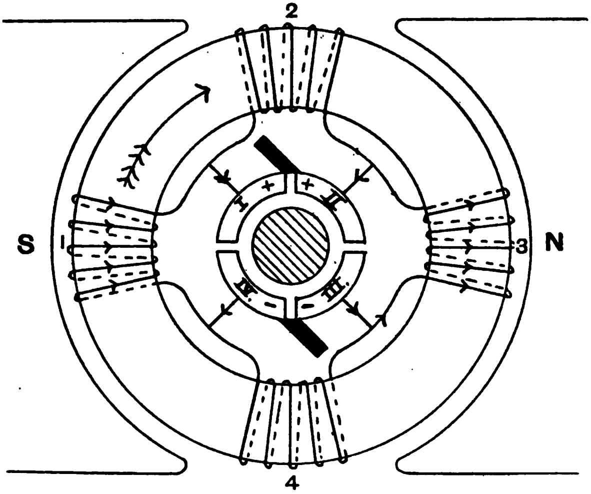


FIG. 27.—Four-coil ring armature.

coils 2 and 3; to the third segment, the junction of coils 3 and 4; and to the fourth segment, the junction of coils 4 and 1. The collecting springs, or, as they are more usually called, the brushes, are again placed diametrically opposite to each other.

Tracing the paths in fig. 28, for the current through the armature, two paths will now be found to be open to it. Commencing at the lower brush, the current passes through this brush to segment III.; it now divides into two parts, half passing through coils 4 and 1, and then to segment I., the other half passing through coils 3 and 2, and also to segment I., where they re-unite and pass to the outer circuit through the upper brush, thence around the outer circuit, and back again to the lower brush. The two paths provided for the current in closed-coil armatures in a two-pole field are thus joined in parallel, in this case at segments III. and I., and the difference of pressure at the brushes,

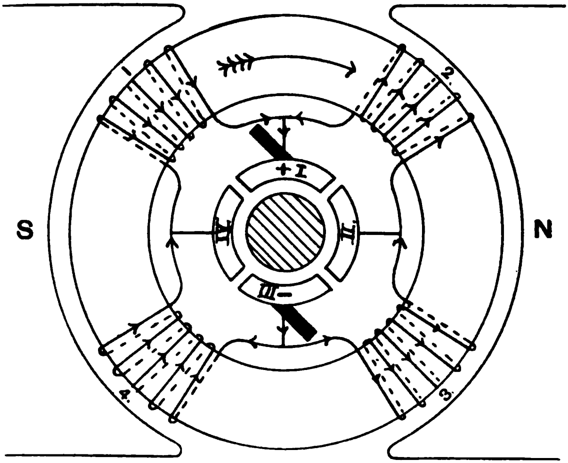


Fig. 28.--Four-coil ring armature.

for the position shown in fig. 28, is that due to the sum of the difference of pressure produced by coils 4 and 1, or by coils 3 and 2—*i.e.*, is proportional to that produced by half the total number of active conductors on the armature in series.

Now, suppose the coils rotated into the position shown in fig. 27. Coils 2 and 4 are now moving practically parallel to the lines of induction, and the difference of pressure generated in these coils will fall to zero; coils 1 and 3 are cutting the lines at right angles, and will generate a maximum difference of pressure. Just, however, as this position is reached, it will be observed that, owing to the surface of the brushes not making contact along a line only, but bearing over a definite portion of the segments of the commutator, the gap between segments I. and II. is bridged by the surface of the upper brush, and at the same instant the gap between segments III. and IV. is also bridged by the lower brush, the circuits of coils 2 and 4 being thus directly closed on themselves by the brush surface, or short-circuited, and the difference of pressure in these coils falls to zero. During the time this short-circuiting lasts, the difference of pressure generated in these coils is reversed in direction, and due to this reversal of the direction of the difference of pressure, a current is started in the opposite direction through these coils, so that when by the further revolution of the armature segments IV. and II. emerge from under the tips of the brushes, the difference of pressure produced by coil 4 is in the same direction as that due to coil 1, and these are added together, these coils being now in series; also, the difference of pressure through coils 2 and 3 is in the same direction, and these being also joined in series, their differences of pressure are also added together. When coils 3 and 1 arrive at the neutral position, they will in turn be short-circuited; the difference of pressure produced by them is reversed in direction; and when segments III. and I. emerge from under the tips of the brushes, coils 3 and 4 are joined in series, also coils 1 and 2 are in series, and so on. At each quarter of a revolution, with four coils on the armature, two coils are short-circuited; the difference of pressure due to them falls to zero, then is reversed in direction, and the coils placed in series again with the other coils having their differences of pressure in the same direction, and the differences of pressure produced by half the total number of active conductors on the armature is thus added together.

The direction of the induced difference of pressure in the armature conductors, in figs. 27 and 28, is indicated by the arrow heads, and, with a closed outside circuit, this is also the direction of the current due to that difference of pressure.

It will next be necessary to inquire as to the variation in the induced difference of pressure with this arrangement of conductors. The curve obtained for a single conductor rotating in a uniform field is shown in fig. 24, page 91, and the curve for each pair of coils on the armature will be very similar in character. But for each pair of coils, as shown in fig. 27, the maximum difference of pressure is generated, while the other pair is short-circuited; and for the position in fig. 28, the difference of pressure produced by two pairs of coils is added together. The total difference of pressure at the brushes will also now be greater, since each coil, as shown, consists of five active conductors. The shape of the curve for the instantaneous value of the difference of pressure will also, for a uniform field, be slightly altered in character, since each coil now covers a certain portion of the circumference of the armature, and all the conductors forming a coil do not attain a maximum rate of cutting lines at the same instant. The general result will be that the curve of instantaneous pressure for each coil will be slightly flatter at the base and crest than if all the conductors of a coil were superposed radially one over the other. The dotted and dashed curves in fig. 29 may be taken to approximately represent the instantaneous values of the induced difference of pressure in the armature coils for a complete revolution.

As previously pointed out, two coils at a given instant are in series, and while so joined the difference of pressure due to one of the coils is diminishing, while that due to the other is increasing; but at 45 deg., 135 deg., 225 deg., and 315 deg., the difference of pressure produced by each coil, with a uniform field, is equal, and the difference of pressure at the brushes is the sum of the difference of pressure due to two coils in series, and at these periods of the revolution attains the maximum value. At 0 deg., 90 deg., 180 deg., 270 deg., and 360 deg. two coils are short-circuited, and the difference of pressure reaches its minimum value, and its value is then the maximum difference of pressure, produced by a single coil.

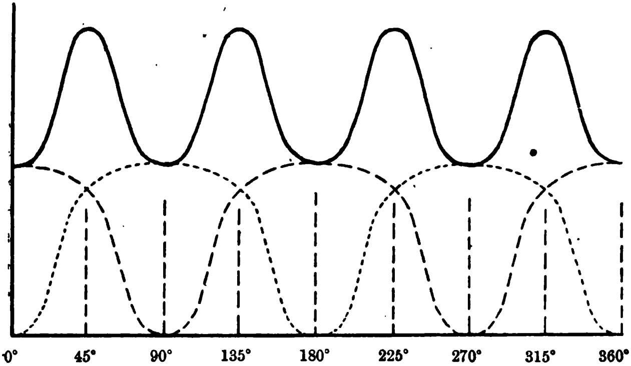


Fig. 29.—Curve representing addition of pressure with four-coil armature.

The full-line curve in fig. 29 represents at any given instant this addition of pressure due to the various groupings of the coils during a complete revolution. It will now be noticed that the curve of pressure undulates four times per revolution, but the undulations are much less marked in character than for a single conductor or single group of conductors; the curve for the difference of pressure at the brushes does not now fall to zero; it is never less than the maximum pressure produced by a single coil.

If the number of coils is doubled, so that eight equi-distant coils are now wound around the ring, and a commutator fitted with eight segments, the undulations in the difference of pressure at the brushes will now be eight per revolution; the minimum difference of pressure produced will be that due to three coils in series, and the maximum that due to four coils in series. Proceeding in this way, by increasing the number of coils on the ring and the number of segments in the commutator so as to sum up the difference of pressure due to the active conductors, the difference of pressure produced at the brushes may be made continuous in direction, and as uniform in value as desired. In practice, the coils, arranged equi-distantly on the circumference of the ring, are made so numerous that the number of active conductors in series is practically a constant number, and the difference of pressure due to them at the brushes is also constant in value. The number of coils and commutator segments in ring armatures rotating at very high numbers of revolutions per minute is rarely less than twenty, and for machines of large size, and running at a smaller number of revolutions per minute, the number of coils and segments has sometimes nearly reached one thousand.

So far the presence of a uniform field, in which the conductors are rotated, has been assumed, but, as now shown, with a sufficiently numerous arrangement of coils and commutator segments, the difference of pressure generated at the brushes is practically constant during every part of a revolution, and the instantaneous difference of pressure generated is automatically averaged or integrated by this method of construction, and has the same value as the average difference of pressure. If, then, the field employed is not uniform, the only difference will be that some of the conductors at a given instant are more active than others. So long, however, as the total number of lines cut per second is a fixed number, it does not matter, so far as affects the final value of the difference of pressure produced at the brushes, how these lines are cut; whether uniformly by all the conductors, or by some at a given instant cutting more than the others. The final result is that these differences of pressure are summed up, and results at the brushes in an average difference of pressure.

To determine the value of this difference of pressure, all that is therefore requisite is to calculate the number of lines cut per second by a given number of conductors joined in series, without any regard being paid to the rate at which individual conductors cut these lines.

CHAPTER VI.

Ring Armature Windings.—Fig. 30 represents diagrammatically a typical two-circuit ring winding for a two-pole field. Twenty coils are shown, each consisting of two turns, the common junctions of the coils being joined to a commutator having twenty segments. The maximum difference of pressure at the brushes will be that due to ten coils in series, and the minimum that due to nine coils in series.

In practice every effort is used to make all the coils of equal length, and to arrange them equi-distantly around the circumference of the armature. Commencing at the − brush, the difference of pressure progressively rises around the two paths of the armature circuit until the + brush is reached. This progressive increase in the difference of pressure can be shown experimentally by a method due to Mordey. One terminal of a voltmeter V is joined to the negative brush, fig. 31; to the other terminal of the voltmeter a flexible wire is attached, and this wire carries at its free end a very thin pilot brush *p*. This brush can be made to bear on the circumference of the commutator at any given points between the negative and the positive brushes. The machine being run at a constant speed, the pilot brush is applied at various determined points on the circumference of the commutator, and the difference of pressure produced between the various positions of the pilot brush and the negative brush of the machine, is thus read directly on the voltmeter. These differences of pressure can then either be plotted in the ordinary way, the volts being taken as ordinates, and the abscissæ as representing degrees of revolution, as in fig. 32, or as suggested by Thompson,* plotted around a circle representing the armature, as in fig. 33.

A modification of Mordey's method, due to Swinburne, avoids some difficulties in its practical application. A uniform wire of high resistance (for 100 volt machine, say 100 ohms) is joined directly to the terminals of the machine. The pilot brush is applied to the commutator, and the free end of the flexible wire, attached to the pilot brush, is joined to the terminal of an ordinary detector galvanometer. The second terminal of the galvanometer has a wire attached

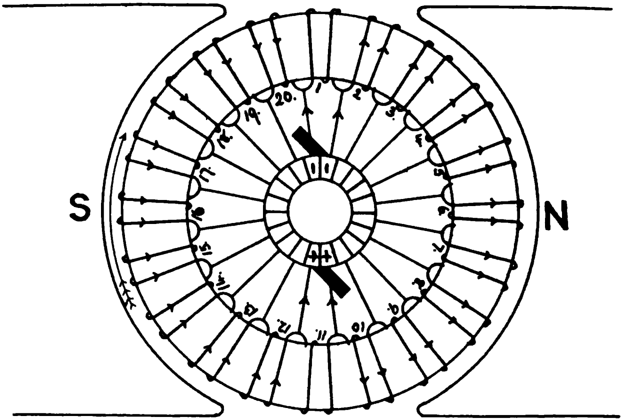


FIG. 30.—Two-circuit single ring winding in two-pole field.

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* Cantor Lectures, 1883.

which can make contact at any point along the length of the high-resistance wire joined to the terminals of the machine. The point of contact on the high-resistance wire is then moved along until, for any given position of the pilot brush, no deflection is observable on the detector needle—*i.e.*, when the difference of pressure at the pilot brush is exactly balanced by the difference of pressure at a point on the high-resistance wire. In this way a series of points can be determined on the high-resistance wire, corresponding to a series of different positions of the pilot brush on the commutator. The difference of pressure is then read as the ratio of the length of the high-resistance wire (from the negative terminal to the sliding contact) to the total length

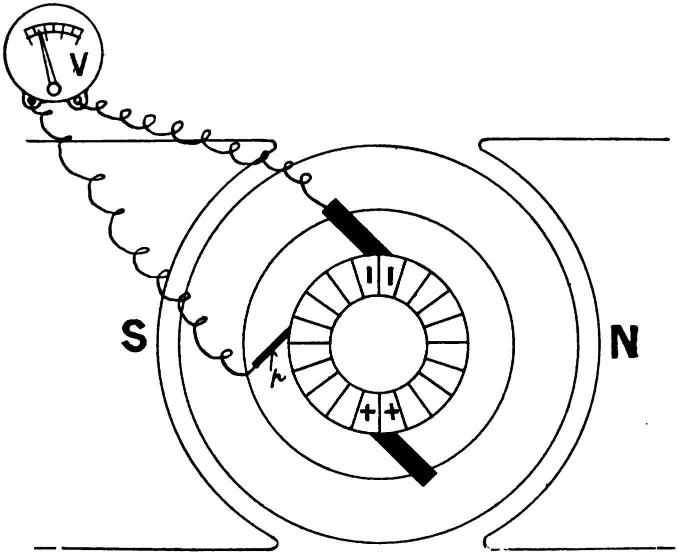

FIG. 31.—Mordey's method of determining the progressive increase of pressure around an armature.

of wire between the machine terminals. The method is thus a zero method, and variations in the speed of the machine do not affect the position for a balance; and when a balance is obtained no sparking occurs at the pilot brush, and contact errors at the commutator do not affect the result.

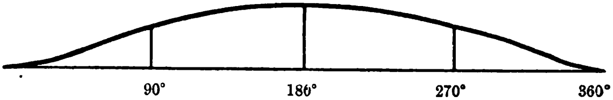

FIG. 32.—Pressure around the commutator of a dynamo.

It is thus possible, by either of these methods, to determine the difference of pressure from point to point around the circumference of the commutator, and since this difference of pressure is proportional to the number of lines cut per second, the distribution of the lines of induction from point to point in the field in which the conductors of ring armatures are rotated can also be calculated.

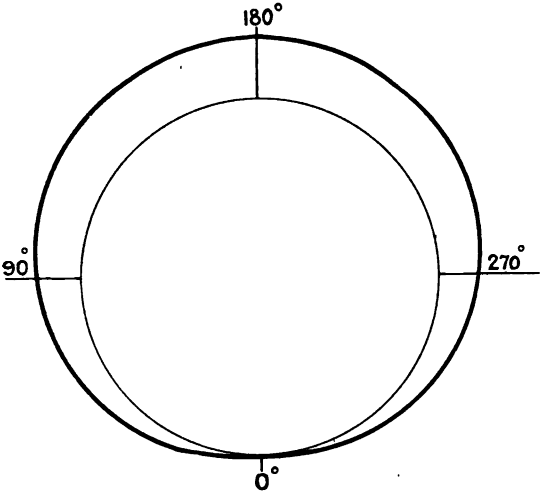

FIG. 33.—Pressure around the commutator of a dynamo.

It is somewhat difficult to give a clear representation diagrammatically of practical armature windings when a large number of conductors are employed. Especially is this the case when some of the more complex ring windings used in multipolar machines have to be delineated. Two methods are used to obviate these difficulties: one to develop the surface of the armature, and on this developed surface to indicate by full and dotted lines the exterior and interior conductors respectively; the other, the winding table method introduced by Kapp,* and for workshop use the latter is certainly to be preferred. All the conductors—or, in the case of groups of conductors, the coils—are numbered successively around the armature. Regarding the winding from the commutator end, conductors wound from the observer on the exterior of the ring, from front to back, are arranged in columns headed D; those coming towards him through the interior of the ring, from back to front, in columns headed U. In the case of coils these letters may be taken as applying to the end wires only of the coils, without regard to the number of complete turns in each coil; denoting exterior conductors by 1, 2, 3, 4,......20, and interior conductors by 1′, 2′, 3′, 4′,......20′. The winding table for the ring winding in fig. 30 would be arranged as follows:—

WINDING TABLE FOR RING ARMATURE.

Two-circuit ring winding for two-pole field.

20 coils each, of 2 turns and 20 commutator segments.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	20	..	1′	− ..	1	..	2′	..	2	..	3′	..	3	..	4′
..	4	..	5′	..	5	..	6′	..	6	..	7′	..	7	..	8′
..	8	..	9′	..	9	..	10′	..	10	..	11′	+ ..	11	..	12′
..	12	..	13′	..	13	..	14′	..	14	..	15′	..	15	..	16
..	16	..	17′	..	17	..	18′	..	18	..	19′	..	19	..	20′
..	20	..	1′	..	..	..	..	..	..	..	..	..	..	..	..

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* Cantor Lectures, 1891.

The letters F and B in the table indicate the connections of the conductors at the front and back ends of the armature respectively. Thus, commencing at conductor No. 20, and following this down to the end of the armature, it is joined at the back to 1′, which comes up towards the observer through the interior of the ring, and is joined at the front to 1; then, down 1, this is joined at the back to 2′; up 2′, joined at the front to 2; down 2, joined at the back to 3′; up 3′, joined at the front to 3′; down 3, up 4′; down 4, up 5′; down 5, and so on, until finally the winding returns up 20′, and this is joined at the front to 20, and the winding made continuous and closed on itself.

The connections to the commutator segments are always made at the front end of the armature, and the position of the brushes on the commutator segments may, for a given position of the ring in the field, then be denoted as shown in the winding table by the signs − and +.

Such a table can also be used to show the progressive increase of pressure around the armature from the − brush to the + brush. To take numbers of conductors or coils and segments more usual in practice, say an armature has 216 turns of conductor and 72 commutator segments—*i.e.*, 72 coils, each of 3 turns. Such an armature, when placed in a two-pole field, has its winding joined by the brushes into two parts in parallel, each part consisting of 36 coils in series. One coil will be short-circuited by each of the brushes, since the brushes in practice bear over rather more than a commutator segment. Adjacent coils to these short-circuited coils will not be very active, since they are moving almost parallel to the lines of induction; so, as an example, 34 coils in series may be taken as being active; and assuming the difference of pressure produced by each active coil is the same (which is, of course, not true in practice; as previously pointed out, some of the coils are more active than others, but these differences of pressure are automatically averaged at the brushes), and that the total difference of pressure at the brushes is 102 volts, each active conductor will then produce 1 volt; and since three conductors form a coil, the difference of pressure between adjacent commutator segments will be on the average 3 volts, and on this assumption the differences of pressure produced by the active coils are added in the winding table below in the columns marked F.

WINDING TABLE FOR RING ARMATURE.

Two-circuit ring winding for two-pole field.

72 coils, each of 3 turns, and 72 commutator segments.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
Volts. 0	72	..	1′	Volts. $\overline{0}$	1	..	2′	Volts. 0	2	..	3′	Volts. 3	3	..	4′
6	4	..	5′	9	5	..	6′	12	6	..	7′	15	7	..	8′
18	8	..	9′	21	9	..	10′	24	10	..	11′	27	11	..	12′
30	12	..	13′	33	13	..	14′	36	14	..	15′	39	15	..	16′
42	16	..	17′	45	17	..	18′	48	18	..	19′	51	19	..	20′
54	20	..	21′	57	21	..	22′	60	22	..	23′	63	23	..	24′
66	24	..	25′	69	25	..	26′	72	26	..	27′	75	27	..	28′
78	28	..	29′	81	29	..	30′	84	30	..	31′	87	31	..	32′
90	32	..	33′	93	33	..	34′	96	34	..	35′	99	35	..	36′
102	36	..	37′	$\overset{+}{102}$	37	..	38′	102	38	..	39′	99	39	..	40′
96	40	..	41′	93	41	..	42′	90	42	..	43′	87	43	..	44′
84	44	..	45′	81	45	..	46′	78	46	..	47′	75	47	..	48′
72	48	..	49′	69	49	..	50′	66	50	..	51′	63	51	..	52′
60	52	..	53′	57	53	..	54′	54	54	..	55′	51	55	..	56′
48	56	..	57′	45	57	..	58′	42	58	..	59′	39	59	..	60′
36	60	..	61′	33	61	..	62′	30	62	..	63′	27	63	..	64′
24	64	..	65′	21	65	..	66′	18	66	..	67′	15	67	..	68′
12	68	..	69′	9	69	..	70′	6	70	..	71′	3	71	..	72′
..	72	..	1′	..	..	..	..	..	..	..	..	..	..	..	..

It will be noticed that no difference of pressure is shown in the table as due to the wires in the interior of the ring, these wires being only connecting or idle wires, and generate no difference of pressure.

The progressive increase of pressure from the – brush around the armature circuits to the positive brush is, with a table of this description, made quite clear. Further, it will be observed that the difference of pressure between adjacent coils is only that due to a single coil, and this constitutes one great advantage of single ring-wound armatures for high-pressure work ; the insulation required between individual conductors has only to withstand a small difference of pressure.

It is not found possible to design satisfactory ring armatures with two-circuit windings much above 15 in. in diameter for use in single-magnet two-pole fields, if the full output which such an armature can give in continuous work is taken from it. In practice serious difficulties arise at the commutator if this size is much exceeded with two-circuit ring windings. Satisfactory two-circuit ring armatures can, however, be made of larger diameter than this for use in double-magnet two-pole fields, but this point will receive attention later.

When a large current output has to be obtained, the size of the armature must be increased to provide the necessary space for conductors of greater cross-sectional area. If, with the same diameter, the length of the armature is increased, and the conductors, for a given voltage, reduced in number, a point is soon reached when difficulties are experienced in securing efficient ventilation in the body of the armature, and as a result of this deficient ventilation, the temperature of the armature is raised beyond what is considered advisable for continuous operation. On the other hand, the requisite number of conductors and commutator segments for a given voltage, if this lengthening is carried to an extreme, may be so few in number that the difference of pressure produced may not be sufficiently uniform in value. The design of the armature shaft is also a difficult matter under such conditions, since the interior space in the ring is limited.

The alternative and more usual course is therefore to increase the diameter of the armature, and this must be accompanied by modifications in the winding. One method, which has been frequently employed, is to wind alternately on the armature two distinct two-circuit windings. Each winding is joined to a separate commutator, one being placed

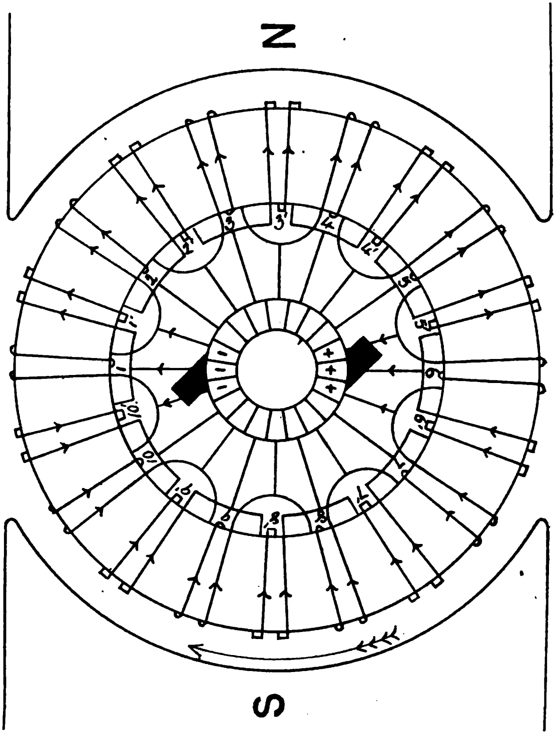


FIG. 34.—Two-circuit double ring winding in two-pole field.

on each side of the armature, and the two windings can be placed in parallel by cables, which connect like brushes together.

One commutator can be dispensed with by increasing the diameter of the remaining one, and doubling the number of segments in it, alternate segments being joined to alternate common junctions of the two windings, as in fig. 34. The thickness and bearing surface of the brushes is then increased, the two windings being thus placed in parallel by the thick brushes. The number of coils and segments on an actual armature would, of course, be many more than shown in fig. 34.

The number of individual windings may be still further increased. Thus three, four, &c., separate windings could be used, the great advantage of thus breaking up the windings being that each coil, when short-circuited, only carries $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, &c., of the current which it would carry in the case of a single winding. The disturbance of the field due to the short-circuiting of the coils is thus reduced in amount, although this disturbance takes place more frequently. Machines of large output, with a triple winding of this description, have been made, which perform quite satisfactorily. The practical limit to the sub-division of the windings in this manner appears to be the difficulty of maintaining the bearing surface of the brushes efficient over the requisite number of segments, and this limit appears to be approached with triple windings, and brushes bearing over more than two commutator segments.

Multipolar Windings (Parallel Wound).—For outputs greater than can be satisfactorily obtained in this manner, the modern practice is to still further increase the diameter of the armature. In place of a two-fold field, the number of poles is then increased to four, six, eight, ten, &c., according to the size and number of revolutions per minute of the machine. Such machines are known as multipolar dynamos.

Fig. 35 shows the single-wound ring armature (previously used in a two-pole field in fig. 30), placed in a four-pole field. In such a field, the coils at four points around the circumference of the armature move practically parallel to the lines of induction, and the difference of pressure induced in these coils falls to zero. A moment later the difference of pressure acting in these coils is reversed in direction. There are now, therefore, four neutral points in the armature winding, and four brushes are provided which divide the winding into four circuits. Brushes of the same sign can then be connected together by cables, and the four circuits placed in parallel. When so arranged, the difference of pressure produced between each pair of brushes is that due to one-fourth the total number of armature conductors

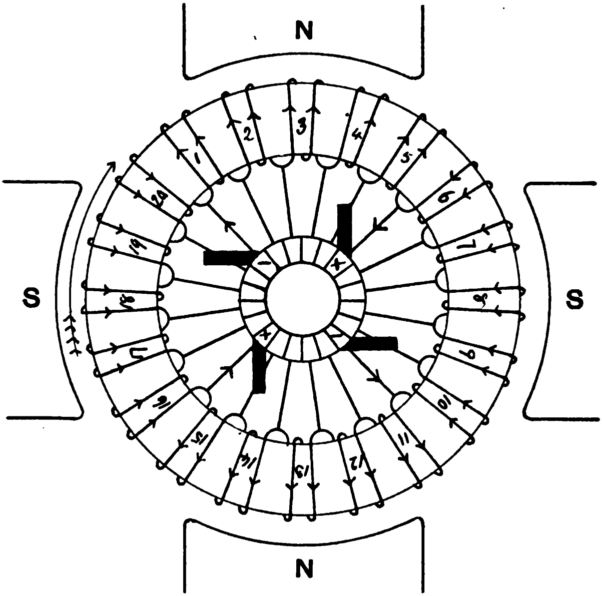


FIG. 35.—Four-circuit single ring winding in four-pole field.

in series, and the total current is the sum of the currents through the four circuits.

With a single winding the number of poles may be thus increased to six, eight, ten, &c., and the armature winding divided into six, eight, ten, &c., circuits by the employment of the same number of brushes as the field has poles, and an output of current of any desired magnitude obtained.

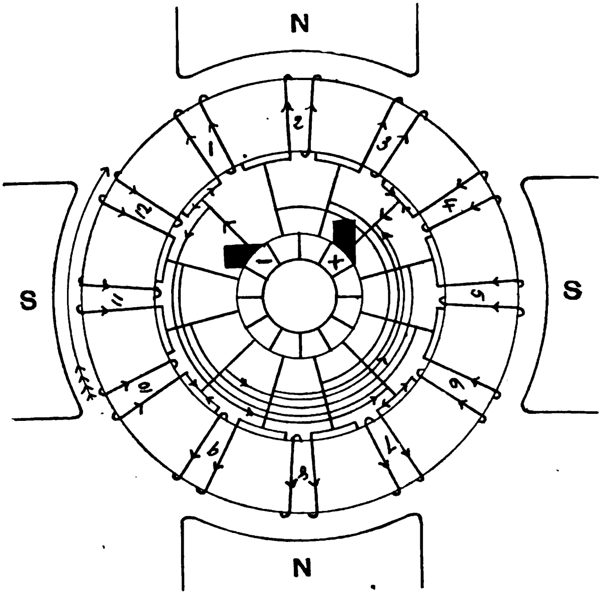


Fig. 36.—Four-circuit single ring winding in four-pole field. Single cross-connections.

In English practice it is common to cross-connect the commutator segments by a method due to Mordey, so as to permit of the employment of two brushes only, as in fig. 36; these cross-connections forming part of the armature, and simply join in parallel coils similarly situated in the various fields. In small machines this is undoubtedly advantageous, but it is questionable if any benefit is derived when the output is large. The difficulty usually is to secure sufficient bearing surface for the brushes on the surface of the commutator for the collection of the current, and if these are reduced to two sets, the length of the commutator must be increased to provide the requisite cross-sectional area of brush contact. This construction would therefore lead to a longer armature shaft, and the length of the machine would be increased. On the other hand, more work is required in the brush holders, &c., when no cross-connections are used, and it is a matter for

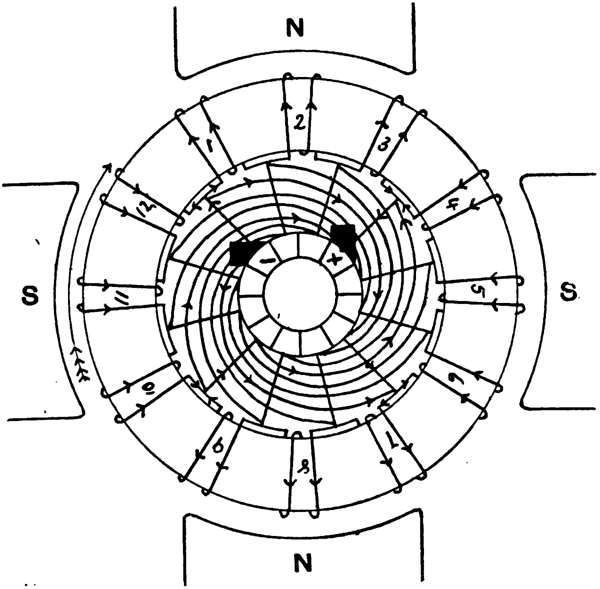


FIG. 37.—Four-circuit single ring winding in four-pole field. Double cross-connections.

careful consideration in any given case which method should be adopted.

The method of cross-connection shown in fig. 36 does not result in a well-balanced armature mechanically, and the modification shown in fig. 37 is preferable in this respect. Double cross-connections are employed, each common junction of the coils being directly connected to a segment, and also cross-connected to the segment directly opposite.

When cross-connected windings are used, the cross-connections should be made of relatively large cross-sectional area, as the cross-connections are in series with one circuit and this is joined in parallel with a circuit which has only the usual short connection to the commutator. The resistance of the two circuits will therefore be unequal unless some care is exercised in designing the cross-connections.

In practical work it is frequently not found possible to obtain the full-current output which can be taken from a given armature, more especially if the load varies very quickly, with a reasonable amount of attention at the brushes and commutator when a single winding is employed, even when placed in a multipolar field. Double or triple windings, each winding being quite distinct, are then resorted to; the number of commutator segments being also increased, and thick brushes join the different windings in parallel, precisely as in the case of the two-pole field, fig. 34. Fig. 38 represents diagrammatically a double-wound ring in a four-pole field. This modification can, of course, also be used with six, eight, &c., poles.

In the windings so far discussed the number of coils and segments is usually made a multiple of the number of poles to preserve a perfectly symmetrical arrangement, and in the double, triple, &c., wound rings, individual windings are quite distinct on the armature; they are only joined electrically at the brushes. If, however, in place of using a number of coils which is a multiple of the number of poles, the total number of coils for a double-wound ring is made an odd number by the addition of an extra coil, and the connection again made as in fig. 38, the end of a given coil being joined to the beginning of the next coil but one, and this carried out right round the armature, it will be found that, after going twice (for a double winding) round the armature, the end of the last coil, as shown in fig. 39, No. 20 is joined to the beginning of No. 1,

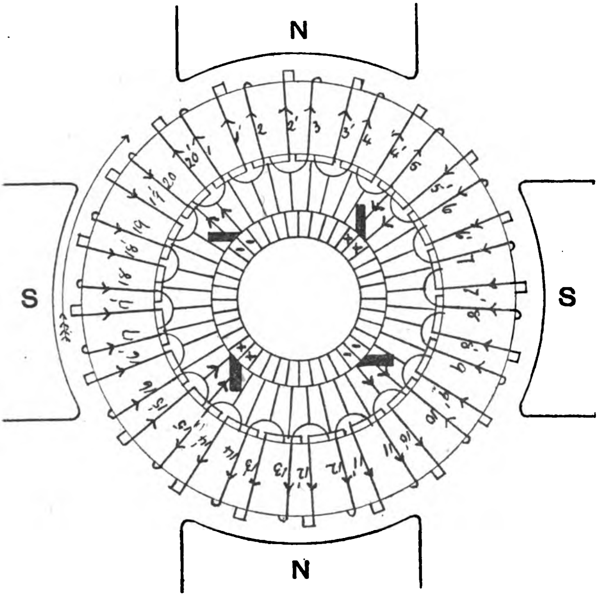


FIG. 38.—Four-circuit double ring winding in four-pole field.

and the winding made continuous. Thick brushes bearing over more than one segment (for a double winding) then join the winding into a four-circuit double winding ; but in this case any given coil does not, as in fig. 38, always form part of a separate winding, but changes four times per revolution from one winding to the other, being short-circuited under a brush at the time this change from one circuit to the other takes place. The departure from a perfectly symmetrical arrangement by the addition of an extra coil is not of very great importance with the large

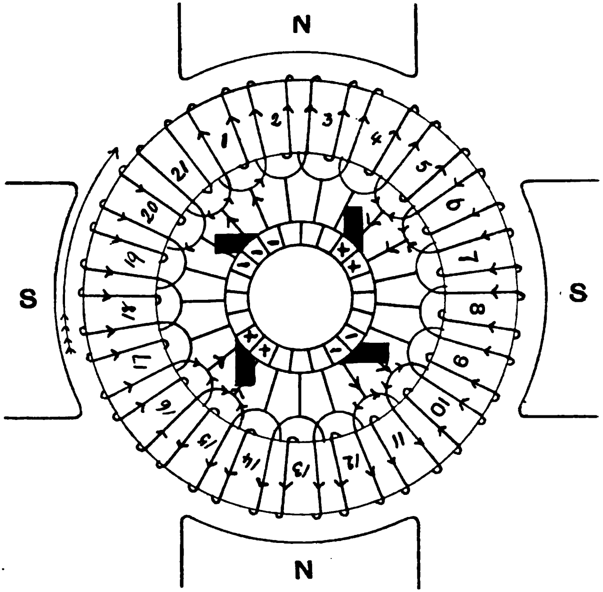


FIG. 39.—Four-circuit single re-entrant double ring winding in four-pole field.

number of coils employed in practice. Such modifications are frequently of use in enabling the number of conductors employed to be adjusted to given conditions of speed and difference of pressure.

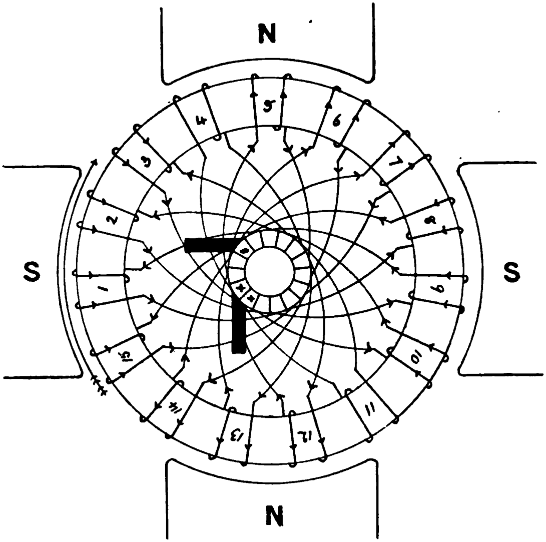
CHAPTER VII.

Multipolar Windings (Series Wound).—For large outputs of current, at a relatively low difference of pressure, parallel wound multipolar ring windings have been very largely used, the total current being the sum of the currents through the separate circuits constituting the armature winding. In place, however, of thus obtaining a large current output, it is possible to so group the ring windings used in multipolar fields that the difference of pressure produced at the brushes is the sum of the differences of pressure due to half the total number of armature conductors in series. Such windings are usually called series windings, and are used when a high difference of pressure is required at the brushes, with a small current, or when a very low speed of rotation is to be employed. Such windings, whether placed in four, six, eight, &c., pole fields, have, as a distinguishing feature from parallel wound rings, only two circuits through the armature winding.

Various methods of connection have been devised for grouping the conductors on multipolar rings so as to provide two-circuit windings, and in practice two methods are now usually followed. In one method coils in fields of the same polarity are connected together, and the conductors in series at any given instant are thus only situated under half the number of field poles—*i.e.*, half the conductors are under N. poles, and the other half under S. poles. The difference of pressure between adjacent coils is, however, small with this method of connection. In the second method, coils under fields of opposite polarity are connected together, and the conductors, joined in series, pass in succession under all the poles of the field. Between adjacent coils so connected the full difference of pressure of the machine periodically exists. A winding of this character, as pointed out by

Swinburne,* eliminates, however, any difficulties which may be caused by the wearing down of the shaft in the bearings, and the unequal loading of the conductors in consequence of the armature rotating about a centre slightly eccentric to the bore of the field poles.

Windings connected on either of these methods are extremely useful when the output of current can be successfully dealt with without having to unduly increase the length of the commutator to provide sufficient bearing surface for the brushes.



Figs. 40.—Two-circuit single ring winding in four-pole field. (Long-connection type)

For the first method of connection, a certain relation must exist between the number of coils and the number of poles. The number of commutator segments is again equal to the number of coils. If the number of pairs of poles employed is an even number, the number of coils must be an odd number; if the pairs of poles are odd, the number of coils may be odd or even. Thus if p = the number of poles in the field, n = the number of coils, and y = the pitch—*i.e.*, the number of coils to be passed over in joining up the coils—then

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* Journal, Institution of Electrical Engineers, vol. xx., page 303.

$$n = \frac{p\,y}{2}. \pm 1.$$

Fig. 40 shows a winding having 15 coils in a four-pole field joined up in this manner: The pitch $y = 7$, and the end of coil 15 is joined to the beginning of coil $15 + 7 = 22$; or, since there are only 15 coils on the armature, to $22 - 15$ = beginning of 7; end of coil 7 to beginning of coil $7 + 7 = 14$; end of coil 14 to beginning of coil $14 + 7 = 21 - 15 = 6$, until, finally, the winding is closed by end of coil 8 being joined to beginning of coil 15. The winding table for this armature, end exterior wires of coils being again denoted by 1, 2, 3, 4, &c., and beginning interior wires by 1′, 2′, 3′, 4′, &c., is as follow:—

WINDING TABLE FOR RING ARMATURE.

Two-circuit series ring winding for four-pole field.

15 coils, each of 2 turns, and 15 commutator segments.

Pitch = 7.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	15	..	7′	− ..	7	..	14′	..	14	..	6′	..	6	..	13′
..	13	..	5′	..	5	..	12′	..	12	..	4′	+ ..	4	..	11′
..	11	..	3′	+ ..	3	..	10′	..	10	..	2′	..	2	..	9′
..	9	..	1′	..	1	..	8′	..	8	..	15′	..	15	..	7′

The pitch y, for multipolar ring windings on this method, may be any number odd or even, provided care is taken, when substituted in the above equation, to add or subtract 1, so as to give an odd number of coils when an even number of pairs of poles is to be used. With the value of y taken, n can be either $\frac{4}{2}\,7+1=15$, or $\frac{4}{2}\,7-1=13$.

When series windings are used, the positions in which the positive and negative brushes can be fixed depend on the number of poles employed. With a four-pole field the angular distance between the brushes is obviously 90 deg. apart, since, if placed 180 deg. apart, both brushes would touch on segments having the same sign. In a six-pole machine, the brushes can be fixed 60 deg. apart, but at 180 deg. the segment has the same sign as at 60 deg. and the brushes could be placed 180 deg. apart, and usually this is done when high pressures are generated. Kapp* has prepared the following table showing the positions in which the brushes may be fixed for multipolar two-circuit windings.

Number of poles.	Angular distance between brushes.				
	Deg.	Deg.	Deg.	Deg.	Deg.
2	180	..	..	..	..
4	90	..	..	..	..
6	60	180	..	..	..
8	45	135	..	..	..
10	..	108	180	..	..
12	..	90	150	..	..
14	..	77	128	180	..
16	..	..	112	158	..
18	..	..	100	140	180
20	..	..	90	126	162

The smaller angles from 10 poles upwards are omitted in this table, since the object is to fix the brushes as far apart as possible for high-pressure machines.

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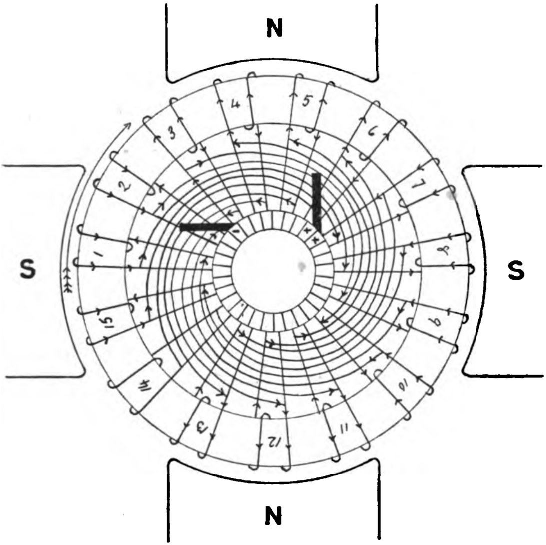
* Dynamos, Alternators, and Transformers, page 206.

With this type of winding the difference of pressure between adjacent segments on the commutator is that due to two coils in series, and two coils in series are alternately short-circuited at the positive and negative brushes, coils Nos. 4 and 11 occupying this position under the positive brush in fig. 40; at a later period coils 7 and 14 in series will be short-circuited under the negative brush, and so on alternately.



Figs. 41.—Two-circuit single ring winding in four-pole field. Number of commutator segments = twice number of coils. (Long-connection type.)

The difference of pressure between the segments can be reduced to that due to one coil only, and one coil only alternately short-circuited at each brush by doubling the number of commutator segments, and joining the two ends of each coil to adjacent segments, and then cross-connecting segments 180 deg. apart for a four-pole field. With a six-pole field, three times the number of segments may be used, and with an eight-pole field, four times the number of segments, or if s represents the number of segments in the commutator, then

$$s = \frac{p}{2} n,$$

and segments

$$\left(360 \div \frac{p}{2}\right)^{\circ}$$

apart must be cross-connected. With a six-pole field, segments 120 deg. apart must be thus cross-connected, and with an eight-pole field segments 90 deg. apart. Fig. 41 shows the two-circuit winding of fig. 40 modified in this manner, 30 segments being employed in the commutator, and coil No. 14 short-circuited under the positive brush. The connections for this winding are very regular, and it has been frequently employed in practice with good results.

In the second method of winding series wound armatures, the relation between the number of poles p, the number of coils n, and the pitch y is represented by the equation :—

$$n = py \pm 2,$$

y being any number, odd or even.

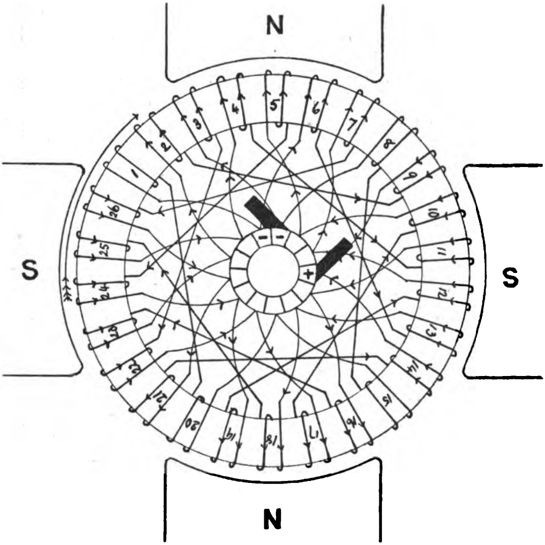
When, however, y is an even number, since n, as found by the above equation, is always even, if the connections were commenced from an even numbered coil, all the odd coils would never be joined in the circuit. It is therefore necessary to use, alternately in this case, pitches differing by 2—*i.e.* $(y - 1)$ and $(y + 1)$. Fig. 42 is an example of a winding of this character. The value taken for p is 4, and $y = 6$ (average).

$$\therefore n = py \pm 2 = 4 + 6 \pm 2 = 26 \text{ or } 22.$$

26 coils are shown, and the number of segments is

$$\frac{26}{2} = 13.$$

As previously remarked, the coils in series are situated under poles of alternate polarity, and in order that the differences of pressure due to the various coils may be added together it is necessary, if all the coils are wound in the same direction, to join the end of a coil situated, say, under a S. pole to the *end* of a coil situated under a N. pole, and the beginning of a coil under a N. pole to the *beginning* of a coil under a S. pole. These connections are usually made at the back and front ends of the armature respectively, and in fig. 42 connections at the back end are shown by straight lines, and at the front end by curved lines.



Figs. 42.—Two-circuit single ring winding in four-pole field.
(Short-connection type.)

In preparing a winding table for this armature, it is sufficient to number the coils successively round the armature, and arrange the table as follows :—

Winding Table for Ring Armature.

Two-circuit series ring winding for four-pole field.
26 coils, each of 2 turns, and 13 commutator segments.
Pitch = 7 and 5.

F	Coil	B	Coil	F	Coil	B	Coil	F	Coil	B	Coil	F	Coil	B	Coil
..	26	..	7	..	12	..	19	..	24	..	5	..	10	..	17
..	22	..	3	$\overset{-}{..}$	8	..	15	..	20	..	1	$\overset{-}{..}$	6	..	13
..	18	..	25	..	4	..	11	..	16	..	23	..	2	..	9
$\overset{+}{..}$	14	..	21	..	26	..	7	..	..	..	..	..	..	..	..

Thus commencing at coil 26, end of this coil is joined at the back to end of coil 7, beginning of coil 7 joined at the front to beginning of coil 12, end of 12 to end of 19 at the back, beginning of 19 to beginning of 24 at the front, until the winding is closed by beginning of coil 21 being joined at the front to beginning of coil 26. Coils 8, 15, 20, and 1 in series, are short-circuited under the negative brush in the position shown in fig. 42.

If the pitch y is chosen an odd number, then it is not necessary to vary the pitch alternately in making the connections. Thus, let $y = 7$, then $n = p\,y \pm 2 = 30$ or 26, and the winding table for 26 coils would be as follow :—

Winding Table for Ring Armature.

Two-circuit series ring winding for four-pole field.
26 coils, each of 2 turns, and 13 commutator segments.
Pitch = 7.

F	Coil	B	Coil	F	Coil	B	Coil	F	Coil	B	Coil	F	Coil	B	Coil
..	26	..	7	..	14	..	21	..	2	..	9	$\overset{+}{..}$	16	..	23
..	4	..	11	..	18	..	25	..	6	..	13	..	20	..	1
$\overset{-}{..}$	8	..	15	..	22	..	3	$\overset{-}{..}$	10	..	17	..	24	..	5
..	12	..	19	..	26	..	7	..	..	..	..	..	..	..	..

Coils 8, 15, 22, and 3 in series, are short-circuited under the negative brush.

To enable all the connections to be made at the front end of the armature and to avoid awkward crossings in the connections, it is better to wind alternate coils in opposite directions and to couple up then as in an ordinary ring the end of one coil to the beginning of the next. More space is, however, available by the former plan, at the front and back end alternately, and it is preferable to make the connections in this manner, as they are somewhat difficult to arrange under either plan.

For larger outputs of current than can be successfully dealt with at the brushes by a series two-circuit winding, it is usual to employ double or triple series two-circuit windings, and to place these windings in parallel by the use of thick brushes. Such windings are connected together as described for the first method of two-circuit series windings, coils under like poles being joined together. They either form two or more distinct windings, as in the parallel case of parallel wound rings shown in fig. 38, or, after passing round the armature a number of times, are finally joined to the beginning of the coil from which the coupling up was commenced and thus form a single closed winding similar to the parallel wound ring shown in fig. 39.

The formula for multiple windings of this type is:

$$n = \frac{p}{2} y \pm m.$$

where p = the number of poles, n = the number of coils, y = the pitch, and m = the number of windings.

Parshall and Hobart* group and express symbolically such multiple two-circuit windings (which they call long connection windings), under three heads as follows:—

I. When y and m are mutually prime, this gives a singly re-entrant winding of m multiple windings.

Thus, if $p = 4, y = 7, m = 4$;

then $n = \frac{4}{2} 7 + 4 = 18$;

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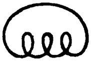
* Armature Windings of Electric Machines, page 47.

and 18 coils within a pitch of 7 would form a singly re-entrant quadruple two-circuit winding, and may be expressed symbolically as

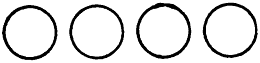


II. When y is a multiple of m, this gives m independently re-entrant two-circuit windings.

Thus, if $p = 4, y = 8, m = 4$;

then $n = \frac{4}{2} 8 + 4 = 20$;

and 20 coils with a pitch of 8 would form four independently re-entrant two-circuit windings, and may be expressed symbolically as



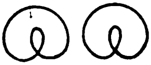
III. When y and m have common factors, this gives a number of independently re-entrant windings, equal to the greatest common factor of y and m.

Thus, if $p = 4, y = 6, m = 4$;

then $n = \frac{4}{2} 6 + 4 = 16$;

and 16 coils with a pitch of 6 would form a quadruple two-circuit winding, consisting of two independently re-entrant windings, since the greatest common factor of 4 and 6 is 2.

This may be expressed symbolically as



In the illustrations given, the number of coils has been taken small in order not to complicate the diagrams. The want of symmetry shown, more especially by the two-circuit series windings, is due in a great measure to this cause. With the large number of coils actually used this is reduced very greatly, and is usually quite negligable.

CHAPTER VIII.

Drum Armatures.—The simplest form of drum armature consists of a single loop, as shown in fig. 25, page 547, vol. xiv., two active conductors situated diametrically opposite to each other, and moving in fields of opposite polarity, being joined together in series by a connecting wire. To secure a difference of pressure constant in value at the brushes, it is again necessary to depart from this simple arrangement, and to arrange a number of conductors symmetrically around the circumference of the drum, so that some of the conductors are cutting lines at the maximum rate—*i.e.*, are moving at right angles to the lines, while others are moving parallel to the lines and cutting no lines of induction; the rate of cutting of intermediate conductors varying from a maximum to zero. These conductors are joined by connectors to form a closed winding, and in a two-pole field half the total number of conductors are joined in series, and the differences of pressure produced by them added together. The conductors are again made so numerous that the number in series is practically a constant number, and results in a difference of pressure, continuous in direction and constant in value, at the brushes.

As a distinguishing feature from a ring armature, the active conductors on a drum armature are joined in series by external connectors, which pass across the front and back ends of the drum respectively; no interior connectors being used. A very large number of modifications have been devised in the methods of winding and joining up the active conductors on drum armatures, in order to form a closed winding. The following examples are intended to represent diagrammatically typical windings, the slight modifications sometimes used being easily derived from those given. The number of conductors and commutator segments is purposely taken small to make the diagrams clear; practically many more are usually employed, and only single conductors, represented by small circles, are drawn, but each loop may, and usually in small machines does, consist of a number of complete turns wound side by side, before attachment to the commutator segments and succeeding loops. The direction of the induced differences of pressure, and with a closed outside circuit the direction of the currents through the active conductors, is shown by a dot representing the point of an arrow coming towards the observer, and a cross representing the tail of an arrow flying from him.

Connectors are shown by curved or straight lines, dotted lines for connectors at the back end of an armature, and full lines for connectors at the front end.

Drum Armature Windings.—Two distinct methods are usually employed in joining up the conductors on drum armatures. In one method the pitch is taken alternately forward at the back end, and backward at the front end. In the second method the pitch is always taken forward, both back and front. The first is usually called the "lap" method, and the second the "wave" method of winding.

Fig. 43 represents a two-circuit lap winding, with the conductors arranged in a single layer around the circumference of the armature in a two-pole field. The number of conductors taken is 16, and the number of loops formed, 8 (half the number of conductors), is an even number. The number of commutator segments is usually equal to, or a multiple of, the number of loops, and with an even number of commutator segments, if conductors diametrically opposite are connected to form loops, the winding produced is unsymmetrical. To obviate this the back end of a given conductor is joined by a connector to the back end of a conductor adjacent to the one which is diametrically opposite to the given conductor. Thus, in fig. 43, the end of conductor 16 is joined at the back to the end of conductor 7. This is joined at the front end to conductor 2, and their common junction connected to a commutator segment. Conductor 2 is joined at the back to conductor 9, and this is joined at the front to conductor 4, and also to the next commutator

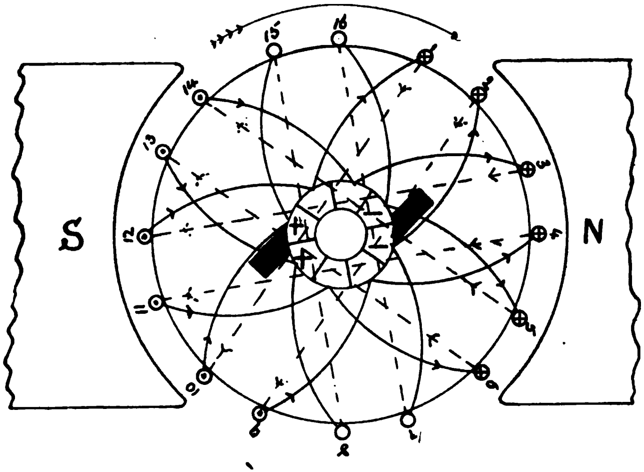


FIG. 43.—Two-circuit single drum winding in two-pole field. (Lap winding in a single layer, with even segments.)

segment, and so on; the pitch at the back end being thus 7, and at the front end – 5. The winding table for this winding follows:—

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single layer lap winding in two-pole field.

16 conductors and 8 commutator segments.

Back pitch 7. Front pitch – 5.

F	D	B	U	F	D	B	U	F	D	B	U
..	16	..	7	− ..	2	..	9	..	4	..	11
..	6	..	13	+ ..	8	..	15	+ ..	10	..	1
..	12	..	3	..	14	..	5	− ..	16	..	..

The loop formed by conductors 15 and 8 is short-circuited under the positive brush, and the loop formed by conductors 16 and 7 under the negative brush. It is, of course, not essential that the connections should be made at the back end by going round in a right-handed direction; the conductors may be numbered in the opposite direction, and the connections made left-handed, if desired.

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single layer lap winding in two-pole field.

100 conductors and 50 commutator segments.

Back pitch 49. Front pitch – 47.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
Volts		Volts		Volts		Volts		Volts		Volts		Volts		Volts	
0	100	0	49	0 –	2	0	51	0	4	0	53	3	6	6	55
9	8	12	57	15	10	18	59	21	12	24	61	27	14	30	63
33	16	36	65	39	18	42	67	45	20	48	69	51	22	54	71
57	24	60	73	63	26	66	75	69	28	72	77	75	30	78	79
81	32	84	81	87	34	90	83	93	36	96	85	99	38	102	87
105	40	108	89	111	42	114	91	117	44	120	93	123	46	126	95
129	48	129	97	129	50	129	99	129 +	52	129	1	129	54	129	3
129	56	126	5	123	58	120	7	117	60	114	9	111	62	108	11
105	64	102	13	99	66	96	15	93	68	90	17	87	70	84	19
81	72	78	21	75	74	72	23	69	76	66	25	63	78	60	27
57	80	54	29	51	82	48	31	45	84	42	33	39	86	36	35
33	88	30	37	27	90	24	39	21	92	18	41	15	94	12	43
9	96	6	45	3	98	0	47	0	100	..	..	..	..	..	..

The chief objection to this winding is that the full difference of pressure generated by the machine periodically exists between adjacent conductors. This can be well shown by a winding table, with the addition of the average difference of pressure generated per conductor. Taking numbers of conductors more usual in practice, say 100 conductors, with a back pitch of 49, and a front pitch of – 47, the following would represent the connections. The differences of pressures are added on the assumption that seven idle conductors exist at each neutral point, and that the average difference of pressure produced by the remaining active conductors is three volts per conductor. It may be as well to point out that these differences of pressure must be placed under both the columns marked B and F for a drum winding, since the conductors in columns D and U are both active and cutting lines of induction.

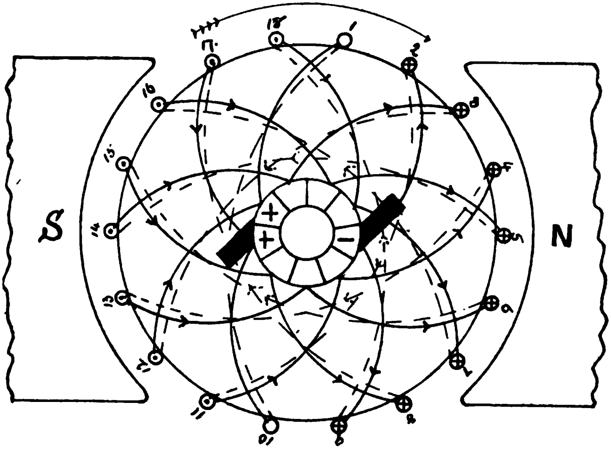


FIG. 44.—Two-circuit single drum winding in two-pole field. (Lap winding in a single layer, with uneven segments.)

As shown by the table, page 130, the difference of pressure for conductor No. 100 is zero, while for the conductor situated next to it, No. 1, the difference of pressure is 129 volts, and as the armature rotates this difference of pressure is in turn developed between all the conductors around the circumference of the armature. The conductors forming the winding must therefore not only be insulated from the core for the full difference of pressure produced, but insulation equally thick must be used to separate individual conductors from each other.

When the number of conductors used forms an odd number of loops, active conductors situated diametrically opposite can then be joined together, and form a symmetrical winding. Fig. 44 shows a winding having 18 conductors, and forming 9 loops with 9 commutator segments.

The winding table is as follows:—

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single layer lap winding in two-pole field.
18 conductors and 9 commutator segments.
Back pitch 9. Front pitch – 7.

F	D	B	U	F	D	B	U	F	D	B	U
..	18	..	9	− ..	2	..	11	..	4	..	13
..	6	..	15	..	8	..	17	+ ..	10	..	1
+ ..	12	..	3	..	14	..	5	..	16	..	7
..	18	..	..	..	..	..	..	..	..	..	..

The same objection applies to this winding, as regards the difference of pressure between adjacent coils, as to the previous one, and more crossing of wires at the ends ensues by winding two sides of a loop diametrically opposite in this manner. Both sides of the short-circuited loop are, however, diametrically opposite, and thus are perfectly symmetrical with the neutral line, and one loop only is short-circuited at the brushes at a given instant; in fig. 44 the loop formed by conductors 1 and 10 at the positive brush, this short circuiting of a loop taking place alternately at the positive and negative brushes.

With the object of reducing the crossing of the conductors at the front and back ends respectively of the armature, Swinburne has used a modification of these windings, the conductors joined together and forming loops, not being situated diametrically opposite or adjacent to the conductor diametrically opposite, but by the use of a smaller pitch than would give either of these forms, conductors at a smaller distance apart are joined to form loops. This form of winding is usually called *chord winding*, and Swinburne has pointed out that the demagnetising effect of the currents in the armature conductors on the field is also less than for the usual windings ; the currents in the conductors between the pole tips being alternately in opposite directions, and the magnetic effects of the currents in these conductors thus neutralise each other. Taking, as an example, 48 conductors, a winding of this character would be obtained by using pitches, say, of 19 at the back and − 17 at the front, in place of the more usual pitches of 23 and − 21.

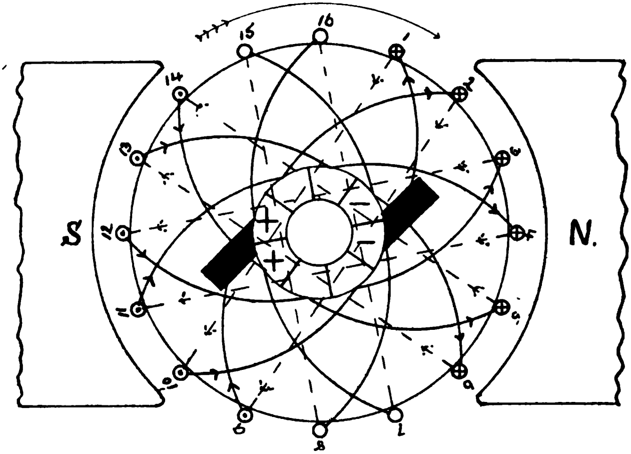


FIG. 45.—Two-circuit single drum winding in two-pole field. (Wave winding i a single layer, with even segments.)

The difference of pressure induced in some of the conductors in circuit with this form of winding is opposite in direction to that induced in the remainder in that circuit, and more loops would therefore be required to attain a given difference of pressure from this differential action of some of the loops. If the pitches are taken small, and the winding carried out over relatively short chords, the short-circuited loops are not situated near the neutral line, and the arc embraced by the pole pieces would probably then have to be made smaller than the usual practice, to obviate sparking at the commutator.

For lap windings, then, the total number of conductors counted all round the armature must be an even number; the pitches back and front must be odd numbers, differing from one another by 2 and opposite in sign, and for an even number of commutator segments the back pitch is usually 1 less than half the number of segments.

With a wave winding the pitch is always forward. Fig. 45 represents a winding of this character, with 16 conductors. The winding table is as follows:—

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single layer wave winding in two-pole field.

16 conductors and 8 commutator segments.

Pitch (back and front) 7.

F	D	B	U	F	D	B	U	F	D	B	U
..	16	..	7	+ ..	14	..	5	..	12	..	3
..	10	..	1	− ..	8	..	15	− ..	6	..	13
..	4	..	11	..	2	..	9	+ ..	16	..	..

The loop formed by conductors 16 and 7 is shown short-circuited under the positive brush, and by conductors 15 and 8 under the negative brush.

This form of winding, with an even number of commutator segments, has a slight structural advantage over an equivalent lap winding, since the connectors at the front and back ends of the armatures are of the same length, whereas with a lap winding the back connectors are longer than the front. For large current outputs these connectors are usually punched strips of copper, and a saving of labour results from all the connectors being of the same length.

This form of winding is subject, however, to the same fault as a lap winding; the full difference of pressure generated by the machine periodically exists between adjacent conductors.

When an odd number of segments are used, and conductors diametrically opposite joined to form loops, the front and back pitches must differ by 2; the pitches both being taken forward. The winding table for 18 conductors is as follows:—

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single layer wave winding in two-pole field.

18 conductors and 9 commutator segments.

Back pitch 9. Front pitch 7.

F	D	B	U	F	D	B	U	F	D	B	U
..	18	..	9	..	16	..	7	..	14	..	5
..	12	..	3	− ..	10	..	1	− ..	8	..	17
..	6	..	15	..	4	..	13	..	2	..	11
+ ..	18	..	..	..	..	..	..	..	..	..	..

On examination, however, this will be found to be exactly the same winding as a lap winding, in which the conductors are numbered in a left-handed direction round the armature, with a back pitch of 9 and a front pitch of − 7. With an uneven number of commutator segments there is therefore no difference between a two-circuit lap winding and a two-circuit wave winding.

The term "single-layer winding," used in describing these drum windings, is not to be taken to imply that the conductors or groups of conductors are arranged in a single layer on the circumference of the armature, or that only single loops, as shown in the diagrams, are formed; but that the winding, when completed, forms a layer over the circumference of the armature, adjacent conductors or groups of conductors being joined, not into the same circuit, but into each of the two circuits into which the armature winding is divided by the brushes on the commutator segments. For small machines these windings usually have two or more turns in a layer, and two or more layers deep before attachment to the commutator and the succeeding loops.

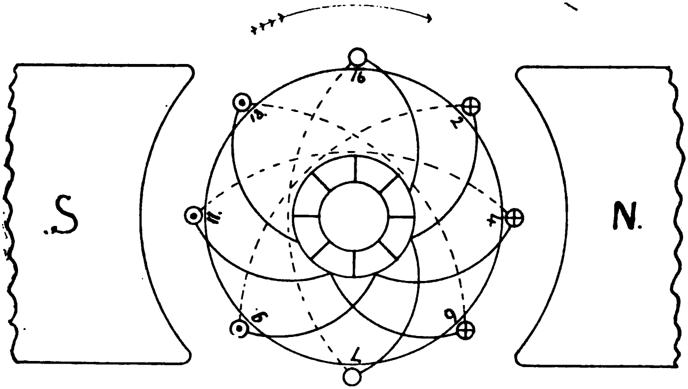


FIG. 46A.—Two-circuit single drum winding in two-pole field. (Lap winding in two layers.)

Two-layer Drum Armature Windings.—As pointed out on page 131, the full difference of pressure generated by the machine periodically exists between adjacent conductors of a drum winding when these are arranged in a single layer. This objection is removed, and, for a given winding, the difference of pressure between adjacent conductors made as small as possible, by arranging the conductors in two layers. In carrying out this method half the conductors are first placed in position, conductors diametrically opposite being usually joined to form loops; the remainder are next arranged directly over the first half, and all then joined up so as to form a closed winding.

Fig. 46A shows half the conductors joined up for this method of winding. Sixteen conductors have been taken in the example, the same number as already used in the single-layer winding on page 129, and shown in fig. 43. The back pitch is again 7, and the front pitch – 5; the winding table for this two-layer winding being exactly the same as that given on page 129 for the single-layer lap winding. Commencing at conductor 16, fig. 46A, this is joined at the back to conductor 7, now placed diametrically opposite; 7 is joined at the front to 2, and their common junction taken to a commutator segment; 2 is joined at the back to 9; 9 is joined at the front to 4, and also to the second segment; 4

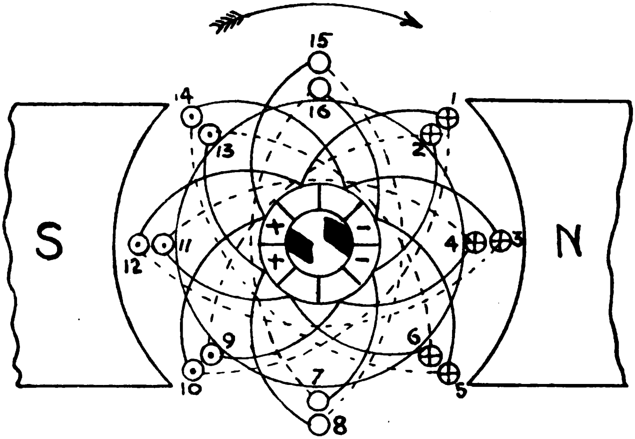


FIG. 46B.—Two-circuit single drum winding in two-pole field. (Lap winding in two layers.)

is joined at the back to 11; 11 is joined at the front to 6, and to the third segment; and 6 is joined at the back to 13. All the conductors in the first layer are now coupled up, and half the commutator segments have been used, precisely as in the case of the single-layer winding shown in fig. 43. Fig. 46B shows the winding completed; 13 (in the inner layer) is joined at the front to 8 (in the outer layer), and to segment 4; 8 is joined at the back to 15; 15 is joined at the front to 10, and to segment 5, and so on, until conductor 5 (outer layer) is joined at the front to conductor 16 (inner layer), and to segment 8, and the winding closed on itself. Instead of the loops for the second half being between those of the first half, as in fig. 43, they are arranged, in fig. 46B, in an outer layer.

The same winding table then can be used either for a single or two layer drum winding, but if the conductors are to be arranged in two layers, with conductors in the same loop diametrically opposite to each other, the total number of conductors must be a multiple of 4, whereas for a single-layer or for a two-layer lap winding this is not requisite; any even number of conductors can be chosen. (See fig. 44.)

The distribution of pressure between the various conductors can also be studied by aid of the winding table on page 130, since the total number of conductors taken, 100, is divisible by 4, and this number could be arranged in either a single layer or in two layers, with diametrically opposite conductors joined to form loops. Numbering the conductors in the manner shown in figs. 46A and 46B, conductors 6 and 8 (in the inner layer) are side by side. The difference of pressure given in the winding table for these conductors is 3 and 9 volts, or that due to a single loop only. Conductors 56 and 58 (in the outer layer) have a difference of pressure of 129 and 123 volts, again only that due to a single loop; and this is so for any two conductors situated next to each other, either in the inner or outer layers. Conductor 1, however, would be directly over conductor 2, and the difference of pressure between these conductors is 129 and 0 volts, the full difference of pressure of the machine.

With a drum winding, then, arranged in two layers, the difference of pressure between adjacent conductors is as small as possible, but the full difference of pressure generated by the machine is periodically developed between all the conductors in the inner and outer layers. The insulation between adjacent conductors is thus only subject to a small difference of pressure; but the two layers of conductors must be thoroughly insulated from each other. This, however, is not difficult to carry out, since, when half the conductors are in place, a thick sheet of insulation can be placed so as to entirely cover the inner layer, and the second layer then placed in position.

Two objections can be urged against a winding arranged in this manner: 1. Twice in each complete revolution of the armature all the conductors in the inner layer, joined in series, are placed in parallel by the brushes, with all the conductors in the outer layer also in series. Now, the outer layer is wound directly over the inner layer, and the conductors constituting the outer layer must therefore have a greater length than those in the inner layer. As the resistance of a conductor is directly proportional to its length, the resistance of the conductors in the outer layer will therefore be greater than that of those in the inner layer. Twice, then, in each revolution the two circuits through the armature joined in parallel by the brushes have unequal resistances, and the current will divide unequally through these two circuits; more passing through the circuit formed by the inner layer of conductors than through the circuit formed by the outer layer.

2. The radial distance of the conductors from the centre of the shaft is greater for the outer than for the inner layer, and hence the outer layer has a higher peripheral velocity than the inner layer. A proposal to eliminate the latter fault is, in making the connections, to join a loop, situated

in the inner layer, alternately with a loop in the outer layer, and so equalise the peripheral velocity. To render a winding of this character symmetrical, it is only possible to employ numbers of conductors which are odd multiples of 4. Thus with 8 conductors, forming 4 loops, the connections would be:—

End of loop 1 (inner layer) to beginning of loop 2 (outer layer).

End of loop 2 (outer layer) to beginning of loop 3 (*inner layer*).

But loop 3 is wound directly over loop 1, and is in the outer layer, and the required connections are thus not possible with an even multiple of 4.

A great objection to this proposed modification is that the connections are difficult to make to loops situated alternately in the outer and inner layers; further, the difference of pressure between adjacent conductors is as great as in a single-layer winding.

The modern method of overcoming these defects in a two-layer winding is to so arrange a loop that one conductor, forming part of that loop, is in the inner layer; and the second conductor, diametrically opposite, forming the other part of the loop, is in the outer layer. Fig. 47 shows this method, 16 conductors being taken, as before, with a back pitch of 7 and a front pitch of −5. To carry out in practice this disposition of conductors several plans have been devised, depending to some extent on the winding required—*i.e.*, whether each loop consists of several complete turns, or only of a single turn, before attachment to the commutator. These will be discussed later, when dealing with armature conductors.

In the two-layer windings so far discussed, diametrically opposite conductors are joined to form loops. This disposition of the separate conductors constituting the loops is, however, not essential for the carrying out of two-layer windings. By the choice of a suitable number of conductors and proper values for the back and front pitches, either a two-layer lap or wave winding would be arrived at. It is scarcely necessary to describe these two-layer windings further, as they are easily derived from those already given. It will suffice to remark that for a lap winding the number of conductors employed must be an even number, the back and front pitches odd numbers, differing by 2, and taken opposite in sign. Thus, using a back pitch of 49, and a front pitch of -47, 102 conductors could be arranged either

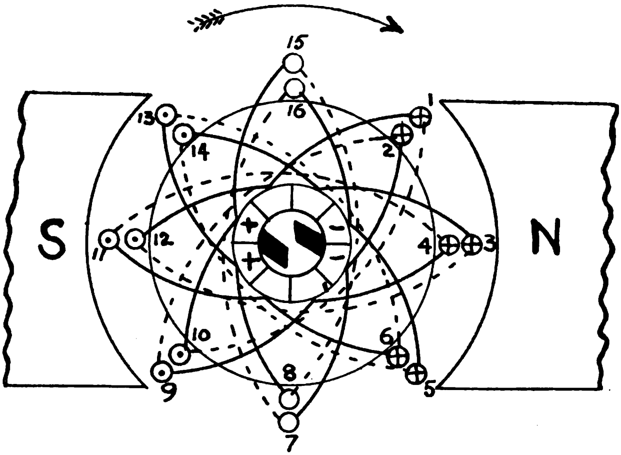


FIG. 47.—Two-circuit single drum winding in two-pole field. (Lap winding in two layers.)

as a single-layer or as a two-layer lap winding, with 51 commutator segments. For a two-pole wave winding the number of conductors n must be equal to $p \,.\, y \,.\, \pm 2$, where p = number of poles and y = the pitch. The pitch must again be an odd number. Sometimes the same pitch back and front is used; at others, back and front pitches differing by 2 (the average pitch being then an even number), but in either case back and front pitches are always taken forward, not alternately forward and backward, as in the case of a lap winding.

In two-layer drum windings the peripheral velocity of the conductors is thus equalised, and the length of the circuits in parallel for every part of a revolution maintained constant; while at the same time the difference of pressure between adjacent conductors in a layer is made small in value.

About the same limit as to diameter, given on page 108 for two-circuit ring armatures, for use in single-magnet two-pole fields, also holds in the case of two-circuit drum windings in similar fields; but from an armature of given diameter, when wound as a drum, about 50 per cent greater output can be obtained, with the same freedom from sparking, than if wound as a ring. If the diameter of the armature is increased much beyond this to obtain a larger output, then the winding must be modified, and two or more distinct drum windings are, in English practice, usually wound side by side on the armature; these separate windings being connected in order to a multiple commutator, in which the segments of the various windings occur in regular succession. Thick brushes then place these two or more windings in parallel. Triple-wound drum armatures of this type, in single-magnet two-pole fields, 24 in. diameter and 36 in. long, were successfully used in the machines made by Messrs. Siemens Bros. and Co. for the lighting of the Naval Exhibition of 1891, giving an output of 180 kilowatts (1,500 amperes and 120 volts) at 350 revolutions per minute.

Multipolar Drum Armatures.—With the extended use in practice of electrical energy for lighting and power transmission, the output of the dynamo machines used as generators has progressively increased, and this increase in output is necessarily accompanied by an increase in size, or in the peripheral speed of the armature conductors. Mechanical considerations tend to limit the peripheral velocity of a composite structure like a dynamo armature, and it does not appear possible, with present methods of construction, to increase, with due regard to safety, the peripheral velocity of the armature conductors to any great extent above those now usually employed. Large outputs are therefore much more usually obtained by increasing the diameter of the armature, so that more pole pieces may be presented to the conductors as they rotate, while still maintaining approximately the same peripheral velocity of the conductors—*i.e.*, the large armature makes a smaller number of revolutions per minute.

Excellent results have been attained in England, as already pointed out, with multiple-wound drum armatures in two-pole fields ; but with this modification it does not at present appear possible to obtain a much greater output than about 250 kilowatts, with a reasonable amount of attention to the commutator to secure freedom from sparking at the brushes, and recent practice has tended to the employment of multiple-circuit drum armatures in multipolar fields. On the Continent, however, large ring armatures with multipolar fields have been frequently employed, some examples (ten-pole) giving an output of about 600 kilowatts at 100 revolutions per minute, with a peripheral velocity of about 3,000 ft. per minute to the armature conductors.

It has usually been urged that ring-wound armatures are more easily wound and repaired (being free from the objectionable crossing of the conductors at the ends), and also offer more surface for ventilation than drum-wound armatures. This was undoubtedly the case for ordinary wire-wound armatures in two-pole fields, but when multipolar fields are employed these objections are practically removed ; and, with modern methods of arranging the armature conductors, drum armatures are quite as easily wound or repaired as ring armatures, while in practice better provision is also generally made for the internal ventilation of the armature core than in a ring-wound armature for the same output. In addition, the drum winding has the very great advantage of giving an output roughly 50 per cent greater than a ring winding from an armature of the same diameter with equal freedom from sparking at the brushes. Examples of such machines of large output are freely employed in

America in the power houses supplying electrical energy for electric traction, several being already in operation with an output of 1,500 kilowatts each.

Signs are not wanting that this type of machine will also be more used in England in the future as the individual output of each machine is increased. Messrs. Siemens Bros. and Co. have already supplied two machines of this type, each giving an output of 500 kilowatts, to the Corporation of Bradford; and a very large machine to be driven by an engine having a maximum indicated horse power of 3,000, at 100 revolutions per minute, is at the present time being built by the Electric Construction Co. for the Corporation of Manchester.

The armature windings used for such machines are, therefore, at the present time well worthy of careful study, and they will be discussed at some length to illustrate recent practice for large outputs. Usually four-pole fields will be used in the illustrations, but this is not intended to imply that only four-pole fields are used, precisely the same type of windings being just as applicable to six, eight, ten, &c., pole fields.

Multipolar Drum Armature Windings (Parallel Wound). The number of circuits through the armature conductors of a parallel-wound multipolar drum armature is equal to the number of poles of the field magnet. For a lap winding, the number of conductors chosen must be an even number; both pitches back and front must be odd numbers, differing by 2, and opposite in sign. The average pitch is usually taken not far different from the number obtained by dividing the total number of armature conductors by the number of poles, or, if n = number of armature conductors, and p = number of poles, then $\frac{n}{p}$ = approximately the average pitch. If the average pitch is taken greater than this, the end connections are unnecessarily long, more costly in copper, and the resistance of the armature winding is increased; hence more loss is caused in the conductors.

If taken shorter than this ratio, a chord winding is obtained, and with this disposition of the conductors forming the loops, the length of the end connectors, as well as the armature resistance, is reduced. So arranged, the current is alternately in opposite directions in the conductors near the neutral point, and hence the magnetic reaction of the armature current on the field is diminished. When the average pitch is taken much less than $\frac{n}{p}$, one (or more) of the conductors under a pole piece will have the difference of pressure induced in it in the opposite direction to that acting in the remainder of the conductors with which it is joined in circuit, hence subtracting from instead of adding to the total difference of pressure.

If the total number of conductors on the armature is made a multiple, odd or even, of the number of poles, each circuit through the armature will contain exactly the same number of conductors. In large machines each loop is usually formed by two conductors (one turn only), these lying under adjacent pole pieces, and the number of commutator segments is then equal to the number of loops. If the number of segments is an even multiple of the number of pairs of poles, commutation takes place at the positive and negative brushes simultaneously; if an odd multiple of the pairs of poles, alternately at the positive and negative brushes.

To secure perfect symmetry, then, the number of conductors employed should be made a multiple of the number of poles. In practice the number of conductors used is usually large, and frequently even numbers of conductors are used which are not exact multiples of the number of poles; and such slight departures from perfect symmetry are not found to cause any great inconvenience in the actual operation of the machine. In some of the windings shown diagrammatically this departure from a perfectly symmetrical arrangement is exaggerated, as, to make the diagrams clear, the conductors used are relatively few in number as compared to the number of circuits through the conductors.

Fig. 48 shows a four-pole parallel winding. In this and succeeding figures the short-numbered radial lines represent the active conductors; the inclined lines, exterior to these, the back-end connectors; and the interior-inclined lines, the front connectors and points of attachment to the commutator segments. Single loops are only shown, but in

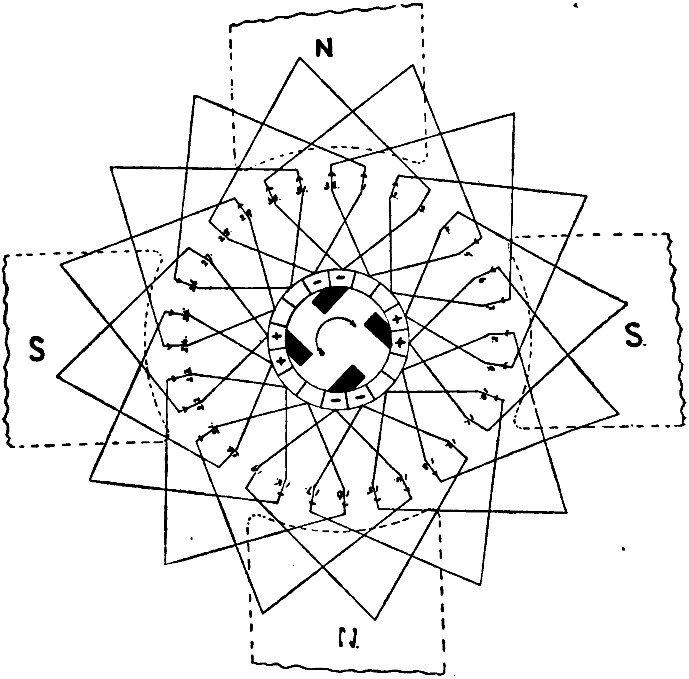


Fig. 48.—Four-circuit single drum winding in four-pole field. (Lap winding in a single layer.

some cases two or more complete loops may be formed before attachment to the commutator. The brushes are placed inside the commutator, so as not to foul the front connectors, and the dotted lines represent the pole pieces. The winding table is as follows:—

WINDING TABLE FOR DRUM ARMATURE.

Four-circuit single-layer lap winding for four-pole field.

32 conductors and 16 commutator segments.

Back pitch 7. Front pitch – 5

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	32	..	7	..	2	..	9	+ ..	4	..	11	+ ..	6	..	13
..	8	..	15	..	10	..	17	– ..	12	..	19	– ..	14	..	21
..	16	..	23	..	18	..	25	+ ..	20	..	27	+ ..	22	..	29
..	24	..	31	..	26	..	1	– ..	28	..	3	– ..	30	..	5
..	32	..	..	..	..	..	..	..	..	..	..	..	..	..	..

Conductors 4 and 11, 12 and 19, 20 and 27, and 28 and 3 are short-circuited in the position shown.

The number of segments, 16, is an even multiple of the pairs of poles, and commutation takes place simultaneously at all the brushes.

Now compare this table with that given for the two-pole winding, fig. 43, page 129. The first portion of the table is exactly the same in both. Conductor 10, however, is joined at the back to conductor 1, whereas now it is so joined to 17. Imagine two such windings as fig. 43, cut across, opened out to fit a larger diameter, and placed end to end. Continue then the connections, and it will be found that 10 is now joined to 1 of the second winding, or to 16 + 1 = 17, if the conductors are numbered round in rotation, and so for all the conductors, until all are joined into a continuous winding. Precisely the same winding as that given above is thus attained, but, as the diameter is doubled, four poles can now be presented to the conductors. A four-pole winding, then, on this method, is nothing more than the conductors on two two-pole windings combined on one armature in regular order, four brushes being now required to collect the current, as the number of commutator segments is also doubled, and four neutral points exist.

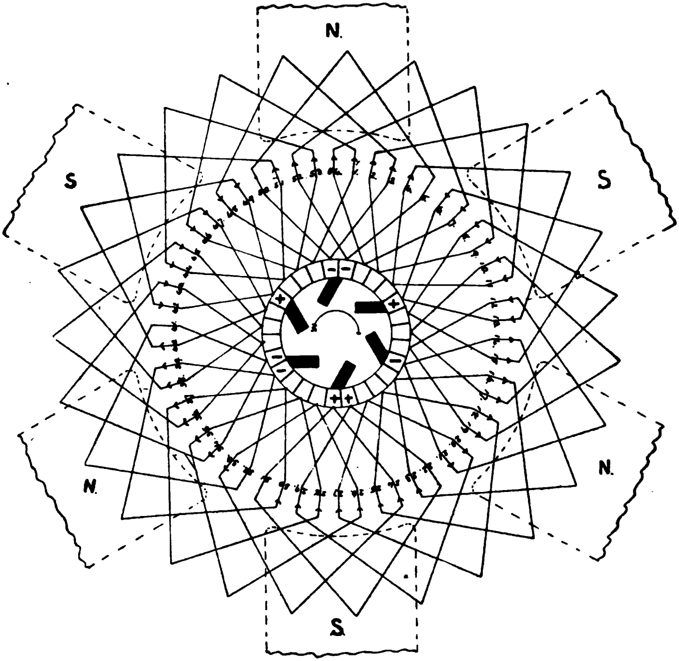


FIG. 49.—Six-circuit single drum winding in six-pole field. (Lap winding in a single layer.)

Fig. 49 shows a six-pole parallel winding. The winding table is as follows :—

WINDING TABLE FOR DRUM ARMATURE.

Six-circuit single-layer lap winding for six-pole field.

54 conductors and 27 commutator segments.

Back pitch 9. Front pitch − 7.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	54	..	9	..	2	..	11	..	4	..	13	+ ..	6	..	15
..	8	..	17	..	10	..	19	..	12	..	21	− ..	14	..	23
..	16	..	25	..	18	..	27	..	20	..	29	+ ..	22	..	31
+ ..	24	..	33	..	26	..	35	..	28	..	37	..	30	..	39
− ..	32	..	41	− ..	34	..	43	..	36	..	45	..	38	..	47
..	40	..	49	+ ..	42	..	51	..	44	..	53	..	46	..	1
..	48	..	3	− ..	50	..	5	− ..	52	..	7	..	54	..	..

Conductors 22 and 31, 32 and 41, and 50 and 5 are short-circuited in the position shown.

This is an example of the number of segments, 27, forming an odd multiple of the number of pairs of poles, and commutation will take place alternately at the positive and negative brushes, any two adjacent brushes being regarded as a pair at any given instant.

This winding is equivalent to three two-pole windings, as shown in fig. 44, page 131, opened out and placed end to end, as is evident on examining the winding table on page 132.

Both the preceding windings are perfectly symmetrical; any circuit through the armature windings contains the same number of active conductors as the other circuits constituting that winding. It is as well to select such windings for machines working on widely varying loads, especially if the total number of armature conductors required is not very great.

Fig. 50 will serve to illustrate the case of unequal numbers of conductors in the different circuits. The winding table is as follows:—

WINDING TABLE FOR DRUM ARMATURE.

Six-circuit single-layer lap winding in six-pole field.

50 conductors and 25 commutator segments.

Back pitch 9. Front pitch – 7.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	50	..	9	..	2	..	11	+ ..	4	..	13	+ ..	6	..	15
..	8	..	17	..	10	..	19	− ..	12	..	21	− ..	14	..	23
..	16	..	25	..	18	..	27	+ ..	20	..	29	+ ..	22	..	31
..	24	..	33	..	26	..	35	..	28	..	37	− ..	30	..	39
..	32	..	41	..	34	..	43	..	36	..	45	+ ..	38	..	47
..	40	..	49	..	42	..	1	..	44	..	3	− ..	46	..	5
− ..	48	..	7	..	50	..	..	..	..	..	..	..	..	..	..

Conductors 4 and 13, 12 and 21, 20 and 29, and 46 and 5 are short-circuited.

For the position shown in fig. 50, the circuits through the armature windings are as follow :—

−	3	44	1	42	49	40	47	38	+
	48	7	50	9	2	11	—	—	
−	30	39	32	41	34	43	36	45	+
	37	28	35	26	33	24	31	22	
−	19	10	17	8	15	6	—	—	+
	14	23	16	25	18	27	—	—	

Showing the irregularity of the circuits joined in parallel by the brushes. This irregularity, as before remarked, is exaggerated by the small number of conductors as compared to the circuits through the armature; but it would be wise to limit the use of such types to very large numbers of conductors.

The last two windings also serve to illustrate that the average pitch is not absolutely fixed by the total number of conductors taken and the number of poles in the field. It may be varied within reasonable limits, and still give a winding which is quite satisfactory in practical work.

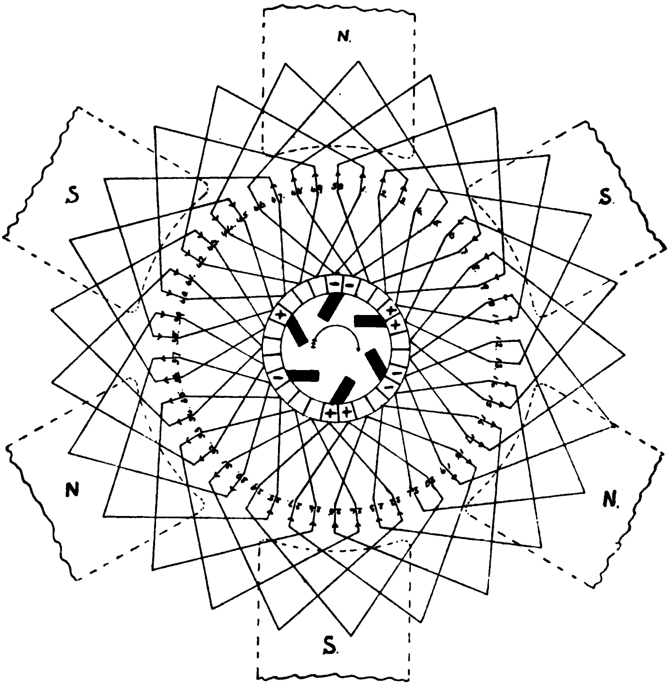


FIG. 50.—Six-circuit single drum winding in six-pole field. (Lap winding in a single layer.)

The three windings in figs. 48, 49, and 50 could also be just as well arranged in two layers. The conditions for small differences of pressure between adjacent conductors in a layer, and equilisation of the peripheral velocity of the conductors in the two layers, have been given in detail for two-pole windings in fig. 47, page 141. To obtain two-layer windings, all that is requisite is to imagine the odd-numbered conductors in figs. 48, 49, and 50 to be moved round and placed directly over the even-numbered ones. Conductors situated in the internal and external layers will then be alternately joined in circuit to form the loops. To show the distribution of pressure between conductors adjacent to each other, or situated in the inner and outer layers, the differences of pressure generated by the conductors are added in the winding table below, assuming that 44 conductors are active between any + or – brush, and that each conductor adds 2·5 volts. This table is also applicable to the case of a single-layer winding, and would then show the distribution of pressure between adjacent conductors in the same layer.

The conductors are supposed to be numbered as in fig. 47, page 141. The full difference of pressure generated by the machine then exists between 1 and 2, 51 and 52, 104 and 105, and 155 and 156, situated respectively in the inner and outer layers, and this difference of pressure will in turn be developed between all the conductors in the inner and outer layers. Adjacent coils in the same layer have only a difference of 5 volts between them—thus 83 and 85 in the inner layer, and 32 and 34 in the outer layer—and this holds good for all the conductors arranged round the circumference of the armature. When, therefore, two or more loops are wound side by side before attachment to the commutator, the insulation between the different conductors forming these loops has only to withstand a small difference of pressure, and will not need to be made thick; while between the inner and outer layers the insulation must be increased to withstand the full difference of pressure generated by the machine.

WINDING TABLE FOR DRUM ARMATURE.

Four-circuit two-layer lap winding in four-pole field.

204 conductors and 102 commutator segments.

Back pitch 51. Front pitch – 49.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
V'lts		V'lts		V'lts		Vl'ts		Vl'ts		V'lts		V'lts		V'lts	
110	204	110	51	110 +	2	110	53	110 +	4	110	55	110	6	110	57
107·5	8	105	59	102·5	10	100	61	97·5	12	95	63	92·5	14	90	65
87·5	16	85	67	82·5	18	80	69	77·5	20	75	71	72·5	22	70	73
67·5	24	65	75	62·5	26	60	77	57·5	28	55	79	52·5	30	50	81
47 5	32	45	83	42·5	34	40	85	37·5	36	35	87	32·5	38	30	89
27·5	40	25	91	22·5	42	20	93	17·5	44	15	95	12·5	46	10	97
7·5	48	5	99	2·5	50	0	101	0	52	0	103	0 –	54	0	105
0	56	0	107	2·5	58	5	109	7·5	60	10	111	12·5	62	15	113
17·5	64	20	115	22·5	66	25	117	27·5	68	30	119	32·5	70	35	121
37·5	72	40	123	42·5	74	45	125	47·5	76	50	127	52·5	78	55	129
57·5	80	60	131	62·5	82	65	133	67·5	84	70	135	72·5	86	75	137
77·5	88	80	139	82·5	90	85	141	87·5	92	90	143	92·5	94	95	145
97·5	96	100	147	102·5	98	105	149	107·5	100	110	151	110	102	110	153
110 +	104	110	155	110 +	106	110	157	110	108	110	159	107·5	110	105	161
102·5	112	100	163	97·5	114	95	165	92·5	116	90	167	87·5	118	85	169
82·5	120	80	171	77·5	122	75	173	72·5	124	70	175	67·5	126	65	177
62·5	128	60	179	57·5	130	55	181	52·5	132	50	183	47·5	134	45	185
42·5	136	40	187	37·5	138	35	189	32·5	140	30	191	27·5	142	25	193
22·5	144	20	195	17·5	146	15	197	12·5	148	10	199	7·5	150	5	201
2·5	152	0	203	0	154	0	1	0 –	156	0	3	0	158	0	5
2·5	160	5	7	7·5	162	10	9	12·5	164	15	11	17·5	166	20	13
22·5	168	25	15	27·5	170	30	17	32·5	172	35	19	37·5	174	40	21
42·5	176	45	23	47·5	178	50	25	52·5	180	55	27	57·5	182	60	29
62·5	184	65	31	67·5	186	70	33	72·5	188	75	35	77·5	190	80	37
82·5	192	85	39	87·5	194	90	41	92·5	196	95	43	97·5	198	100	45
102·5	200	105	47	107·5	202	110	49	110	204	..	..	..	..	..	..

The brushes shown for these drum multipolar parallel windings are equal to the number of poles in the field. Two brushes only can be used for any number of poles, by the device of cross-connecting together such commutator segments as are at the same difference of pressure, as already described on pages 112 and 113, for ring multipolar parallel windings. The remarks there made as to the difficulty of securing sufficient cross-sectional area of brush contact on the commutator are equally applicable to drum multipolar windings. If cross-connections are used, the number of conductors chosen must be a multiple of the number of poles.

For parallel-wound drum, multipolar single or two-layer wave windings, the number of conductors n must be equal to $p\,y \pm p$, where p = the number of poles and y = the pitch. Thus, if $p = 6$, and $y = 7$, then $6 \times 7 \pm 6 = 48$, or 36 conductors would be employed. As an example, a winding table is given below. It is scarcely necessary to illustrate it separately, as the only difference is that the pitch is always taken forward.

WINDING TABLE FOR DRUM ARMATURE.

Six-circuit single-layer wave winding in six-pole field.

48 conductors and 24 commutator segments.

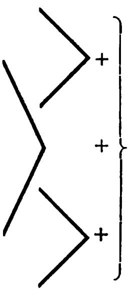
Pitch (front and back) = 7.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	48	..	7	+ ..	14	..	21	+ ..	28	..	35	..	42	..	1
..	8	..	15	− ..	22	..	29	− ..	36	..	43	..	2	..	9
..	16	..	23	+ ..	30	..	37	+ ..	44	..	3	..	10	..	17
..	24	..	31	− ..	38	..	45	− ..	4	..	11	..	18	..	25
..	32	..	39	+ ..	46	..	5	+ ..	12	..	19	..	26	..	33
..	40	..	47	− ..	6	..	13	− ..	20	..	27	..	34	..	41
..	48	..	..	..	..	..	..	..	..	..	..	..	..	..	..

Conductors 14 and 21, 30 and 37, 46 and 5 are short-circuited under the positive brushes; and 22 and 29, 38 and 45, and 6 and 13 under the negative brushes.

The circuits through the winding and the distribution of pressure between the conductors are as follow :—

	5	10	15	20	25	30	Volts.
−	15	8	1	42	35	28	
	36	43	2	9	16	23	+
−	20	27	34	41	48	7	+
	47	40	33	26	19	12	
−	31	24	17	10	3	44	+
	4	11	18	25	32	39	



Conductors situated vertically below one another in the above being at the same difference of pressure, and each is supposed to add 5 volts to the pressure. The distribution between adjacent conductors for a single layer is periodically that of the full difference of pressure generated by the machine; if arranged in two layers between adjacent conductors, a maximum of that due to a single loop, but the full difference of pressure periodically between inner and outer layers.

Back and front pitches, differing by 2, may also be used, but always taken in the positive direction. The pitch to be inserted in the above equation for number of conductors is then the average of front and back pitches.

Multiple-wound, Multipolar Drum Armature Windings.—For large and widely varying outputs of current, it is sometimes not found possible to secure, with a given armature and number of revolutions per minute, the requisite freedom from sparking at the brushes as the load varies, when employing the single-wound multipolar drum windings already described. To surmount this difficulty double, triple, &c., drum multipolar windings are then frequently resorted to. Parshall and Hobart* classify these lap windings thus:—

Let m = the number of windings (double, triple, &c.); then these m windings may form one re-entrant winding; m independent re-entrant windings; or a number of re-entrant windings equal to some factor of m, each of which re-entrant windings is composed of two or more components. They give the following rule for the proper number of conductors: If m equals the number of windings, and n equals the number of conductors, then the number of independently re-entrant windings will be equal to the greatest common factor of $\frac{n}{2}$ and m.

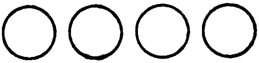
For lap windings of this character the number of conductors, n, must be an even number; but as is evident from the rule given above, the number of windings, m, for these multiple-wound multipolar drums, is independent both of the number of poles and of the pitch.

Back and front pitches must both be odd numbers, and must differ by $2m$, where m equals the number of windings (double, triple, &c.).

As examples, let a quadruple winding be required; then $m = 4$. With $n = 304$, the greatest common factor of

$$\frac{n}{2} = \frac{304}{2} = 152, \text{ and } m = 4,$$

is 4; and the quadruple winding will consist of four independent windings, or, expressed symbolically,



With $n = 302$, the greatest common factor of

$$\frac{n}{2} = \frac{302}{2} = 151, \text{ and } m = 4,$$

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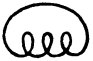
* "Armature Windings of Electric Machines."

is 1; and the quadruple winding will consist of a singly re-entrant quadruple winding; or, symbolically


.

With $n = 300$, the greatest common factor of

$$\frac{n}{2} = \frac{300}{2} = 150, \text{ and } m = 4,$$

is 2; and the quadruple winding will consist of two

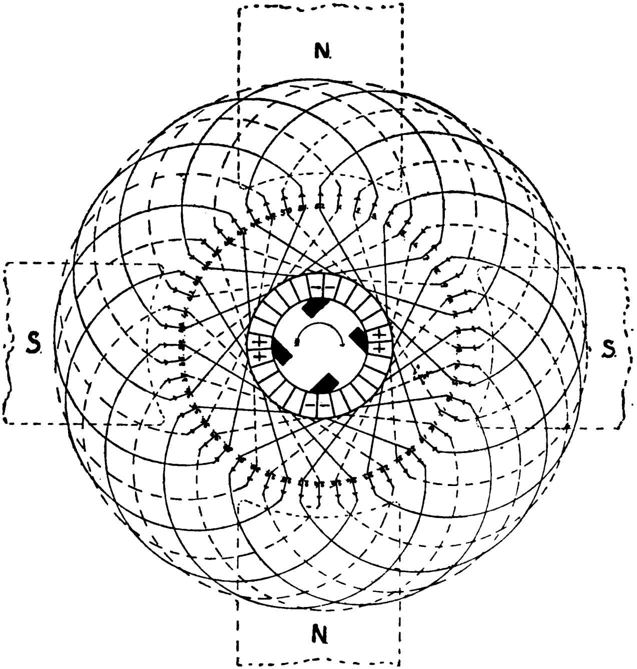
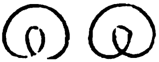


FIG. 51.—Four-circuit double drum winding in four-pole field. (Lap winding in a single layer.)

independent double windings, each of the double windings being re-entrant; or, symbolically,



The number of conductors, n, should be chosen so that the number of poles, multiplied by the average pitch, is somewhere nearly equal to n; and in practice, to make the end connectors short, and to keep down the resistance of the winding, the number of conductors, n, is usually taken equal to, or preferably rather more than, the number obtained by multiplying the average pitch by the number of poles.

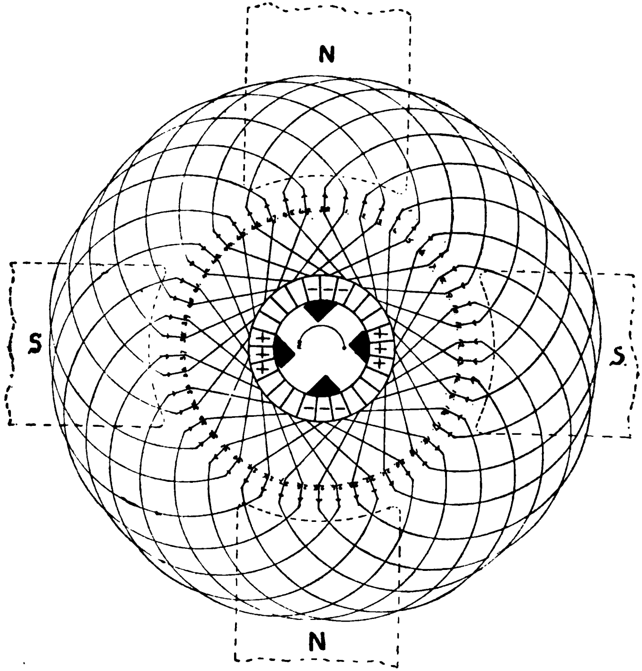


FIG. 52.—Four-circuit triple drum winding in four-pole field. (Lap winding in a single layer.)

Fig. 51 represents a four-circuit double winding. The radial lines again represent the active conductors. The back connectors are shown by the semicircles outside the radial lines, and the interior inclined lines show the front connectors and the points of attachment to the commutator segments. Each winding is quite distinct on the armature, but joined in parallel by the thick brushes on the commutator, and the two separate windings are distinguished by the front and back connectors being made in full lines and in dotted lines. The winding shown is not symmetrical as regards each circuit through the winding containing the same number of conductors, and this want of symmetry is, of course, much exaggerated by the small number of conductors taken. To secure perfect symmetry in this respect it would be necessary to use numbers of conductors, such that $\frac{n}{2}$ (for a double winding) is a multiple, odd or even, of the number of poles. The winding table is as follows :—

WINDING TABLE FOR DRUM ARMATURE.
Four-circuit double winding for four-pole field
52 conductors and 26 commutator segments.
Back pitch 15. Front pitch – 11.

Re-entrancy ○ ○

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	52	..	15	..	4	..	19	+ ..	8	..	23	..	12	..	27
..	50	..	13	..	2	..	17	+ ..	6	..	21	..	10	..	25
..	16	..	31	− ..	20	..	35	..	24	..	39	..	28	..	43
..	14	..	29	..	18	..	33	− ..	22	..	37	..	26	..	41
+ ..	32	..	47	..	36	..	51	..	40	..	3	..	44	..	7
..	30	..	45	+ ..	34	..	49	..	38	..	1	..	42	..	5
− ..	48	..	11	..	52	..	..	..	..	..	..	..	..	..	..
− ..	46	..	9	..	50	..	..	..	..	..	..	..	..	..	..

The position of the brushes in the above table will serve to emphasise the irregularity of the winding with the number of conductors used. With the large numbers usually employed in practice, this defect would be much reduced.

Fig. 52 shows a four-circuit triple winding. The winding table is as follows :—

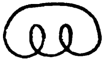
WINDING TABLE FOR DRUM ARMATURE.

Four-circuit triple winding for four-pole field.

50 conductors and 25 commutator segments.

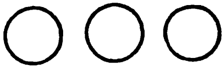
Back pitch 15. Front pitch − 9.

Re-entrancy



F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	50	..	15	+ ..	6	..	21	..	12	..	27	− ..	18	..	33
..	24	..	39	+ ..	30	..	45	..	36	..	1	.	42	..	7
− ..	48	..	13	..	4	..	19	+ ..	10	..	25	..	16	..	31
− ..	22	..	37	..	28	..	43	+ ..	34	..	49	..	40	..	5
− ..	46	..	11	..	2	..	17	+ ..	8	..	23	..	14	..	29
− ..	20	..	35	..	26	..	41	+ ..	32	.	47	..	38	..	3
− ..	44	..	9	..	50	..	..	..	..	..	..	..	..	..	..

It is not possible to arrange a perfectly symmetrical singly re-entrant triple winding. If in any given case, from the comparatively small number of conductors required and the conditions under which the machine is to work as regards freedom from sparking under varying loads, perfect symmetry in the different circuits through the armature winding is deemed advisable, this condition would be perfectly attained by choosing a triply re-entrant triple winding, symbolically



the conductors in each winding being made a multiple of the number of poles.

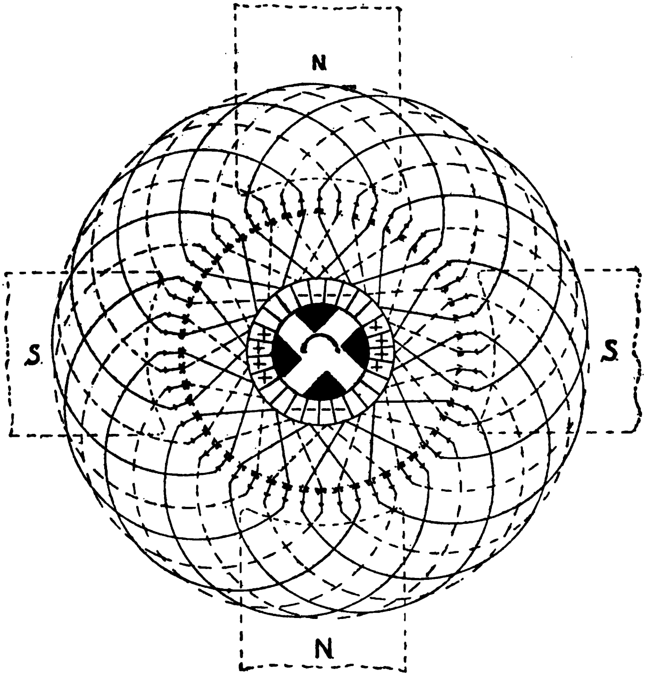


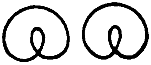
FIG. 53.—Four-circuit quadruple drum winding in four-pole field. (Lap winding in a single layer.)

Fig. 53 represents a four-circuit quadruple winding, and the winding table corresponding to this is given on page 162.

WINDING TABLE FOR DRUM ARMATURE.

Four-circuit quadruple winding for four-pole field.
52 conductors and 26 commutator segments.
Back pitch = 15. Front pitch = 7.

Re-entrancy



F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	52	..	15	+ ..	8	..	23	..	16	..	31	– ..	24	..	39
..	50	..	13	+ ..	6	..	21	..	14	..	29	– ..	22	..	37
+ ..	32	..	47	..	40	..	3	– ..	48	..	11	..	4	..	19
..	30	..	45	+ ..	38	..	1	– ..	46	..	9	..	2	..	17
+ ..	12	..	27	– ..	20	..	35	..	28	..	43	+ ..	36	..	51
+ ..	10	..	25	..	18	..	33	– ..	26	..	41	+ ..	34	..	49
..	44	..	7	– ..	52	..	..	..	..	..	..	..	..	..	..
..	42	..	5	– .	50	..	..	..	..	..	..	..	..	..	..

Since, in this example,

$$\frac{n}{2} = \frac{52}{2} = 26,$$

and

$$m = 4,$$

and the greatest common factor of 26 and 4 is 2; a doubly re-entrant quadruple winding results. The two distinct windings are shown in fig. 53 by the full lines and dotted lines representing the end connectors.

In the examples given in figs. 51, 52, and 53, the numbers of conductors used have been purposely taken, so as to illustrate unsymmetrical windings. As already pointed out, any even number of conductors can be used; but when the total number required is comparatively small, then the type of winding and the number of conductors should be chosen, so as to give a symmetrical winding. For very large numbers of conductors, however, this slight want of symmetry is usually not of very great importance.

These windings can again be arranged on the surface of the armature in a single or double layer as required. It is not necessary to deal further here with these modifications; they have already been fully dealt with in the examples previously given of single windings.

In practical work, at the present time, double windings are frequently employed. Triple windings have also been used with success, but this appears to form the practical limit to the employment of these multiple windings at present. The difficulty is to keep the thick brushes bearing over the requisite number of segments on the commutator. As the surface of the brush slowly wears away in use, the brush holders, carrying the brushes, are caused to partially rotate on their supporting spindles, by means usually of springs, and thus preserve the contact of the brush with the commutator. This slight circular rotation of the holders causes the brush to bear on its toe, and its heel to move out of contact with the surface of the commutator, and sparking takes place under the brush. If some means could be devised of mounting the brushes so that they advance radially to the surface of the commutator, this defect would be removed, and it should be possible to employ more windings placed in parallel in this manner. At present, however, no simple and reliable method of effecting this appears to have been introduced.

Multipolar Drum Armature Windings (Series Wound).—All the multipolar drum windings so far given have as many circuits through the winding as the field has poles, the number of conductors joined in series from brush to brush depending on the number of poles employed, being one-fourth, one-sixth, one-eighth, &c., of the total number used.

As in the case of ring multipolar windings, page 117, so with drum multipolar windings, it is again possible to group the conductors so that only two circuits are formed and half the total number of conductors are placed in series, independently of the number of poles employed. For a winding of this character, the pitch back and front is always taken forward, a conductor under one pole piece being joined to a conductor situated under the next pole piece, and this, in turn, to a conductor under the third pole piece, and so on, a conductor under every pole piece being joined in circuit before a second conductor under the first pole piece is again joined in circuit. This method of connection is continued round and round the armature until all are joined in circuit, and a closed wave winding is obtained.

These windings have also the advantage already pointed out on page 118, for equivalent ring windings, of eliminating any unequal loading of the conductors by the wearing down of the shaft in the bearings; the conductors joined in series being acted on by all the poles of the field. For small current outputs with a high difference of pressure, or when a very low number of revolutions per minute is desired, these windings have been frequently and successfully employed, two sets of brushes being then only used (see table, page 120). With very large current outputs, the length of the commutator must either be increased when only two sets of brushes are used, to provide the requisite brush area for the collection of the current, thus greatly increasing the cost of the commutator; or as many sets of brushes must be employed as the field has poles. In the latter case, notwithstanding the equalisation of the currents by this construction through the conductors forming the armature circuits, sparking difficulties are experienced from the unequal division of current between different sets of brushes of the same sign. If the density of the fields could always be made exactly equal at all the poles, and equality in the contact resistance of the brushes on the surface of the commutator could always be maintained, equal sub-division of the current amongst the various brushes of the same sign would be assured. In practical work these conditions are never perfectly realised, and independent adjustment of the various sets of brushes of the same sign relatively to each other is required; the difficulty of such individual adjustment of the sets of brushes of the same sign relatively to each other becoming more troublesome to carry out as the number of poles employed in the field is increased. With two-circuit multipolar drum windings, neither the magnetic reaction on the field of the current in the armature conductors, or the fall of pressure from the resistance of the conductors, assists in preventing this unequal division of the current through brushes of the same sign; whereas, with multiple circuit windings both these effects tend to prevent unequal division through the different circuits, and thus also through brushes of the same sign. Probably for these reasons, with very large current outputs, multiple circuit armatures are in present practice usually used, although more costly than two-circuit armatures. For a two-circuit multipolar winding, the number of conductors is $\frac{2}{p}$ times the number required for a multiple-circuit winding, p representing the number of poles used; the smaller number of large conductors required for a two-circuit winding costing less for insulation than a larger number of smaller sectional area, and the number of commutator segments is also reduced in the same proportion as the number of conductors.

For series-wound multipolar drum armature windings, with a single winding, the relation between the number of conductors n, the number of poles p, and the pitch y is expressed by the equation—

$$n = py \pm 2.$$

The pitch y is always taken forward, and frequently has the same value at the back and front. It must be an odd number, and if the same pitch is used back and front, this tends to restrict somewhat the available number of conductors that can be employed. This difficulty can be obviated by using back and front pitches differing by 2, so that the average pitch is an even number. No benefit is derived from this device with four-pole fields, but with six, eight, &c., poles the choice is not so limited. As an example, take a six-pole field, and pitches respectively of 20, 21, 22, 23, 24, 25, 26; then, substituting these values in the equation $n = py \pm 2$, the number of conductors that can be used are—

$y = 20, \quad n = 6 \times 20 \pm 2 = 118$ or 122.
$y = 21, \quad n = 6 \times 21 \pm 2 = 124$ or 128.
$y = 22, \quad n = 6 \times 22 \pm 2 = 130$ or 134.
$y = 23, \quad n = 6 \times 23 \pm 2 = 136$ or 140.
$y = 24, \quad n = 6 \times 24 \pm 2 = 142$ or 146.
$y = 25, \quad n = 6 \times 25 \pm 2 = 148$ or 152.
$y = 26, \quad n = 6 \times 26 \pm 2 = 154$ or 158.

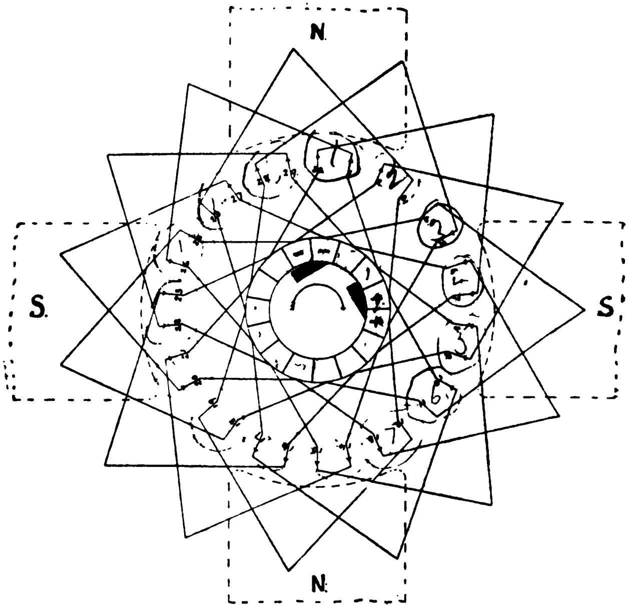


FIG. 54.—Two-circuit single drum winding in four-pole field. (Wave winding in a single layer, with equal pitches, back and front.)

Thus with a six-pole field, and the same pitch back and front, the number of conductors that could be used would be limited to those given by $y = 21$ (124 or 128), $y = 23$ (136 or 140), $y = 25$ (148 or 152); whereas, by using back and front pitches differing by 2, the numbers given by $y = 20$ (118 or 122), $y = 22$ (130 or 134), $y = 24$ (142 or 146), and $y = 26$ (154 or 158), could also be employed, if desired.

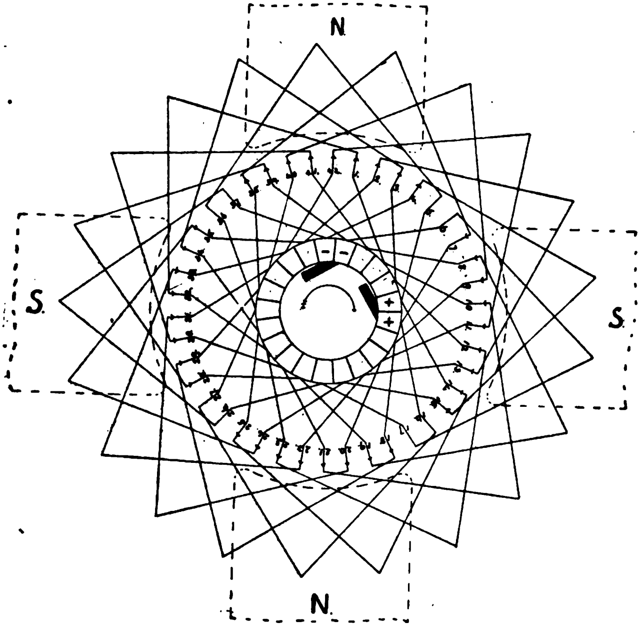


FIG. 55.—Two-circuit single drum winding in four-pole field. (Wave winding in a single layer, with unequal pitches, back and front.)

Fig. 54 shows a two-circuit single-layer winding in a four-pole field. The pitch y (back and front) = 7, and

$$n = 4 \times 7 \pm 2 = 26 \text{ or } 30,$$

and 30 conductors are shown. The winding table is as follows :—

Winding Tables for Drum Armature.

Two-circuit single winding for four-pole field.
30 conductors and 15 commutator segments.
Pitch (front and back) = 7.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	30	..	7	..	14	..	21	..	28	..	5	+ ..	12	..	19
..	26	..	3	+ ..	10	..	17	..	24	..	1	..	8	..	15
..	22	..	29	..	6	..	13	..	20	..	27	− ..	4	..	11
..	18	..	25	− ..	2	..	9	..	16	..	23	..	30	..	..

Conductors 12, 19, 26, and 3 in series are short-circuited under the positive brush, and 4, 11, 18, and 25 in series under the negative brush.

To make the end connections as short as possible, the pitch should be taken less than $\frac{n}{p}$; and if an even number results, then one pitch should be taken $(y + 1)$ at one end and $(y - 1)$ at the other.

As an example, fig. 55 is given, in which $y = 10$ and $p = 4$;

$$\therefore n = 4 \times 10 \pm 2 = 38 \text{ or } 42,$$

and 42 conductors are used.

Winding Table for Drum Armature.

Two-circuit single winding for four-pole field.
42 conductors and 21 commutator segments.
Back pitch = 11. Front pitch = 9.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
..	42	..	11	..	20	..	31	..	40	..	9	..	18	..	29
..	38	..	7	+	16	..	27	..	36	..	5	+	14	..	25
..	34	..	3	..	12	..	23	..	32	..	1	..	10	..	21
..	30	..	41	..	8	..	19	..	28	..	39	−	6	..	17
..	26	..	37	−	4	..	15	..	24	..	35	..	2	..	13
..	22	..	33	..	42	..	..	..	..	..	..	..	..	..	..

Conductors 16, 27, 36, and 5 in series are short-circuited by the positive brush, and 6, 17, 26, 37 in series by the negative brush.

Only two brushes, situated 90 deg. apart, have been shown in figs. 54 and 55, this being the usual arrangement with series-wound multipolar machines of small current output in four-pole fields. The next two examples are intended to illustrate the effect of employing as many brushes as poles, this being essential with very large outputs to provide the requisite cross-sectional area of brush contact, while securing a reasonable cost of the commutator.

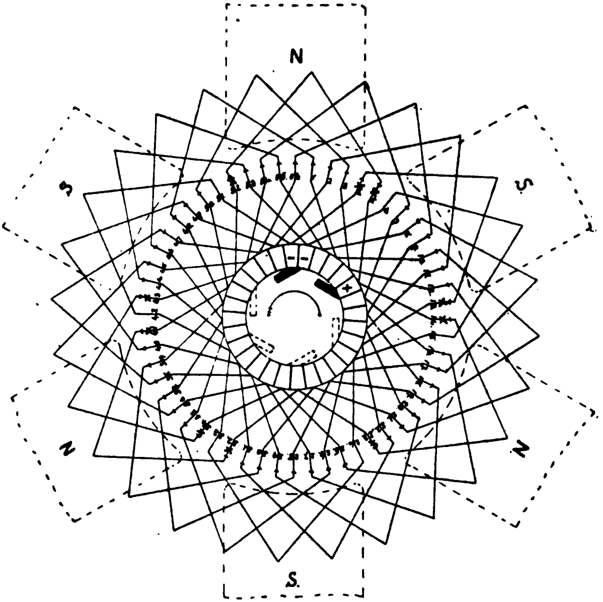


Fig. 56.—Two-circuit single drum winding in six-pole field. (Wave winding in a single layer.)

Fig. 56 shows a two-circuit single-layer winding in a six pole field. The pitch $y = 9$;

$$\therefore n = 6 \times 9 \pm 2 = 52 \text{ or } 56,$$

and 56 conductors are taken. The winding table is as follows:—

Winding Table for Drum Armature.

Two-circuit single winding for six-pole field.

56 conductors and 28 commutator segments.

Pitch (back and front) = 9.

F	D	B	U	F	D	B	U	F	D	B	U
..	56	..	9	..	18	..	27	..	36	..	45
..	54	..	7	..	16	..	25	..	34	..	43
..	52	..	5	+	14	..	23	..	32	..	41
..	50	..	3	..	12	..	21	..	30	..	39
..	48	..	1	..	10	..	19	..	28	..	37
..	46	..	55	..	8	..	17	..	26	..	35
..	44	..	53	—	6	..	15	..	24	..	33
..	42	..	51	—	4	..	13	..	22	..	31
..	40	..	49	..	2	..	11	..	20	..	29
..	38	..	47	..	56	..	..	..	..	..	..

If only the two brushes shown black are used, conductors 6, 15, 24, 33, 42, 51 in series are short-circuited under the negative brush, and the circuits through the winding are:—

$$\longrightarrow - \left\{ \begin{array}{l} 53-44-35-26-17-8-55-46-37-28 \\ \quad -19-10-1-48-39-30-21-12-3 \\ \quad -50-41-32-23-14 \\ \qquad\qquad \times \quad \times \quad \times \quad \times \\ 4-13-22-31-40-49-2-11-20-29 \\ \times \quad \times \\ \quad -38-47-56-9-18-27-36-45-54 \\ \qquad 7-16-25-34-43-52-5 \\ \qquad\qquad\qquad\qquad \times \quad \times \quad \times \quad \times \end{array} \right\} + \longleftarrow$$

If four more brushes, shown in fig. 56 by dotted lines, are added, certain other conductors, for the position shown, are short-circuited, two in series between brushes of like sign, and these are marked above and in fig. 56 by crosses. The number of circuits through the armature is just the same as when two brushes only are used, but certain additional conductors are short-circuited, and these would therefore not be active if six brushes are used.

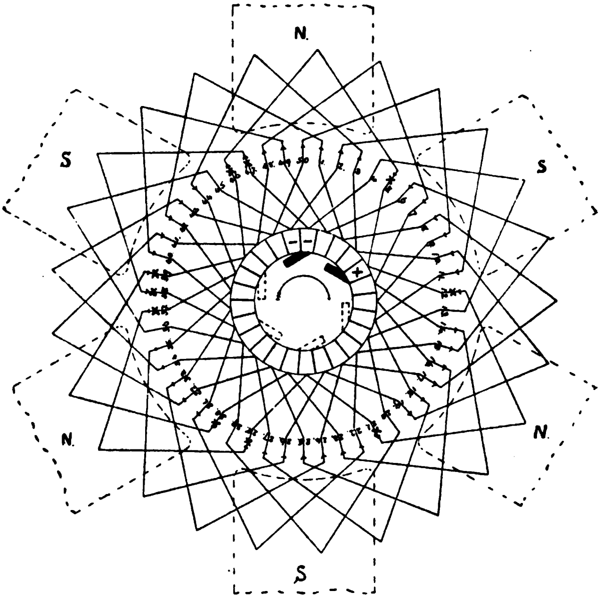


FIG. 57.—Two-circuit single drum winding in six-pole field. (Wave winding in a single layer, with unequal pitches, back and front, and as many brushes as poles.

Fig. 57 shows the case with back and front pitches differing by 2. $y = 8$, and $p = 6$;

$$\therefore n = 6 \times 8 \pm 2 = 46 \text{ or } 50,$$

and the following is the winding table for 50 conductors :—

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit single winding for six-pole field.

50 conductors and 25 commutator segments.

Back pitch = 9. Front pitch = 7.

F	D	B	U	F	D	B	U	F	D	B	U
..	50	..	9	..	16	..	25	..	32	..	41
..	48	..	7	..	14	..	23	..	30	..	39
..	46	..	5	+	12	..	21	..	28	..	37
..	44	..	3	..	10	..	19	..	26	..	35
..	42	..	1	..	8	..	17	..	24	..	33
..	40	..	49	..	6	..	15	..	22	..	31
..	38	..	47	—	4	..	13	..	20	..	29
..	36	..	45	—	2	..	11	..	18	..	27
..	34	..	43	..	50	..	..	..	..	..	..

With the two black brushes in use, conductors 4, 13, 20, 29, 36, and 45 in series are short-circuited under the negative brush, and the circuits through the winding are :—

$$\longrightarrow - \left\{ \begin{array}{l} 47-38-31-22-15-6-49-40-33-24 \\ \times \\ \qquad -17-8-1-42-35-26-19-10-3 \\ \qquad -44-37-28-21-12 \\ \qquad\qquad \times \quad \times \quad \times \quad \times \\ 2-11-18-27-34-43-50-9-16-25 \\ \qquad -32-41-48-7-14-23-30-39- \\ \qquad 46-5 \\ \qquad \times \quad \times \end{array} \right\} + \longleftarrow$$

Adding four more brushes, shown in fig. 57 in dotted lines, the conductors marked with crosses are short-circuited, two in series between brushes of the same sign, and these would therefore in addition be rendered inactive when six brushes are used.

These windings can also be arranged in two layers, precisely in the manner described for parallel-wound multipolar drums, page 151, and the winding table below is given to show the distribution of pressure between conductors adjacent to each other for a single-layer winding, or situated respectively in the outer or inner layers in a two-layer winding, 50 conductors in series being supposed active, and each furnishing, on the average, two volts.

WINDING TABLE FOR DRUM ARMATURE.

Two-circuit winding for four-pole field.

118 conductors and 59 commutator segments.

Pitch (back and front) = 29.

F	D	B	U	F	D	B	U	F	D	B	U	F	D	B	U
Volts		Volts		Volts		Volts		Volts		Volts		Volts		Volts	
100	118	100	29	100 +	58	100	87	100	116	100	27	100 +	56	100	85
100	114	100	25	100	54	93	83	96	112	94	23	92	52	90	81
88	110	86	21	84	50	82	79	80	108	78	19	76	48	74	77
72	106	70	17	68	46	66	75	64	104	62	15	60	44	58	73
56	102	54	13	52	42	50	71	48	100	46	11	44	40	42	69
40	98	38	9	36	38	34	67	32	96	30	7	28	36	26	65
24	94	22	5	20	34	18	63	16	92	14	3	12	32	10	61
8	90	6	1	4	30	2	59	0	88	0	117	0 –	28	0	57
0	86	0	115	0 –	26	0	55	0	84	2	113	4	24	6	53
8	82	10	111	12	22	14	51	16	80	18	109	20	20	22	49
24	78	26	107	28	18	30	47	32	76	34	105	36	16	38	45
40	74	42	103	44	14	46	43	48	72	50	101	52	12	54	41
56	70	58	99	60	10	62	39	64	68	66	97	68	8	70	37
72	66	74	95	76	6	78	35	80	64	82	93	84	4	86	33
88	62	90	91	92	2	94	31	96	60	98	89	100	118	..	..

The full difference of pressure generated by the machine is thus shown to be periodically developed between all the conductors as the armature rotates. Thus in the position, taken this difference of pressure is for conductor 58, 100 volts, and for conductor 57, 0 volts. Between alternate pairs of conductors, however, the difference of pressure is only that due to p conductors in series, p again representing the number of poles. Thus for conductor 77, 74 volts, and conductor 75, 66 volts; also for conductor 12, 52 volts, and conductor 14, 44 volts; or, since p in this example = 4, and each conductor is assumed to add 2 volts, $4 \times 2 = 8$ volts between alternate pairs of conductors. With conductors arranged in a single layer, insulation must thus be provided between every conductor capable of withstanding the full difference of pressure generated by the machine.

For the smallest difference of pressure between adjacent conductors in a layer with a two-layer winding, all the odd numbered conductors must be arranged in one layer, and all the even numbered in the other, a thick layer of insulation being interposed between the outer and inner layers.

Disc Armatures.—The core in this type of armature has the form of a flat ring, and the pole pieces are arranged on each side instead of, as in ring and drum armatures, round its curved circumference. The active parts of the winding are consequently radial. In nearly all cases the disposition of the poles is such that opposing poles are of the same sign, positive and negative occurring alternately in passing round the circumference, so that at any one section of the core the magnetic lines are entering it on both sides, or leaving it at both. Thus if wound, for instance, with a simple ring winding (fig. 58), it will be seen that, as the direction of motion is in the same direction and the lines cut are in the opposite directions, the electro-motive forces act in the same sense from end to end of the coil within the region under the influence of two like poles, and that the lengths A, A of a turn on the disc is equivalent to the length B, B running from the front to back end on the outside of a cylindrical core. This being so, it follows that any of the winding schemes discussed in the preceding chapters, either rings or drums, are equally applicable for discs in so far as the electrical design is concerned. The ring winding, however, is that usually employed, and in this the disc armature presents the advantage that the inactive portions of a turn are considerably shorter than in the usual cylinder.

With the arrangement of poles described, the connections of a drum-wound disc pass between the ends of conductors situated on the same side of the disc.

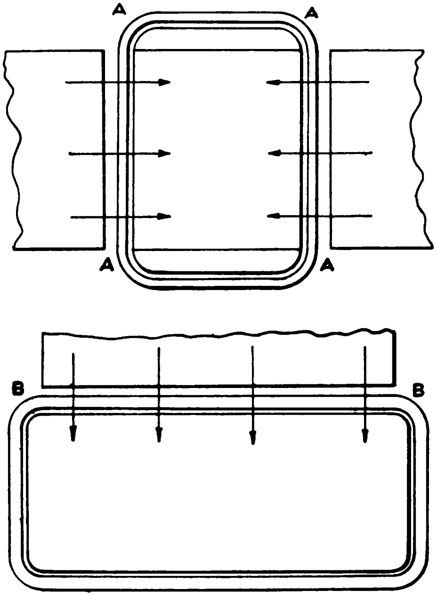


FIG. 58.

In some cases the arrangement of poles, however, brings those of opposite sign to face each other, the lines passing in at one side of the core and out at the other. When such a field is employed, the connectors of the armature conductors must obviously pass between the ends lying on opposite sides of the disc. Fig. 59 is an illustration of such, being a two-circuit single-wave winding, for which the table is as follows:—

WINDING TABLE FOR DISC ARMATURE.

Two-circuit single winding for six-pole field.

44 conductors and 22 commutator sections.

Front pitch = back pitch = 7.

U	Out.	D	In.	U	Out.	D	In.	U	Out.	D	In.	U	Out.	D	In.
+	1	..	8	..	15	..	22	..	29	..	36	..	43	..	6
..	13	..	20	..	27	..	34	..	41	..	4	..	11	..	18
..	25	..	32	..	39	..	2	..	9	..	16	..	23	..	30
..	37	..	44	..	7	..	14	..	21	..	28	..	35	..	42
..	5	–	12	..	19	..	26	..	33	..	40	..	3	..	10
..	17	..	24	..	31	..	38	..	1						

It will be noted that for the case of a disc, the notation has been slightly modified. Here "out" denotes radially outwards on the one side of the disc, "in" radially inwards on the other side.

Open Coil Armatures.—The characteristic feature of all the windings hitherto considered is that, starting from any point, and passing continuously along the conductors, the whole winding is passed over and the starting point again reached. Of course, in the case of double and triple wound cores this applies, not to all, but to each winding separately. As a consequence of this, each conductor is carrying current when the machine is running on a closed circuit.

In an open coil armature, on the other hand, the ring winding is divided into sections disconnected electrically from one another; each of which is during some part of a revolution entirely cut out of the current-carrying circuit. The most primitive form of this type is the old shuttle-wound armature, now of little importance, and manufactured chiefly as a toy. In this, except for the fact that the wires lie in grooves slotted lengthwise in a cylindrical core, the arrangement is identical with that shown in fig. 26, and the electro-motive force induced varies from zero to the maximum, as indicated by the full line curve of page 91. A variation of this extent would render a commercial machine impossible, and by means to be mentioned shortly it has been reduced. Nevertheless, the large fluctuation in voltage during a revolution is a characteristic of this type of machine.

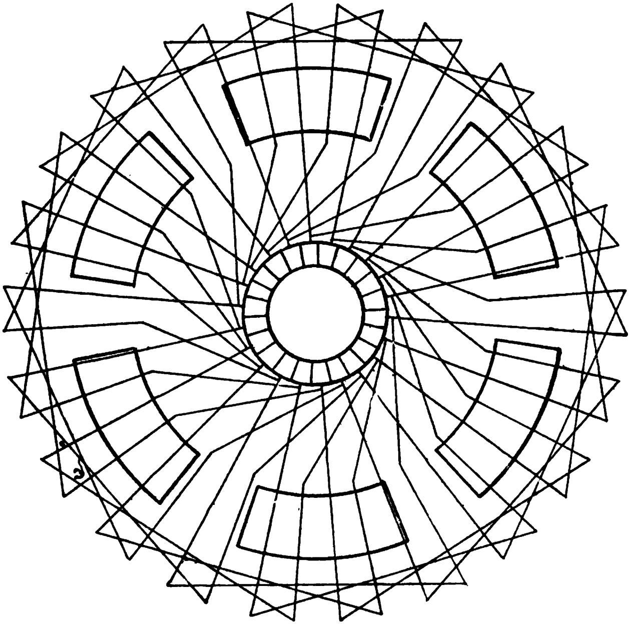


Fig. 59.

The actual variations of electro-motive force when a shuttle-wound armature rotates in an ordinary bipolar field are somewhat as shown in fig. 60, the coil being fully opposite the pole pieces at 90 deg. and 270 deg., when the maximum electro-motive force is induced. If the commutator segments are cut down to span only 90 deg. instead of 180 deg., the curve rises and falls abruptly at 45 deg., 135 deg., 225 deg., and 315 deg., as indicated by the dotted lines. A second coil may now be wound at right angles to the first, and the curve for this will be similar to that of the first, but bridging over its intervals from 315 deg. to 45 deg., and 135 deg. to 225 deg. The continuous upper curve furnished by the two coils is that of the electro-motive force resulting from the combination, and, as will be seen, has a far less percentage variation than either singly. In a similar manner, by further narrowing the commutator segments, and winding three coils, the fluctuation may be made still smaller.

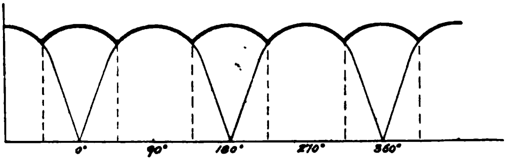


FIG. 60.

Very nearly the same result, however, will be attained if, instead of the angular width of the commutator segments being reduced, a second commutator is placed on the spindle beside the first for the added coil, and the brushes of the same polarity on each are connected. A combination of the two methods gives four shuttle-wound coils and two commutators, each of four segments, displaced relatively to one another through an angle of 45 deg.

The arrangement arrived at is practically that adopted in the Brush machine, which, with that manufactured by the Thomson-Houston Company, is one of the best known examples of this class of dynamo. In this a ring winding is employed, two diametrically opposite coils in connection corresponding to a single coil shuttle-wound. In fig. 61 an eight-coil winding is shown, and the two commutators are represented as within each other, though actually of the form shown in fig. 62. It is to be noted that the segments are so shaped as to overlap in such a way that a brush is on two segments every alternate eighth of a revolution, and that each pair of coils is thus cut out of circuit entirely for the same period when in its inactive position. By tracing the course of the current through the armature, and taking note of the fact that in this machine the brushes of each commutator are coupled in series, it is readily seen that the

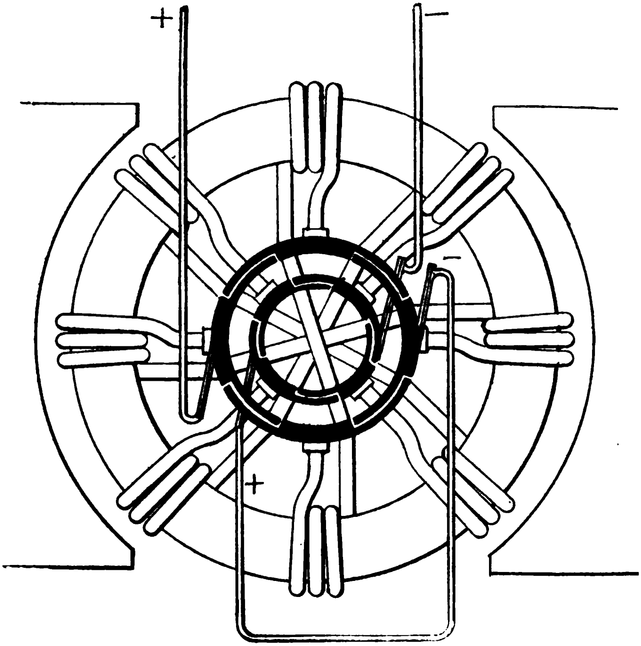


FIG. 61.

entire current flows through that pair of coils which are in their most active position, and divides, flowing through two pairs in parallel which are less active. Thus, for the position shown in fig. 61, the current enters by the negative brush A, passes through two pairs of coils in parallel to brush B, thence by an external connection to C, and through another pair to the positive brush D. The armature resistance is thus one and a half times that of a single pair of coils. This machine is also made with twelve coils and three commutators, the idle coils in this case being cut out during one-twelfth of a revolution, and since the current flows through a second pair of coils in parallel with each other, the armature resistance is twice that of a single pair.

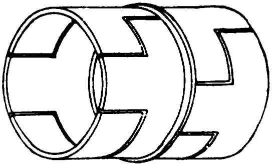


FIG. 62.

In the Thomson-Houston machine' two methods of winding are employed; the one used in the larger sizes being a ring winding, calling for no special description, the other a drum winding laid on an ellipsoidal core, and forming when complete a spherical armature. Both, however, are remarkable for consisting of but three parts—*i.e.*, three coils in the drum, and three pairs of coils in the ring. The mean planes of the coils in the drum make with each other angles of 120 deg., and the coils themselves overlap, forming a considerable thickness of copper round the core, though in fig. 63, for the sake of clearness, each is shown with but few turns.

It will be seen that one end of each coil is brought to a common connection, so leaving three free ends which are led to a three-segment commutator; and that each coil is in consequence cut out of circuit for nearly one-third of each revolution with the usual arrangement of brushes. It cannot be regarded, however, as inactive during so great a part of a revolution; indeed, during a considerable fraction of the time the electro-motive force induced by it would be more than sufficient to drive the current against its own resistance, and it is advantageous for it to be in circuit. In order, therefore, to diminish and regulate the idle period, a device equivalent to a brush of variable angular width is applied, an increase or reduction of the width respectively cutting the coil out for a shorter or longer time. Under normal circumstances, the greatest contact angle of the brush is 60 deg., and then no coil is cut out at all, the current passing through one single coil in series with the other two in parallel.

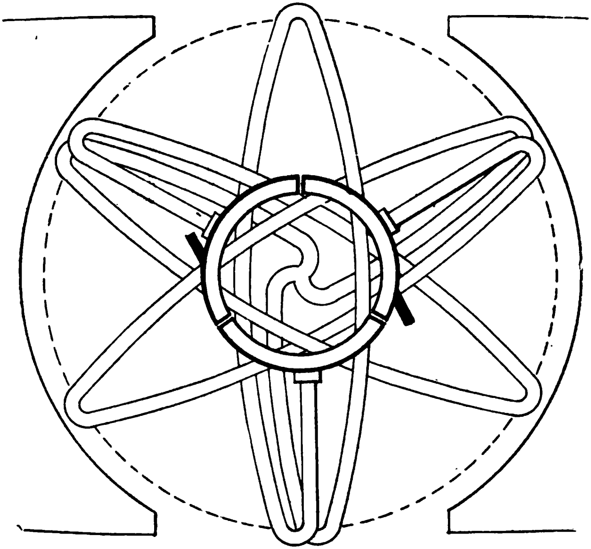


FIG. 63.

As the contact angle is reduced, a coil is cut out, and three times in each revolution the current flows through two coils in series. The obvious objection to this winding is the inaccessibility of the inner conductors in the case of a fault arising in the insulation. As, however, the machine appeared in the quite early days of electrical engineering, and is still manufactured, the conclusion is that a proportionally greater amount of care is given to this point in the construction.

As has been remarked, a characteristic of these machines is the fluctuation of their voltage. The two described, nevertheless, differ widely in the extent to which the statement applies; for while in a twelve-coil brush machine it may be as little as 2 per cent above and below the average value, it may in the Thomson-Houston be as great as 20 per cent of the same.

A more detailed account of the actual arrangement of poles and details in these machines is reserved for fuller treatment in a more appropriate chapter.

The Mechanical Construction of Armatures.—In order to obtain a strong magnetic field in the region through which the conductors of an armature rotate, it is necessary that the path of the magnetic lines from pole piece to pole piece should be through a highly permeable medium, and on this account the core upon which the winding is supported is, except in one or two quite abnormal cases, of iron. It will be apparent, however, that exactly as an electro-motive force is generated in the winding by its motion in the magnetic field, so an electro-motive force will be produced in the iron core, the outer surface of which cuts the identical lines at the same rate, and, if of solid metal, the resistance will be so small that even with a small potential difference between its ends a large current will be created. As such currents, usually referred to in this connection as "Foucault" or "eddy" currents, would be doubly harmful in wasting power and in greatly heating the armature, the iron is so sub-divided as to present a considerable resistance in an end to end direction; in other words, it is divided in planes parallel to the magnetic lines and perpendicular to the direction of the induced electro-motive force.

In early forms of core this was often accomplished by winding charcoal iron wire of about one-sixteenth of an inch diameter upon a gun-metal cylinder furnished with end flanges, the smallness of the surfaces in contact between neighbouring turns, and the film of oxide sometimes present, producing considerable resistance. Owing to the expense of the gun-metal, the relatively low electrical resistance, and the discontinuity of iron along the path of the magnetic lines, together with the fact that only about ·785 of the core's volume was actually iron, this method has become entirely superseded by that of building the core up of thin stamped-iron discs or rings. These stampings are usually about one-fortieth of an inch thick, and each is insulated from those on either side either by a film of oxide (which is unreliable), a coating of varnish, or by having its sides covered with thin paper. With such discs, well pressed together, from 85 to 90 per cent of the volume is actual iron, the percentage depending to some extent upon the flatness of the stampings. These may now be obtained, stamped in one piece from the sheet, in all sizes up to 5 ft. diameter; for larger sizes it is usual to stamp separate segments, which are subsequently united into complete rings. Two examples of the manner in which this may be done are shown in figs. 64 and 65. In the first, due to Kapp, the segments of alternate rings break joint, the whole core being clamped by bolts passing through provided lugs. The latter necessitate a certain waste in stamping the segments. In the other method, adopted by the Oerlikon works, the segments are locked together by dovetailing and clamped by bolts passing through the core itself.

These discs, when clamped and connected with the necessary driving arrangement to the armature spindle, should be turned with a sharp tool in order that the edges of adjoining discs may not be burred over into contact.

In building up a core the entire length is frequently divided into sections separated by air discs, which, by permitting air to pass from the inside of the armature outward, to some extent cool the core and dissipate the heat produced by hysteresis and eddy currents. These air discs consist of two wrought-iron discs, one-eighth of an inch or so in thickness, separated by a number of radial non-magnetic distance pieces, often of vulcanised fibre, to which they are riveted. The air space thus produced is usually from a quarter to half an inch wide, according to the size of the armature.

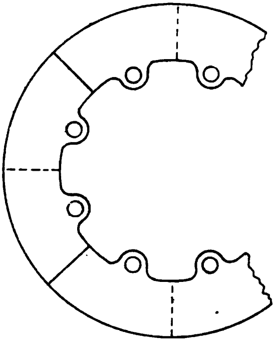


Fig. 64.

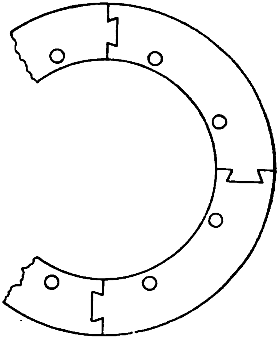


Fig. 65.

The fact, however, should not be lost sight of that, while promoting ventilation, their thick plates increase the eddy current loss.

The ends of the core are supported by thicker plates capable of withstanding the stress of the clamping device without bulging, an effect which would result if the force were applied directly to the thinner discs. The simplest method of connecting the core with the shaft is that often employed in small armatures, of turning the hole in the centre to fit the shaft and driving by an ordinary key or feather. This is, of course, only applicable in the case of drum-wound armatures, and has the disadvantage that it precludes all possibility of internal cooling. In larger drum armatures the discs are partly cut away inside, leaving air passages, or are simple rings driven by an intermediate piece, as must always be the case with ring windings. A convenient form of disc for moderate sizes is shown in fig. 66, where three air tunnels exist. For larger sizes, where the discs become mere rings, the methods of driving are very much the same for both drum and ring windings; but whereas for drum windings there is no objection to the inner hub and drivers being of iron or steel, it is absolutely essential in the case of ring windings that this should be of gun-metal or some other non-magnetic material. The reason for this will be obvious when it is remembered that all magnetic lines crossing from side to side of a ring-wound core internally are cut across by the inside wires, and so produce an electro-motive force opposed to that induced in the external layer. A few examples of driving arrangements employed in modern machines are given in figs. 67—72. In fig. 67 the arms of the gun-metal hub are machined at the ends, and engage in slots in the core, the clamping being effected by extensions of the arms at the commutator end of the core, and by a claw plate and nut at the other.

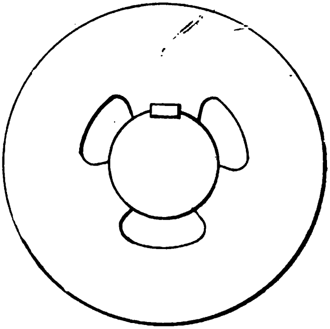


FIG. 66.

The nut itself when tightened is secured by a set screw. A somewhat similar arrangement applied to drums is shown in fig. 68. Here the clamping cast-iron end plates have the support for the end connections of the conductors secured to them. Among others, the hub, or "spider" (fig. 69), is employed by the Oerlikon Company. This consists of two parts, each furnished with six arms. The alternate arms are prolonged, as a consequence of which both spiders engage with the central core discs, and any relative rotation is prevented. The clamping is obtained by the bolts, which also secure the connector carriages. Examples from larger machines are (fig. 70) employed by Parshall for traction generators, the commutator being supported on the extension of the hub, and (fig. 71) adopted by the General Electric

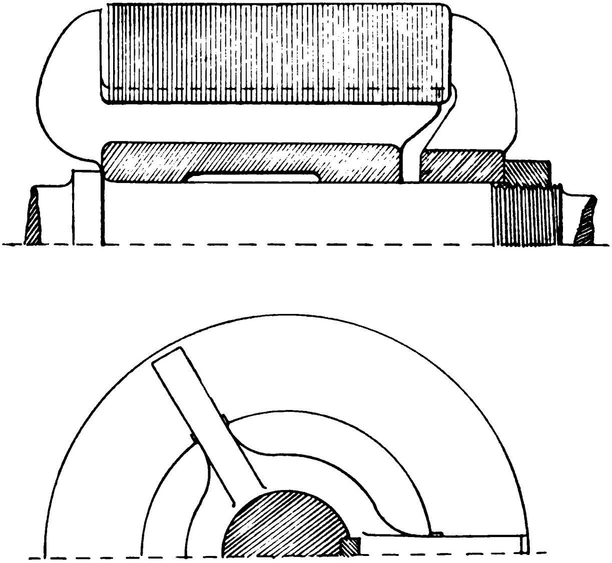


FIG. 67.

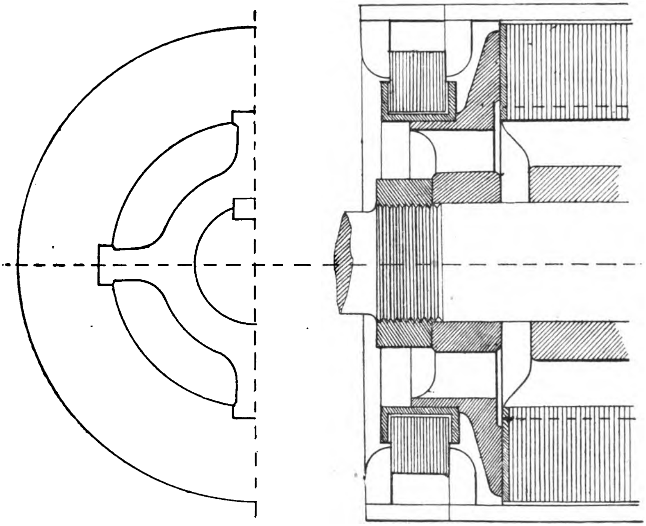


FIG. 68.

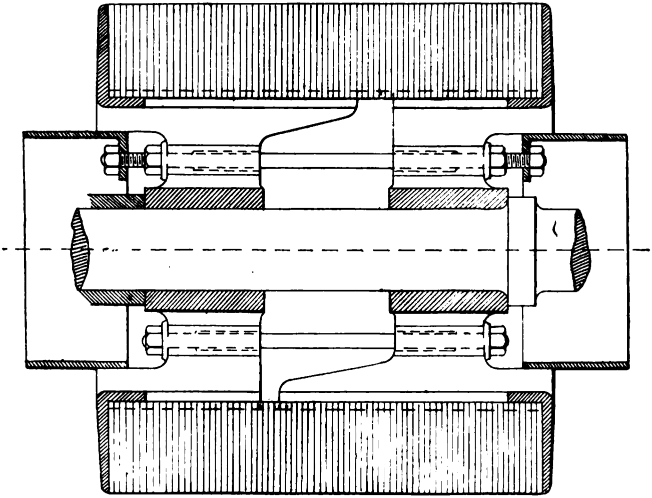


FIG. 69.

Company for large multipolar machines. In large ring-wound armatures, to minimise expense, cast-iron hubs are sometimes used, intermediate non-magnetic arms transmitting the power (fig. 72).

Where clamping bolts pass through the core itself they should be insulated if the saturation of the core is at all high. This point particularly demands attention in the case of small bipolar machines, but it is of smaller importance with large multipolar machines when the density of the lines in the core is usually less.

Disc armature cores are composed of iron strip wound in layers to the required depth, and secured by radial bolts passing through the core and the rim of the wheel upon which it is built.

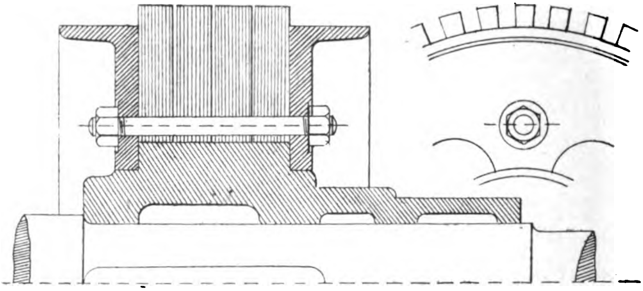


FIG. 70.

Though numerous modifications of the driving hubs described are found in modern practice, these will afford a sufficient idea of their general character, and attention may now be turned to the arrangement of the winding upon the core. In this respect cores may at once be classified in two groups—smooth or surface-wound cores in which the winding lies upon the smooth circumference, and slotted cores in which it is collected into slots separated by the teeth (fig. 73). The width of tooth and slot are usually about equal, while the depth averages two to three times the width. Excessive

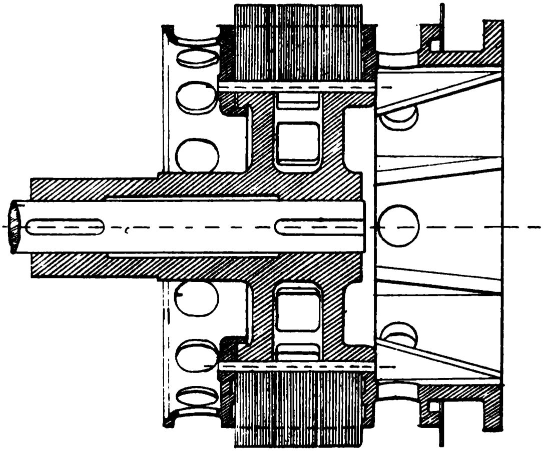


FIG. 71.

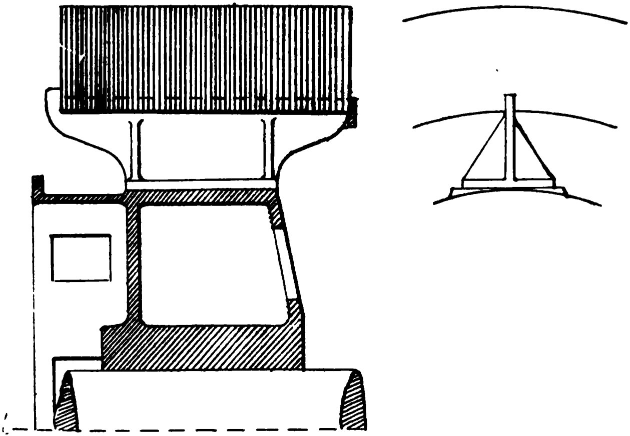


FIG. 72.

width, however, leads to the production of eddy currents in the polar faces of the magnets, due to tufts of magnetic lines following the teeth round in their rotation. A modification of the simple slot is shown in fig. 74, where the groove is partly closed. With this form the wires are often secured in place by a strip of wood driven in above them. A further development of this idea leads to the tunnelled core, originally employed by Brown, but not widely used, in which the conductors, surrounded by an insulating tube, are entirely buried beneath the surface of the core (fig. 75).

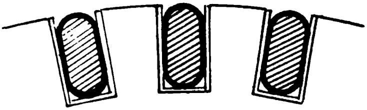


FIG. 73.

In all such slotted cores the conductors are not only greatly protected from injury by the teeth, but are also positively driven by them. In smooth cores, on the other hand, unless reliance is placed entirely on the friction and tightness of winding for driving, which is only permissible in the case of the very smallest armatures, it is necessary to provide a certain number of thicker discs furnished with projecting driving horns. Where present, the sides of the air discs usually carry such horns.

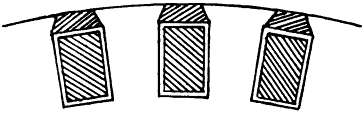


FIG. 74.

Another method of driving is by means of pegs of non-magnetic material having high resistance, driven into holes cut in the core surface, and this, for large armatures, is preferable to the previous device.

Preparatory to receiving their winding smooth cores are wrapped over with an insulating layer of varnished paper, micanite, vulcanised fibre, or tape, an extra thickness being put over all edges and corners. The wire, if of small size, is then wound on by hand. This winding is most easily performed when the entire number of external conductors can be arranged in a single layer. Frequently, however, two or even three layers (external layers) are wound on externally. Since the available circumferential space inside a ring-wound core is less than that outside, it is clear that, if the latter is fully utilised, the internal wires must be arranged in a greater number of layers. Thus with a single external layer there are commonly two internally, with two externally there are three or four internally, and so on. Frequently the inner wires do not form a whole number of complete layers, and when this is so those which have to overlap should be chosen at equal distances apart circumferentially. When the wire is of large size it is advantageous for it to have a rectangular section, so utilising more fully the winding space. To wind such on the core directly by hand is all but impossible, and the usual method is to cut lengths sufficient for one turn, to bend these on a special shaper, and then spring them over the core, finally riveting and soldering consecutive turns. A convenient shaper for this purpose is illustrated in fig. 76. The cast-iron body turns about a pin secured to the bench, and the length of wire is placed in a slot round its edges. It is held there by clips, and bent round the corners either by blows from a mallet, or by a roller and lever if of larger size. Where, as is usual, there are two internal layers, two such shapers are, of course, required.

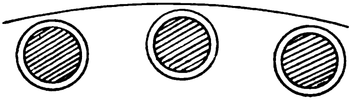


FIG. 75.

The device of laying square external bars and voussoir-shaped internal and end conductors has been employed, and has the advantage of fully using the winding space, but the multiplication of soldered joints is to be deprecated from both an electrical and mechanical point of view.

The winding, being completed, is bound down by bands of German silver or other high-resistance wire wound at intervals on the external circumference. These prevent the conductors from being thrown off by the centrifugal force when rotated, and are conveniently laid in grooved fibre or wooden pieces slipped over the driving horns for the primary purpose of preventing the horn driving directly against the insulation of the conductors, a condition which would speedily result in the latter being cut through. These bands should be of fine strong wire, separated from the underlying conductors by an extra thickness of insulation, and, that eddy currents may be prevented, not more than 1 in. wide. Each should be of one length of wire, and have the ends well soldered to the adjacent turns. A part of the circumference of such an armature is shown developed out in fig. 77. At the ends of armatures, outside the polar field, these bands are often of strip metal, and furnished with a tightening screw.

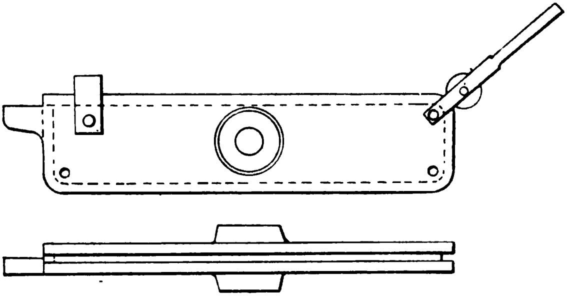


Fig 76.

The above remarks apply, in most cases, equally to ring-wound slotted cores. In these, however, it is, of course, not necessary to wrap the whole external circumference with insulating material, but only to line the slots. The binding wires, when required, may very conveniently be sunk in shallow channels turned in the teeth, by which means the over-all diameter of the armature is slightly reduced.

The operation of winding a drum is, except so far as the end connections are concerned, a much less troublesome business than winding rings, since the conductors are merely laid on the outer surface, or in the slots, and secured as explained above. The end connectors themselves in most cases involve a considerable amount of labour. Broadly speaking, they fall into two groups :—

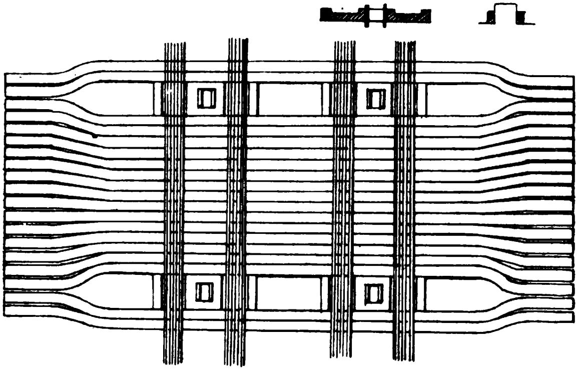


FIG. 77.

(*a*) Including all arrangements whereby the connectors are formed by bendings of the winding, and are consequently of the same section, and in one piece, with two inductor bars.

(*b*) Comprising all those arrangements in which the connectors are distinct parts, connected to the inductor bars ; but not necessarily, or indeed usually, of the same section.

The very simplest example of the first group (*a*) is that in which the wires are wound by hand across the ends of the core, swerving from a straight line only so far as to permit the emergence of the spindle. When completed, a rounded mass of overlapping wires adorns each end of the armature, the bunching appearance of which amply justifies the term "chignon," not infrequently applied to this class. For obvious reasons the method is undesirable, and quite inadmissible, except in the case of the smallest machines.

The regular and orderly arrangements with this type of connection are obtained by shaping the wires as shown in figs. 78 and 79, the former for wave, the latter for lap winding. It will be evident that such connections need not necessarily lie in a plane perpendicular to the shaft, but may be disposed on any conical surface having the circular end of the core as its base. The limit of this modification is reached when they lie upon the cylindrical surface of the core produced. Such "laid-on" or "barrel" windings are not uncommon, and their shaping is simple, but they occupy a greater length than those figured, so increasing the length of the machine. A simple form of shaper for a laid-on winding is shown in fig. 80, and the contrivance of more elaborate ones for other shapes presents no great difficulty.

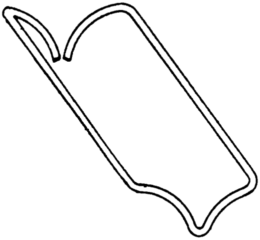


FIG. 79.

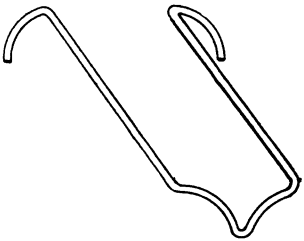


FIG. 78.

As compared with connections in group (*b*), those in group (*a*) have the advantage of fewer soldered joints; but, on the other hand, in case of damage a wire is not so readily moved. Where the conductors are small the bending of the ends can be accomplished by hand on the core as the winding proceeds, especially if a non-metallic ring be used as a connector core. In this case the total labour is less than with separate connectors. When the loops consist of numerous turns (page 128), moreover, the latter arrangement is not feasible.

The second (*b*) type of connectors permit of the armature inductors being cut in simple straight lengths, and further allow of any one or more being easily removed in the event of their being damaged. The connectors themselves are cut from sheet copper, insulated by wrappings of tape or silk, and, after being bent to shape, are collected together by a device which clamps the complete set for one end of the armature. The Kapp connector, fig. 81, permits the over-all length of the armature to be kept down, but is somewhat wasteful of copper in cutting. The lugs are bent up and down respectively at right angles to the connector, the angular length of which is the mean of the front and back pitches, and the requisite increase or reduction is obtained by bending the lugs. The set is built up on a channel-shaped rim (shown in fig. 68), carefully lined with insulation, round which the connectors pass spirally in such a way that while the lugs at one end are attached to short-ended armature bars, those at the other are soldered to bars with prolonged ends; an outer layer of insulation and

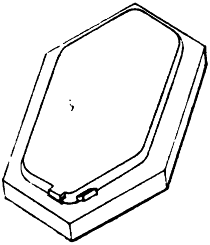


Fig. 80.

binding wires prevents any dislocation. A form which, with small modifications in the clamping hub, is widely employed, is shown in fig. 82. In this the strips are bent in opposite direction, making connection with long and short ended bars, while the complete set may form a compact overlapping mass, or have a more open structure, according to the radial depth at disposal. A similar connector, obtained by the folding of continuous strips, is shown in fig. 83. The construction of the supporting hubs will be sufficiently apparent from figs. 84 and 85.

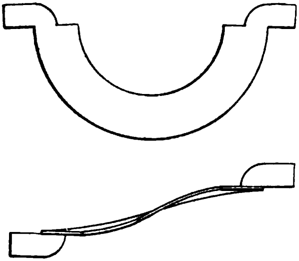


Fig. 81.

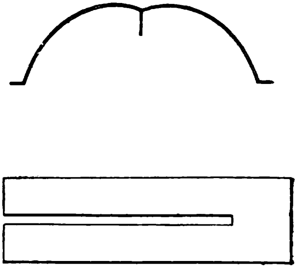


Fig. 82.

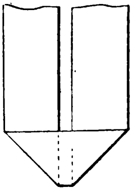


Fig. 83.

The simplest method of attaching the armature bars and connector strips is to solder the latter in saw cuts in the bars; but a better, though more laborious, way is to fold the lugs round the bars and solder. Somewhat intermediate between end connections of types (*a*) and (*b*) is the expedient of attaching rectangular bars, bent into a V shape, to the ends of the armature conductors, the point of the V usually entering a commutator segment, and its ends being secured to the bars on the armature.

Equaliser rings are commonly fitted to large parallel-wound multipolar armatures to counteract the effect of unequal resistance in the several circuits, and of unequal flux density between different pole faces. The unrepressed effect of these irregularities, which by reason of soldered joints and wearing brasses respectively are unavoidable, would be a constantly varying current in the bushes, productive of sparking. This is obviated by connecting together points in the winding symmetrically situated with regard to the poles, which may be accomplished by discs furnished with the same number of lugs that there are pairs of poles, or by a number of curved bars carried on insulators at the back of the armature, as in the British Thomson-Houston generators, fig. 86.

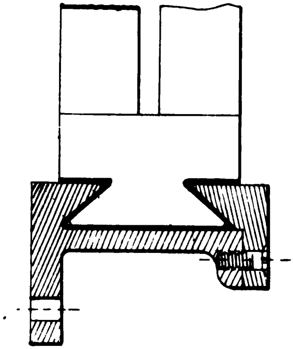


FIG. 84.

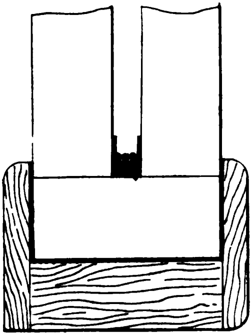


FIG. 85.

These devices, however, while preventing the sparking, do not prevent a very uneven distribution of current in the winding if the flux density under different poles varies, with a consequent waste of energy in overheating. It is therefore of importance that the armature should be accurately centred originally, and maintained so in running by careful attention to the bearings, for, as the heating is proportional to the square of the current density, a slight inequality is productive of considerable loss.

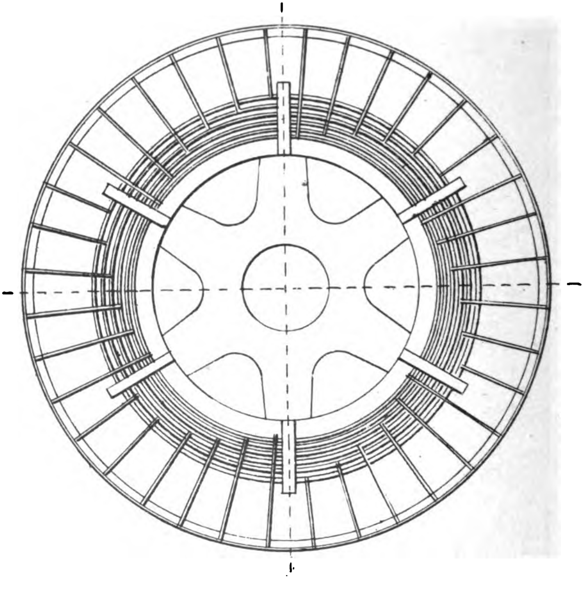


FIG. 86.

The armature conductors in the smallest machines necessarily consist of round wire. In most cases, however, the alternative is presented of employing some rectangular section. For large machines the latter is usually adopted, as utilising the winding space more fully, and so reducing the resistance of the armature. For ring windings, as also for "laid on" or "barrel" drum windings, in machines with wire of smaller section, the point for consideration is, whether the increased difficulty of winding a rectangular wire is compensated for by the larger section, or whether it is preferable to use round wire, much more easily wound, but filling only 78 per cent of the available space.

In many cases a stranded conductor is used on account of the immunity thereby obtained from eddy currents. Such bars are hydraulically compressed to any required section, but are expensive, and have only about 80 per cent of their cross-section actually copper. As, however, the eddy loss would be considerable in stout solid bars, it is necessary that when a heavy current is to be carried, the conductor should at least be built up of several smaller bars bound together by an insulating cotton braiding. The actual thickness of the cotton covering should naturally vary according to the potential difference of the conductors insulated by it. In practice, the size of the bar has much to do with the question. For ordinary voltages, up, for example, to 300 volts, an average thickness of 7·5 mils. for small round wires, and of 10 to 20 mils. for larger bars, may be taken as adequate (1 mil. = ·001 in.). The suggestion may present itself that a thickness sufficient for a drum would be superfluous for the corresponding ring, but it is to be remembered that it is mechanical abrasion rather than any electrical stress which is apt to break the insulator down; hence the relation between thickness and the size of bar.

Whatever its thickness be, the insulation of a bar abruptly bent is impaired at the bend, which should consequently receive a covering of silk tape.

Previously to being wound the covering of the wires should be treated with shellac varnish, or one of the many compositions sold for the purpose, and, after completion, the whole armature should be baked for some hours to expel moisture, and liberally varnished all over. The effect of this varnishing is to render the insulation impervious to moisture, and to materially raise the temperature at which charring occurs.

Commutators.—The commutators of modern machines invariably consist of a number of wedge-shaped segments built into a hollow cylinder, and clamped by sleeves into a compact self-contained component. In early days these segments were frequently of cast gun-metal or phosphor-bronze, but the inevitable want of homogeneity in such metal produced sparking and rapid deterioration, leading to the present uniform adoption of hard-drawn copper strips for the purpose. In some few cases, however, the segments are individually stamped in moulds.

It is, of course, essential that each segment should be insulated from the others and from the clamping sleeves.

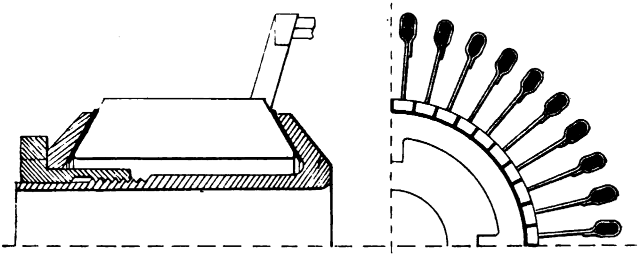


Fig. 87.

For the former purpose sheets of mica are employed; for the latter, rings of micanite or fibre. For large commutators, sheets of micanite are sometimes substituted for the more expensive mica, but when this is done it is advisable to heat the commutator at the time of clamping up. The mica for this purpose should be about a thirtieth of an inch thick for ordinary voltages, and of a soft variety, in order that it may wear down equally with the copper under the brush friction, in which respect the Indian is superior to the Canadian mica. The advantageous feature of micanite is the ease with which it may be moulded into plates, rings, &c., of any required form, when heated, thereby saving labour in turning, and at the same time giving a higher insulation than fibre.

The clamping action of the sleeves, as will be noted from the figures, is commonly produced by drawing the segments radially inwards by an application of the inclined plane. A form suitable for the smallest sizes is shown in fig. 87. In this it will be seen that, as is usually the case, the ring, pressing the segments, does not itself rotate upon tightening up, any such rubbing being likely to injure the insulation; but is carried on a nut, which is ultimately secured by a set-screw. The pitch of the thread should be small, eleven threads to the inch being convenient for medium and small sizes. The Oerlikon Company have constructed commutators of the pattern shown in fig. 88, while fig. 89 is a form devised by Kapp. The latter allows the ends of the segments to be cut square off, and for the same working length is shorter over-all than most other commutators; the insulation, too, between clutches and segments is more substantial. The channel cut in the ends of the segments is lined with mica, and the fibre ring inserted is cut into three parts, so permitting radial contraction upon tightening up. For larger machines the commutator sleeves are carried upon a

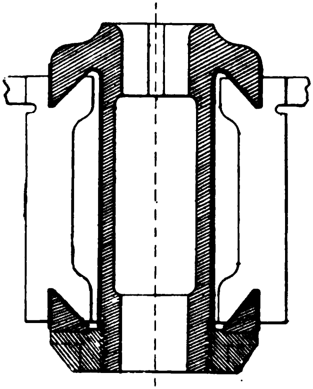


FIG. 88.

boss, or attached to a projecting portion of the armature casting, fig. 93. A commutator adapted for machines of larger size is given in fig. 90, and a form adopted by Messrs. Bergtheil and Young, wherein displacement of the segments, such as is sometimes caused by expansion on heating or centrifugal force, is prevented by their interlocking, is illustrated in fig. 91.

In most commutators it is impossible to remove one or more segments without unbuilding the whole. Fig. 92, however, shows a small pattern in which this may be done, while fig. 93, suitable for larger sizes, has the same advantage. The matter is of no great importance in small machines, but in large multipolar generators the unbuilding of a commutator is an expensive operation, and the provision alluded to is commonly specified.

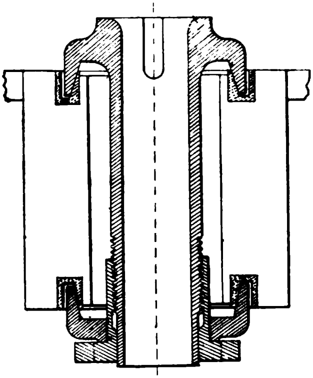


Fig. 89.

Though multipolar parallel-wound machines are now almost always furnished with as many collectors as there are circuits, occasionally cross-connections are fitted to the commutator and a single pair of collectors used. Such cross-connections have at times been placed inside the commutator, but the practice is not commendable, owing to the want of

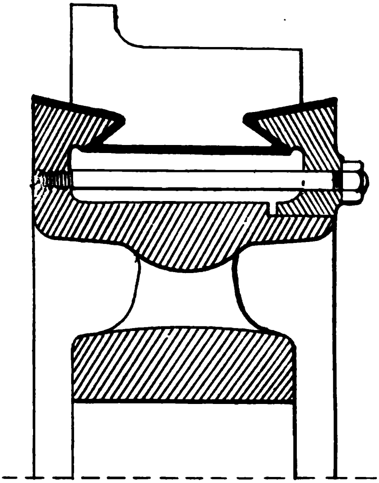


Fig. 90.

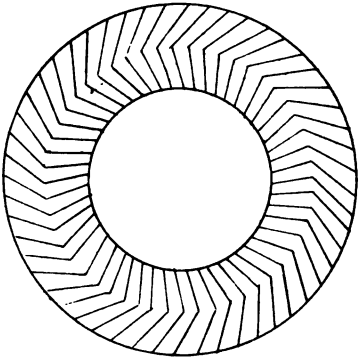


Fig. 91.

ventilation and consequent high temperature rise. Another device resembling the old Edison armature connectors, now obsolete, is to employ insulated copper discs furnished with lugs attached to the commutator segments. These discs, threaded side by side on the shaft, form a cylindrical mass between the commutator and armature. The reasons leading to the abandonment of the Edison connectors would apparently apply equally to this contrivance. The most satisfactory method is that of employing connectors similar to those described for the armature bars, placed close to the inner end of the commutator. The length of the working surface of a commutator is determined by the width and number of the brushes. The maximum current per inch width for a copper brush of $\frac{1}{4}$ in. to $\frac{3}{8}$ in. thickness may be taken at 100 amperes. In no case, however, should there be

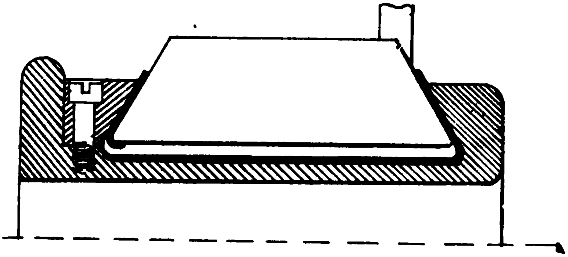


FIG. 92.

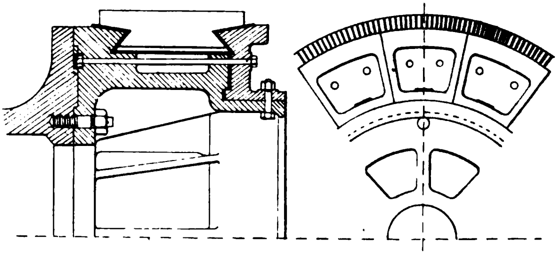


FIG. 93.

less than two brushes on each brush bar, as it may be necessary to raise and adjust one while the other carries the full current. The arrangement should be such that the brushes may be placed at any point on the segment, so preventing the grooving that would ensue if any part could not be brought under them. For a similar reason it is well to cut a groove round the commutator just by the lugs, as shown in fig. 88. With stamped segments the connections to the armature conductors are sometimes made by bringing the latter down and securing them directly to the segments by screws, but the practice is, for obvious reasons, unsatisfactory. More usually, the connectors on the armature are soldered into saw cuts in the segments, or copper strips, secured in the same manner, are carried out and soldered round the armature conductors themselves, which are prolonged for this purpose.

In constructing large commutators especial attention must be given to both the security of the clamping and the strength of the rings, as the centrifugal force is often considerable and has on occasions burst the commutator, scattering the segments with dangerous violence. On this account the sleeves and rings, which when small are of gun-metal, and when larger of cast iron, or preferably of steel, should be substantial and free from any sharply cut re-entrant angles. In construction, the segments, with their mica insulation, are assembled in a temporary clamping frame and turned at the ends and inside, after which the sleeves are tightened on them, and the wearing surface is turned and polished. Perfect trueness is essential, as any irregularity, such as may be produced by a segment subsequently rising, causes jumping of the brushes with consequent sparking and pitting of the surface.

Certain abnormal forms of commutator are fitted to open-coil machines, and will be described when dealing with this class of generator. A commutator with radial strips attached to the side of a disc upon the spindle, and so having the contact surface at right angles to the latter, would appear feasible and under certain circumstances advantageous, but, so far as the author is aware, has never been adopted.

It may be mentioned, in concluding these constructional remarks, that much copper dust is produced by the mutual wear of commutator and brushes, which, finding its way into the armature, tends to facilitate short circuits. To prevent the entrance of this and other unnecessary matter, it is the practice of some makers to swathe the armature with presspahn, oiled cloth, or other like material.

Armature Losses and Reactions.—The energy losses in an armature demand attention not only on account of their lowering the efficiency, but because they also are a transformation of mechanical and electrical energy into heat, which must be got rid of with as small a temperature rise as possible. Chief among these losses are—

(*a*) Loss due to ohmic resistance of the conductors.
(*b*) Hysteresis loss in the core.
(*c*) Eddy-current losses in core and conductors.

Resistance loss (*a*) is, in watts, equal to the product of the current in amperes and the electro-motive force required to drive it through the conductor. By Ohm's law the latter is equal to the product of current and resistance in ohms. Hence the loss equals C^2R. If there be n parallel circuits in the armature, each has 1-nth of the total winding resistance R; likewise 1-nth of the total current C flows in each circuit. The loss, therefore, is $C^2R \div n^3$. The internal drop of voltage has also to be considered; this is evidently the total loss divided by the current per circuit, or $C\,R \div n^2$. In particular, with a bipolar machine, the voltage drop due to resistance is one-fourth of C R, and the energy loss one-eighth. The specific resistance of pure copper is 1·6 microhms; the resistance of an inch length of one square inch sectional area is ·66 microhms. Hence, if

l = length per $\begin{cases}\text{turn in a ring winding} \\ \text{external conductor in a drum winding,}\end{cases}$

s = sectional area of conductor,

T = number of external wires round armature, we have in centimetre measure—

$$\left.\begin{aligned} R &= \frac{l\,T}{s} \times 1\text{·}6 \times 10^{-6} \\ \text{or in inches}\quad R &= \frac{l\,T}{s} \times \text{·}66 \times 10^{-6} \end{aligned}\right\}\text{ for ring windings;}$$

and, if l^1 denote the mean length of a connector, and s^1 its sectional area, in centimetre measure,

$$\left.\begin{aligned} R &= \left(\frac{l}{s} + \frac{l^1}{s^1}\right) T \times 1\text{·}6 \times 10^{-6} \\ \text{or in inches}\ R &= \left(\frac{l}{s} + \frac{l^1}{s^1}\right) T \times \text{·}66 \times 10^{-6} \end{aligned}\right\}\text{for drum windings.}$$

The quantity $\frac{R}{n^2}$ is usually termed the armature resistance. As the specific resistance of copper increases with a rise of temperature by about ·38 per cent per degree Centigrade, and the running temperature of a machine may be taken as about 70 deg. Cen., the resistance obtained as shown should be multiplied by 1·26 to get its real working value.

(*b*) The hysteresis loss. It has been shown in preceding pages that when a volume of iron is carried through a complete magnetic cycle a certain amount of energy is expended on it, reappearing as heat in the metal. The actual amount so lost is dependent upon the magnetic qualities of the iron; for the best wrought iron, however, the loss in watts for various inductions is given in the table appearing on the following page.

To obtain the loss in watts, the number in the column, which refers to one cycle per second, must be multiplied by the number of magnetic cycles per second; *i.e.*, for the core of a bipolar machine by once, for a four-pole machine by twice, etc., the revolutions per second.

According to Steinmetz, if V be the volume of iron in cubic centimetres, N the number of cycles per second, and η a numerical coefficient,

$$\text{Loss in watts} = \eta\, B^{1\cdot 6} N\, V$$

For wrought iron η may be taken as ·003, for cast steel ·006, and for cast iron ·016.

Since the loss increases more rapidly than the induction, the loss in metal when the latter is not uniform will be somewhat greater than that calculated on the basis of the average induction; but when the variation of density is not great this difference may be neglected.

C.G.S. lines per square cm.	Watts per cubic cm.	C.G.S. lines per square inch.	Watts per cubic inch.
3,000	·00008	19,400	·00131
4,000	·00012	25,800	·00197
5,000	·00017	32,300	·00278
6,000	·00022	38,700	·00360
7,000	·00028	45,200	·00459
8,000	·00034	51,700	·00557
9,000	·00042	58,100	·00689
10,000	·00050	64,600	·00820
11,000	·00058	71,100	·00950
12,000	·00067	77,600	·01147
13,000	·00076	84,000	·01246
14,000	·00087	91,000	·01427
15,000	·00093	97,000	·01525
16,000	·00105	103,300	·01720
17,000	·00116	110,000	·01900
18,000	·00128	116,100	·02100
19,000	·00140	122,800	·02300
20,000	·00157	129,200	·02600

(*c*) Eddy or Foucault currents. If the number of magnetic lines passing through a mass of metal be in any way altered, transient currents are produced thereby in the metal tending to prevent the change. Thus, if the north pole of a bar magnet be brought up near to a metal plate, a current is generated tending to convert that part of the plate opposite to the magnet into a north pole; while when the magnet is withdrawn a reverse current arises. If the magnet be moved across the face of the plate, the action is equivalent to bringing it up toward those parts it is passing over, and withdrawing it from those parts just passed. These parts will therefore be the seats of currents tending to produce patches of opposite polarity in the plate, and following the path of the magnet. A characteristic of such eddy currents is that they are confined to the superficial parts of the conductor, dying away rapidly as they pass to the interior. The strength of such currents is directly proportional to the rate at which the number of magnetic lines is increased or diminished in the region where they flow—that is, in the case mentioned, proportional to the velocity with which the magnet moves. But the energy connected with the current is evidently proportional to its strength squared and its time of flow, and thus the loss is proportional to the square of the velocity. Since the phenomenon depends only upon the relative motion, an identical effect is produced if the metal move and the magnetic lines be stationary; in a dynamo the eddy currents are created sometimes by the one, sometimes by the other, of these conditions. In the faces of the pole pieces, they are produced by the teeth of slotted cores rotating past; in the core, by its own rotation in a field of varying intensity. The effect in the latter case is minimised by the expedient, already discussed, of laminating the iron along planes parallel to the lines and to the direction of motion.

According to Professor Fleming, the loss in watts for disc-built cores in which the discs are of not more than 1 millimetre thickness is given by

$$\text{Watts lost} = 16\,\text{V}\,(d\,\text{N}\,\text{B})^2 \times 10^{-12}$$

where V = volume of core in cubic centimetres.
N = frequency per second = revolutions per second $\times p$.
B = maximum induction in C.G.S. lines.
d = thickness of discs in centimetres.

Unfortunately the expression is of limited utility in determining the actual loss under this head in an armature; for with discs of the usual thickness the calculated loss is almost negligible, while in the constructed machines it may amount to as much as that produced by hysteresis, or in exceptional cases even more. This discrepancy is due to both the stout clamping plates at the ends of the core and the air discs in it, and even more to the eddies in the conductors of a smooth winding. This last component is roughly proportional to the width, measured along the circumference, of the bars, and reaches its maximum in any particular bar when entering or leaving the fringe of lines beyond the edges of the pole pieces. To reduce the effect large bars should be subdivided, or preferably stranded. The device of giving the bars a twist through 180 deg. midway in their length has been adopted, but is a troublesome expedient.

With regard to this particular loss, slotted cores have a great advantage, as very little of the magnetic flux enters the grooves, and the lines spring from tooth to tooth, without penetrating down through the winding, and so producing eddy currents. On the other hand, the teeth draw bands of lines across the pole faces in their rotation, so diverting to some extent the loss to another portion of the machine.

Finally, it may be said that the total loss is in most cases roughly from three to four times that calculated for the discs by Fleming's formula; while, for a given machine, it varies as the square of the speed.

Armature reaction is an effect produced by the current flowing in the coils which, while not producing any liberation of energy appreciably in the armature, causes a drop of voltage with increasing current over and above that due to ohmic resistance. In this place the general nature of its cause alone will be considered, further treatment being reserved until it arises in connection with sparking. As a first step, let the case of a current-carrying armature be considered, and the brushes be situated on the neutral axis—

i.e., upon a diameter at right angles to the line passing through the centres of the polar surfaces. If, looking upon the end of the armature, and referring to external conductors, the current flows up in the left-hand conductors and down in those on the right, it creates magnetic lines flowing upwards through the core, and downwards outside through the magnets. This will be the more apparent when it is noted that the currents in those bars on either side of a brush between the pole horns are in opposite directions, so giving no resultant vertical field.

These lines will be in the same direction as those of the magnets under the trailing horns, and in opposition to them under the other two; but, since the pole horns are nearly saturated by the magnet's lines, it follows (page 240) that the weakening of the intensity under the leading will exceed the strengthening under the trailing horns, and that the total flux of lines through the core will be reduced, with a resulting drop of voltage. To a very slight extent the hysteresis is also increased by the want of uniformity in the density. For reasons to be mentioned later, it is necessary to shift the diameter of commutation round through a certain angle, in the direction of rotation with a generator, backwards with a motor, the brushes then lying nearer to one pole horn than to the other. Denoting this angle by θ, there are then in the interpolar gap of a generator a number of conductors, occupying an angle 2θ, which give rise to magnetic lines directly opposed to the main flux, and so reduce it by the number of lines they themselves produce. This, of course, causes a further voltage drop in addition to that due to the lateral bars. The falling of the armature winding into groups of "cross" and "back" turns will be clear from fig. 94.

It may again be pointed out that, unlike losses (*a*), (*b*), and (*c*), these reactions produce only a voltage drop, not a loss of watts heating the armature.

As a result of these losses, the temperature of the armature rises when the machine is running until a stage is reached at which heat is lost from the external surface at the same rate as it is produced internally. This rejection of heat consists in part of radiation to the cooler pole faces and other surrounding parts, but more considerably of cooling by the air currents created by the rotation. In order that this latter effect may operate directly on the inner parts of the armature, air discs driving air by centrifugal force from inside the core outwards, or small fans carried at the ends of the core, are frequently adopted, while in certain cases a separate special fan and air duct, directing a blast from the end of the armature to the interior, have been used.

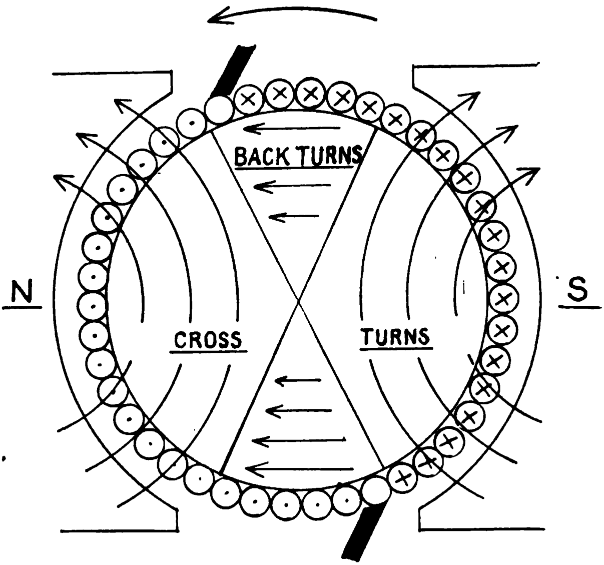


FIG. 94.

The rate of heat loss will, neglecting the comparatively small effect of special cooling devices, depend upon three quantities: (1) The area of cooling surface, which may be regarded as the area of the outer curved surface and ends of the armature; (2) the excess of temperature in the armature above that of the air; (3) the circumferential velocity. To the first two of these it is directly proportional, increasing in some smaller ratio with the last.

Since the temperature of an armature must obviously be below that sufficient to damage the insulation, and all the losses are more or less augmented by an increase of current, the ohmic resistance loss, which is by far the greatest, increasing as its square, the output of an armature is limited by its cooling surface and speed, or, so far as the current is concerned, for all practical purposes by the cooling surface.

Though exact calculation can scarcely be possible in a matter of this nature, a formula has been deduced by Esson which gives approximate results when carefully applied, viz.:—

Degrees (Cen.) rise of temperature

$$= \frac{225 \times \text{watts lost}}{\text{cooling surface in sq. cms.}}$$

$$= \frac{35 \times \text{watts lost}}{\text{cooling surface in sq. in.}}.$$

No account, however, is taken of the effect produced by a variation of speed in this expression. Messrs. Zimmermann, taking this into account, give—

Degrees (Cen.) rise of temperature

$$= \frac{645 \times \text{watts lost}}{(\text{cooling surface in sq. cms.})\ (1 + \cdot 3\ \sqrt{v})}$$

where v = circumferential velocity in metres per second.

Taking V as feet per second, and cooling surface in square inches, this becomes—

Degrees (Cen.) rise of temperature

$$= \frac{100 \times \text{watts lost}}{(\text{cooling surface})\ (1 + \cdot 5\ \sqrt{\text{V}})}.$$

Usually a temperature rise of 40 to 50 deg. Cen. is allowed in open machines above that of the surrounding air, assumed to be at 25 deg. Cen. In partially cased-in motors this is sometimes increased, and in those entirely closed, as for traction work, as far as an excess of 60 deg., but in these, of course, the full load is not on for long continuously.

In practice, as the temperature of the air rises above 25 deg. Cen. that of the armature increases more rapidly, the excess becoming greater; and in machines for ship-lighting, where in tropical climates the engine room temperature may rise to 45 deg. Cen., or higher, regard must be paid to this fact. The Admiralty, having it in consideration, specify a maximum rise of 70 deg. Fah. after a continuous full-load run of 6 hours, in a room at 120 deg. Fah., with the even more exacting condition that no accessible part of the machine shall be at a higher temperature than 150 deg. one minute after stopping.

As a general rule, an allowance of 1 square inch of cooling surface per watt lost gives a temperature rise which is well within due limits. In enclosed compact motors, however, it is often necessary to limit the allowance to little more than half this quantity, reckoned on the maximum load.

The temperature of the commutator is commonly as much as five or more degrees higher than that of the armature, but it is of less consequence, as the mica insulation is less likely to be injured; indeed, the point deserving consideration is the effect of heating and cooling in expansion and contraction, which tend to distort and dislocate segments.

The Calculations in Armature Design.—Like many other matters in electrical work, the best dimensions of an armature satisfying given conditions can only be determined by the method of trial and error, certain dimensions being assumed, those depending upon them calculated, and the resulting design considered with a view to altering the dimensions where necessary. A new design may then be roughed out, the modifications reduced or increased if found requisite, and so a satisfactory design arrived at. It is generally possible, however, to fix upon appropriate dimensions for trial, either from experience of previously constructed machines, or by the aid of formulæ suggested by Snell, Fisher-Hinnen, and others.

The original data are the voltage at the terminals, the current, and frequently the revolutions per minute. The volts drop at full load on account of armature resistance and reaction must firstly be estimated, and, unless the magnets are shunt-wound, the drop in the series magnet coils added. The sum of these and the terminal voltage is the armature voltage induced internally. In determining the resistance drop in the armature it is necessary to increase the specified current by the amount passing through the shunt in shunt and compound wound machines. For this purpose the resistance drop in the armature may be taken as between 2 and 5 per cent of the terminal voltage, and that in the series coils of the field as about half the amount. The current in the exciting coils of a shunt machine may be 2 to 10 per cent of the full-load current. The actual final percentages vary according to circumstances, but must ultimately be adjusted to satisfy any efficiency conditions. The sum of the armature resistance and reaction drop is on an average two to three times that of the former alone.

Let now E = internal armature voltage.
C = total current in amperes.
N = revolutions per minute.
T = total number of external conductors on the armature.
n = number of parallel circuits in the winding.
p = number of pairs of poles.
Z = total magnetic flux in C.G.S. lines from one pole.
D = diameter of core in centimetres.
L = length of core in centimetres.

Various formulæ have been proposed for making an approximate estimate of the size of the core. According to

Steinmetz, $\frac{\text{dia.} \times \text{length}}{\text{kilowatts}}$ is about equal to 4 for average machines of moderate size. Others have given the average size as furnished by

$$D^2L = \frac{\text{watts}}{\text{revs. per min.}} \times k.$$

where k varies from 125 in small, to 45 in large machines, being on the average 65. The above expressions refer to the dimensions in inches.

The number of turns in the winding must now be calculated, and the sectional area permitted by the space at disposal found. It has been seen (page 89) that the electromotive force induced in a conductor is given by

$$\frac{2\,Z\,N}{60} \times 10^{-8}$$

where Z is the total magnetic flux of a bipolar machine. If there be T external conductors, there will be $\frac{T}{2}$ in series for such a machine, and

$$E = \frac{T\,Z\,N}{60} \times 10^{-8}$$

or generally, for any machine, bi- or multi- polar—

$$E = \frac{T\,Z\,N}{60} \cdot \frac{p}{n} \times 10^{-8}$$

whence

$$T = \frac{60 \,.\, E \,.\, 10^8}{Z\,N} \cdot \frac{n}{p}.$$

It is now evident that before T can be calculated a value must be assigned to Z; also that if B denote the induction (in C.G.S. lines per square centimetre) in the air gap between the pole face and core,

$$Z = B \times \text{area of the pole face.}$$

The area of the pole face is usually about equal to

$$\frac{\pi D}{p} \cdot \frac{125}{360} \cdot L.$$

The density of the lines in the air gap, as in the remainder of the magnetic circuit, varies greatly in machines of different manufacture. From observations made on a number of machines, B = 6,000 appears as an average density, though in some cases it falls below 3,000. The tendency in modern design, however, is to employ a "stiffer" field, and Prof. Short has recently suggested that 12,400 is a very satisfactory value to adopt.* In the common bipolar machine it is possible to give the pole faces a relatively greater area than in other forms, and B = 5,500 may be assumed for the present purpose, in which case

$$T = \frac{E\,n}{N\,D\,L} \cdot 10^6.$$

With a smooth core the external winding space in which the T conductors must lie is equal to π D minus the length of circumference occupied by driving horns, &c.; and, according as it is proposed to dispose them in a single or double layer, the width over insulation of the wires is the space divided by T or $\frac{T}{2}$. For round wires it is only necessary to deduct twice the insulation thickness, and the net diameter of copper, from which the sectional area can be calculated, is obtained. With rectangular wire or bars the sectional area, of course, depends on the depth, which may be from one to three times the width.

For small ring-wound armatures the length of winding per external conductor may now be taken as 2 L + ·5 D, but for large sizes it is necessary to find the depth of core before the length of the end portions can be estimated. For drum-wound armatures the length per conductor is about L + length of one end connector, and the latter can be estimated without difficulty if the number of poles be decided on. The armature resistance can now be found (p. 207), and the C^2 R loss calculated.

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* Institution of Electrical Engineers (Manchester Section), January, 1901.

For the latter purpose it may be noted that the resistance of a conductor of pure copper having 1 square centimetre sectional area is ·00023 ohms per metre at a temperature of 70 deg. Cen., or, if of 1 square inch section, ·00001 ohms per foot length at the same temperature.

With a slotted core, each slot as a rule contains either one or two bars in large machines; in smaller armatures a pair of slots may contain one or more coils, each consisting of many turns of fine wire. In other words, the number of commutator segments is from one to three times the number of slots.

The width of slot should be about equal to, or somewhat less than, that of the tooth, and the depth may be taken as between two and three times the width. A deduction of 1 millimetre upwards, according to voltage, being made for insulation, the cross-section of the winding may be arrived at.

Should the $C^2 R$ loss, so found, be in excess of that allowed by efficiency conditions, it is necessary to judge by how much R must be reduced, and to adopt a larger core, or reduce T by increasing N or B. If not apparently excessive, the next point is to make trial of the temperature limit (p. 213), and, in the case of a ring, to sketch a portion and ascertain how the conductors may be arranged internally. For this purpose the depth of the core must be determined, on the basis of some selected core induction B_c. This varies from 17,000 in bipolar, to 10,000 or less in large multipolar machines. If d denote the core depth, and L′ the length of the armature which is net core discs,

$$d = \frac{Z}{2\,B_c\,L'}.$$

In slotted cores the depth d is taken from the bottom of the slots to the inner surface. The density in the teeth themselves is commonly from 19,000 to 23,000.

It may now well happen that in the internal circumference, rendered even less by the insulation on the core, and reduced by the space required for the driving arms and their insulation, the conductors cannot conveniently be got in with a reasonable number of layers, and a larger core is seen to be necessary. This will particularly be found to be the case with small rings, in which the depth of core is proportionately greater. With drums the difficulty does not, of course, present itself, and a lower value of B_c may conveniently be adopted in the smaller sizes.

Should the conditions of space and heating be satisfied, it yet remains to ascertain whether there is any possibility of the cross armature turns when carrying the full current reversing the direction of the magnetic lines under the leading pole tips.

The cross turns may be taken as equal to $\frac{N \phi C}{360\, n}$ where ϕ is the angle subtended by the pole piece. If this be taken as $\frac{125}{360\, p}$ the magnetising force is given by $\frac{\cdot437\, N\, C}{n\, p}$. This force produces lines passing as shown in fig. 94, and as the iron section of their path is large, almost the whole resistance consists of the air gaps. Denoting the length of the air gap by δ, the strength of the reversing field,

$$\frac{\cdot437\, N\, C}{2\, \delta\, n\, p}, = \frac{\cdot22\, N\, C}{\delta\, n\, p}, \text{ approximately,}$$

and it is essential that this should be less than B, the impressed field in the air gap.

The absolute necessity for the above relation, and the ready manner in which it may be obtained by reducing the number of conductors under a pole face, is one of the points rendering multipolar machines advantageous for large outputs. Thus, it is common to employ four poles for outputs of more than 50 kilowatts, though the type lends itself well also to smaller sizes. The other alternative is to employ a stronger field, so increasing B, and at the same time reducing N.

The choice of speed, so far as the machine itself is concerned, depends upon the limiting peripheral velocity, and is determined by the constructive details. English manufacturers have commonly adopted a speed of 2,000 ft. to 3,000 ft. per minute for the circumferential velocity, but in American and Continental machines it may be sometimes found as high as 6,000 ft. or even 7,000 ft. per minute. The rotational speed is in many cases determined largely by the nature of the driving power, extremely high speeds being reached in turbo-generator sets, and some very low ones with large generators driven by marine-type engines.

In the course of the preceding calculations the extent to which the originally assumed dimensions may be reduced, or must be increased, will have been ascertained, and probably an alteration made to that value of B originally chosen. At this stage, the commutator may receive attention, as under some circumstances commutation may necessitate an alteration in the winding. The number of segments should be as great as is practicable from a mechanical point of view within the limits $\frac{T}{2}$ in the case of drums, and T in the case of rings, these being the maximum numbers possible respectively. It is often permissible to employ fewer, but in high voltage machines the number should be sufficiently great to prevent the potential difference between adjacent segments exceeding 45 volts. The diameter should be as small as possible consistent with accommodation of the segments, the width of which, for all but the smallest machines, may be taken as about $\frac{3}{8}$ in., and the peripheral speed should not exceed 2,600 ft. per minute, as with velocities greater than this it is difficult or impossible to prevent sparking. The length of the commutator is determined entirely by the current to be collected, and by the current density allowed in the brushes. The point will be discussed fully when dealing with brush gear, and some remarks are also to be found at page 204.

A résumé of the operations may now be given, and the preceding methods illustrated. These are—

(1) Assumption of external core dimensions.
(2) Calculation of pole-piece area, and so of Z.
(3) Calculation of T and section of conductor.
(4) Consideration of magnetic density in the teeth, and determination of core depth.
(5) Comparison of $C^2 R$ loss and cooling surface.
(6) Consideration of armature reaction.

Let the case of an armature for a four-pole machine, to give 240 amperes with a terminal pressure of 210 volts and a speed of 500 revolutions be taken—the core to be slotted, and the magnets to be shunt wound. For a machine of this output—50 kilowatts—a loss of about 2½ per cent may be allowed, both for the field and for armature resistance. This gives a loss of 1,250 watts in the exciting coils, the shunt current thus being about

$$\frac{1250}{210} = 6 \text{ amperes},$$

and the total armature current 246 amperes. If the armature be parallel wound, the current in any conductor is

$$\frac{246}{4} = 61{\cdot}5 \text{ amperes},$$

and as the loss is to be 1,250 watts, the resistance of the whole winding must be

$$\frac{1250}{(61{\cdot}5)^2} = {\cdot}33 \text{ ohms}.$$

The armature resistance (page 207) is thus

$$\frac{{\cdot}33}{4^2} = {\cdot}02,$$

and the voltage drop over it is

$${\cdot}02 \times 246 = 4{\cdot}92.$$

The internal voltage must consequently be

$$210 + 5 = 215.$$

Assuming a core of 18 in. diameter, and of the same length, the area of pole piece is

$$\frac{18\,\pi}{2} \cdot \frac{125}{360} \times 18\,;$$

giving the area of the air gap when fringing is allowed for as 170 square inches. With

$$\mathrm{B} = 8000, \text{ or } 53500 \text{ per square inch}\,;$$
$$\mathrm{Z} = 9100000\,;$$
$$\mathrm{T} = \frac{220 \times 60 \times 10^8 \times 2}{9100000 \times 500 \times 2} = 284.$$

The length of a single conductor may be taken as about 22 in., and the resistance of one connector as equal to that of about 8 in. of the conductor. The sectional area of the latter, then, must be

$$\frac{284\ \ 30 \times 66}{\cdot 33 \times 10^6} = \cdot 017 \text{ square inch.}$$

If two bars be placed in each slot, 142 slots are required, the width of a slot and tooth being

$$\frac{18\,\pi}{142} = \cdot 4 \text{ in.}$$

If ·1 × ·17 be taken as the dimensions of the bars, the width of slot, allowing for insulation, is ·18, and the mean width of tooth ·20 in. Allowing for insulation between the discs, the net iron in the cross-section of a tooth is ·20 × 18 × ·87 = 3·15 square inches, and the number of teeth under a pole may be regarded as the nearest integer to

$$\frac{142}{4} \cdot \frac{125}{180} + 1 = 36 \text{ or } 37.$$

The total lines therefore pass into 36 × 3·15 = 113 square inches of iron, and the induction in the teeth is 80,000 per inch. If the slots are half an inch deep, sufficient room is given for the bars; and a depth of 4½ in. for the disc below these gives a net iron section of $4\frac{1}{2} \times 18 \times {\cdot}87 = 70$ square inches. The induction in the core is then 65,000 per inch.

The cooling surface of the armature is $(18\,\pi \times 18) + \pi\,81 = 1{,}270$ square inches; the C^2R watts lost in it are 1,250; there is then sufficient cooling surface, and it only remains to test the armature reaction.

Taking the air gap as ·3 cm., the reversing field is

$$\frac{{\cdot}22 \times 246 \times 284}{2 \times 4 \times {\cdot}3} = 6350.$$

and as the impressed field is 8,000, no trouble arises in this respect.

The iron losses on account of hysteresis are in the teeth

$$1{\cdot}575 \times 142 \times \frac{1000}{60} \times {\cdot}0012 = 44 \text{ watts (see page 207)},$$

and in the core

$$3160 \times \frac{1000}{60} \times {\cdot}009 = 474 \text{ watts},$$

or in all some 520 watts.

The centrifugal force tending to throw the bars out of the slots is, for each bar, given by

$$\frac{(\text{weight of bar})\,(\text{linear velocity in feet per second})^2}{32\,(\text{radius of core in feet})} \text{ lbs.},$$

and the stress on the binding wires, neglecting friction, is equal to $\frac{1}{2\,\pi}$ the total force on all the bars. In this case the volume of a bar is ·375 cubic inches, and, since a cubic inch of copper weighs ·32 lb., its weight is ·12 of a pound,

$$\frac{284\,({\cdot}12)\left(\pi\,1{\cdot}5 \times \frac{500}{60}\right)^2}{32 \times 2\,\pi \times {\cdot}75} = 350 \text{ lbs.}$$

With regard to the relative merits of the different forms of core and winding, it should be noted that slotted cores afford greater protection from mechanical injury to the conductors than smooth cores, besides shielding them from eddy currents, and that, owing to the smallness of the air gap, less exciting power in the field is required to produce the same magnetic flux. On the other hand, for the same external diameter as a smooth core the internal diameter is smaller, which is frequently a serious disadvantage for ring windings. Their armature reaction and the self-inductance of their coils are both much greater than with smooth cores. Ring windings have an advantage over drums for high-voltage machines, inasmuch as the difference of voltage between adjacent coils is merely that lost over their resistance, while in a drum it reaches the full voltage of the machine. It is sometimes, too, convenient that the commutator may have as many, or even twice as many, segments as there are armature bars. The bad points of the ring are the resistance of, and room required by, the internal wires, and the necessity for non-magnetic hubs. Drum windings are free from the objections mentioned, and can be more simply applied to multipolar machines. The core can be made of any depth, or solid on the spindle, so minimising hysteresis loss. Owing to this and the shorter winding, armature reaction, self-inductance, and heating are reduced, so permitting, for the same output, the drum to be some 10 per cent or more smaller than the ring. The occasional disadvantages may be inferred from the remarks on rings. There still remains the question of a lap or wave winding. Lap windings are only applicable to parallel-wound armatures, while wave windings may be employed for both these and series-wound armatures. The latter may, moreover, have an equal pitch for their connectors at each end; but lap windings require a difference of pitch between the two. On the whole the lap winding is more convenient and more commonly employed.

CHAPTER IX.

Field Magnets and Exciting Coils.—Except in the case of extremely small dynamos used for exploding mines and a few other purposes, the magnets which produce the magnetic flux through the armature are excited by coils receiving their current from the machine itself. The question of how, under these circumstances, the action of the machine can begin may present itself perhaps as a difficulty; but, as may be seen from fig. 14, the magnets retain a certain amount of "residual" magnetism after the exciting current is removed, and this is usually sufficient to induce an electro-motive force in the armature on starting. The current thus produced and the excitation then increase together until the normal running conditions are established.

According to the manner in which the coils receive their current they are classified as shunt or series coils. A shunt is connected across the terminals of the machine, and its exciting current must thus be carried by the armature over and above that supplied to the external circuit. Since there is the full electro-motive force of the machine driving the current through, and the latter must necessarily be small—from about 10 per cent of the whole in a 1 kilowatt to about 1 per cent in a 500 kilowatt machine—the resistance must be considerable, and a conductor of small section is employed; while to secure the necessary exciting power with the small current the coil must contain many turns. A series winding is connected between a brush and terminal, and is consequently traversed by the entire current flowing in the circuit; it requires, therefore, but few turns, but in order that the voltage drop over it may be small—from 10 to 1 per cent for the outputs mentioned above—it must be large and of low resistance.

For reasons to be discussed later, it is often advisable to furnish the magnets with both shunt and series coils, together forming a "compound" winding. In these cases the shunt may be either connected across the brushes or across the terminals, being a "short" shunt in the former case, a "long" shunt in the latter.

It has been seen that electric and magnetic circuits always exist together, like two interlinked rings. There is, nevertheless, this difference between the two, that while the current can be compelled to flow in the path followed by the conductor, the magnetic flux issuing from one end of the coil can only to a limited extent be compelled to pass in a particular path externally to the other end. With tolerable completeness it may be led through any magnetic circuit of high permeability, but some leakage always occurs (except with a closed ring wound all over), pursuing other paths, and this is more markedly so where the permeable path provided is interrupted by air gaps, as is the case between the pole faces and the core of a dynamo.

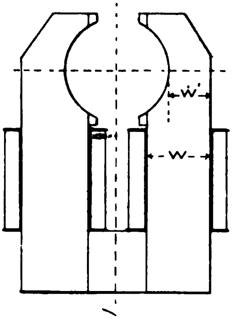

FIG. 95.

For the magnetic circuits of dynamos nearly every conceivable form has at one time and another been adopted, but certain shapes, approved by practice, are more or less becoming standard types. With British manufacturers the single-circuit bipolar field has long been favoured, even to the extent of employing it for outputs for which the multipolar would be more appropriate. In the "over-type" form, fig. 95, it is commonly employed for belt-driven, and as the "under-type," fig. 96, for direct-driven machinery, the lowness of the spindle in the latter being adapted to the height of the shaft in vertical engines.

In order that the induction in the air gap may be fairly uniform, it is necessary to make W′ somewhat greater than half of W; and since the flux becomes densest along the path of least resistance—that is, if the core is central, in the region nearest to the coils—it is well to have the armature eccentric to a slight degree in over-type machines. This expedient at once tends to promote uniform density of flux, and to allow of the bearings wearing down somewhat without too great reduction of the armature clearance. If too eccentric, the inequality of density is reversed, but the weight of the armature is partly taken from the spindle by the upward magnetic pull. For obvious reasons, mostly the reverse of the preceding, this arrangement is not advisable for an under-type machine.

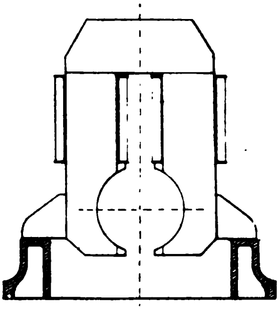

FIG. 96.

The magnet limbs are usually of wrought iron, but in over-type machines their yoke is frequently cast with the bed plate. As the proportions do not admit of a pole face of 120 deg. being machined from the solid, in most cases wrought or cast iron pole horns are attached.

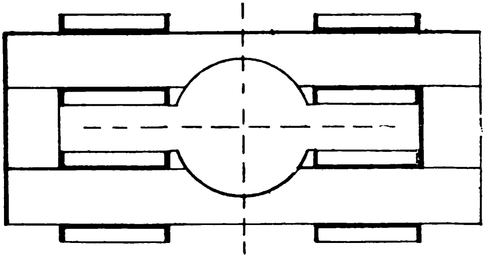

FIG 97.

To prevent considerable magnetic leakage, it is necessary that under-type magnets should be carried, and separated from the iron base plate, by brackets of non-magnetic material, fig. 96, usually gun-metal or zinc. The gap between the magnets and iron bed should be from 4 to 7 inches for machines of 12 to 24 inches diameter of armature, the length in any particular case depending upon the general design.

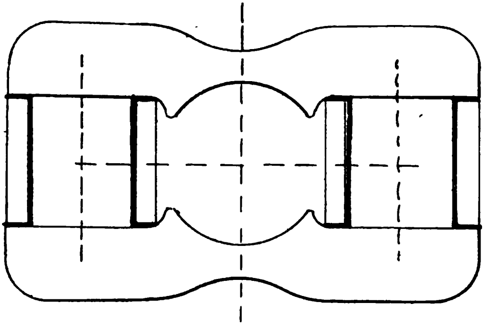

FIG. 98.

For armatures in which the diameter is much greater than the length, a lighter but somewhat more expensive field may be obtained by providing a double magnetic circuit, fig. 97. The type is employed both vertically and horizontally, and no magnetic insulation is required for its support. Another bipolar field, the "Manchester" type, originally used by Messrs. Mather and Platt, is shown in fig. 98. It has a better appearance than the others mentioned, but a greater weight and leakage. The cores are of wrought and the yokes of cast iron; where weight is objectionable the latter, however, may be of wrought iron. Fields in which the coils are surrounded by the magnetic circuit, and in which consequently there is no leakage around the machine, figs. 99 and 100, are termed "ironclad,"

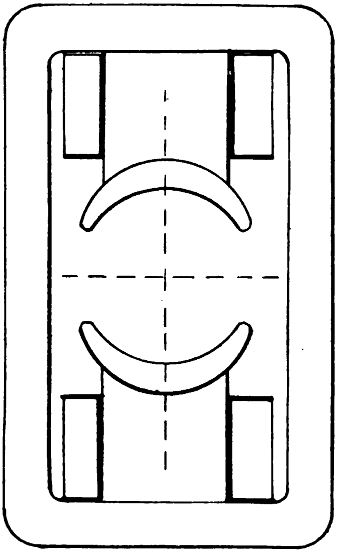

FIG. 99.

and are often preferred for ship lighting. Other bipolar fields are shown in figs. 101 and 102.

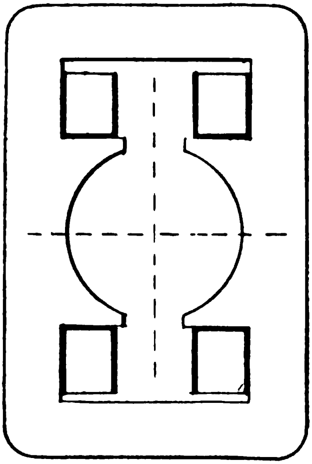

Fig. 100.

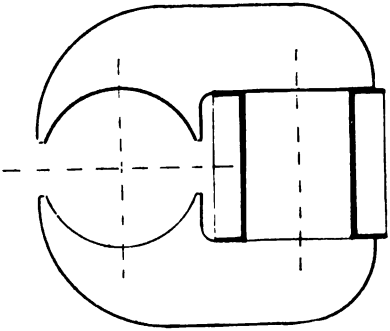

Fig. 101.

Though bipolar machines have been constructed for outputs of 180 or more kilowatts, there is a limit at which the

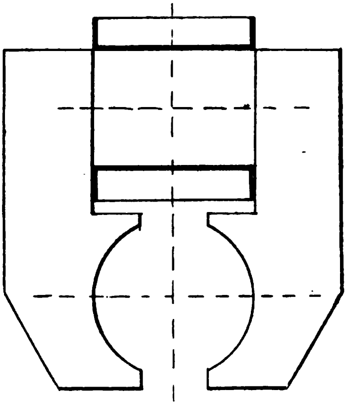

FIG. 102.

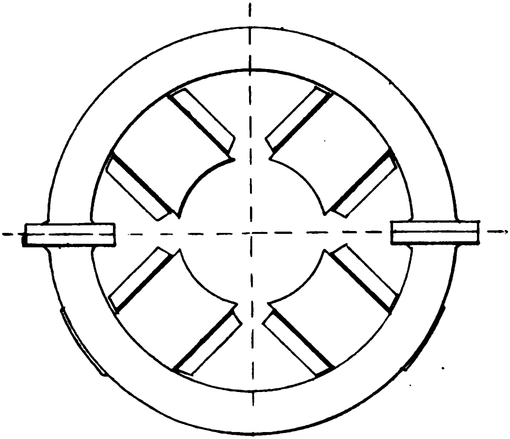

FIG. 103.

adoption of a four-pole field is more economical. The most common form, fig. 103, consists of a cast yoke ring, within which the wrought-iron cores are secured by screws. Figs. 104—106 show other forms, the last being under the disadvantage of requiring magnetic insulation from its bed.

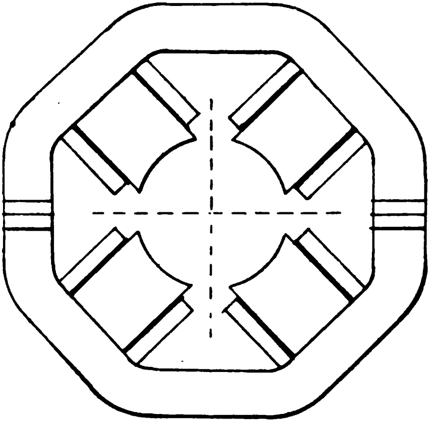

FIG. 104.

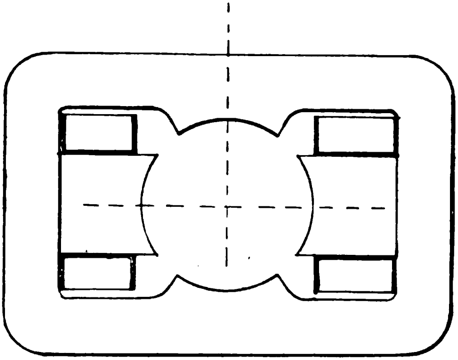

FIG. 105.

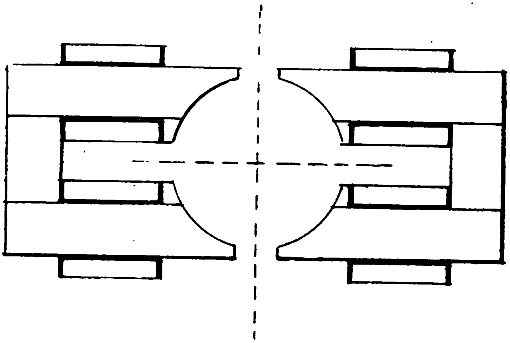

Fig. 106.

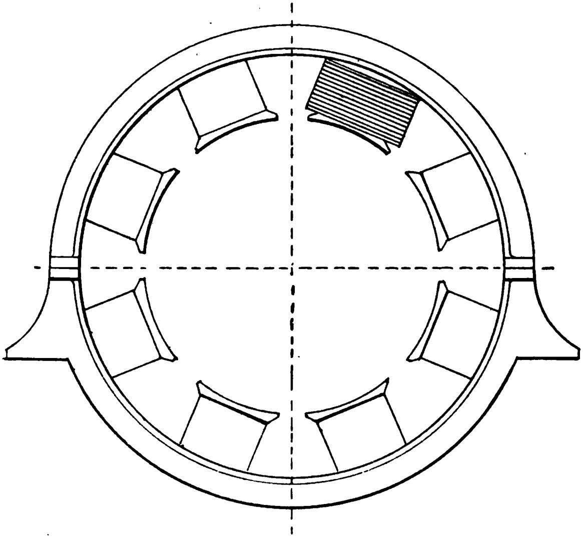

Fig. 107.

Multipolar fields, having six, eight, ten, or any greater number of poles, may advantageously be adopted for machines of increasing output. The form which is becoming standard for such is shown in fig. 107, and consists of a cast-steel yoke ring, having facing pieces cast inside for the poles, divided into halves bolted together on the horizontal diameter, and so allowing the upper part to be removed for access to the armature. The lower part carries the feet, whereby the whole is secured to the base plate. The cores are usually of wrought iron, circular or rectangular in section, fitted with separate pole pieces, which are bored after erection. By some manufacturers the cores are built up of laminated sheet iron, and set in the yoke-ring mould, so that the ring is cast solid round the core ends, making them undetachable.

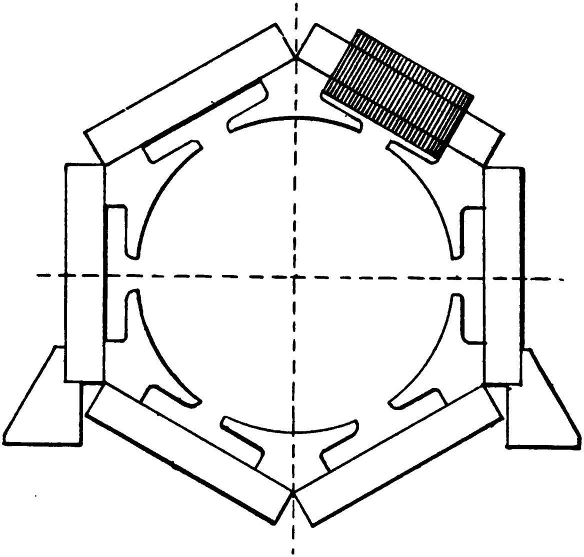

FIG. 108.

Fig. 108 is a multipolar design frequently used by continental makers, but there is little in its favour; it is expensive, but the coils are more exposed, so keeping cooler, than when radial, and a non-magnetic bracket is necessary for its support. The "over-hung" type of armature has also, to a limited extent, been employed on the Continent in conjunction with an inside field, fig. 109; a satisfactory mechanical design, however, presents much difficulty.

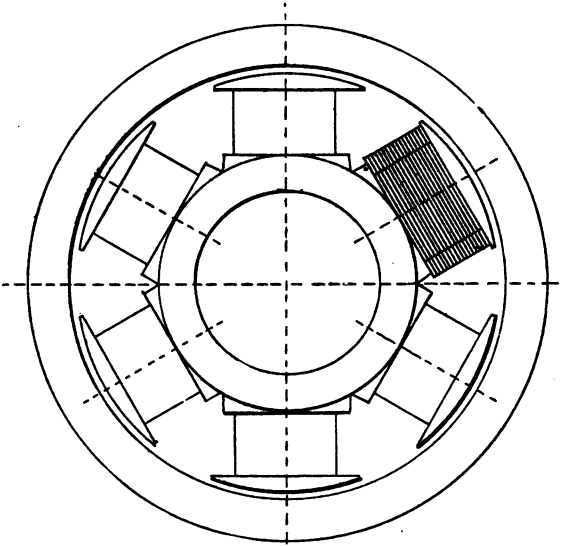

FIG. 109.

The fields for disc machines usually consist of a number of cylindrical cores, fitted with trapezoidal pole pieces, and attached parallel to the axis round yoke rings of cast iron. As before mentioned, alternate poles on one ring are of the same polarity, similar poles on the two rings being opposite each other. In a few machines, however, a positive pole on one side is opposite a negative pole on the other. In either way, the tracing out of the magnetic circuit presents no difficulty. A few small machines are made with "unipolar" or "homopolar" fields, having a single pole face round the armature. In these the magnetic flux is produced by a coil surrounding the spindle, and passes through the latter to the core, fig. 110, thence by the cast-iron casing back to the spindle, which must be of large diameter to prevent saturation. The pole in this type of machine may embrace a greater arc than 125 deg., an arc of 180 deg. not being excessive.

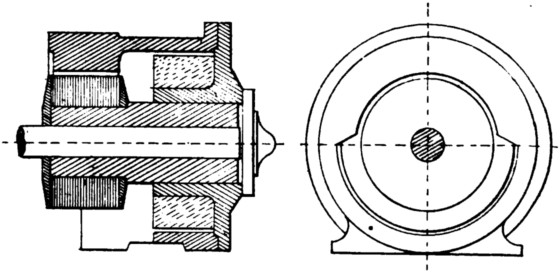

FIG. 110.

For a given armature diameter and flux, the weights of the fields shown have approximately the following ratios :—

Fig. 95	1
Fig. 97	·87
Fig. 98	1·15
Fig. 99	·9
Fig. 105	1·1
Fig. 106	1

For traction purposes, where it is essential that minimum weight, compactness, and efficiency should be combined, the four-pole field, with the cast-steel box serving also as the yoke, has become the standard for the motors.

In manufacture, the magnets, when of regular form, are usually rough planed or turned, all corners running lengthwise being cut to a small radius; they are then bolted to the yoke, fitted with the pole horns, and bored to a radius allowing mechanical clearance for the armature. In the multipolar type, fig. 107, the magnet cores, when detachable, may be fitted into holes in the yoke, or simply screwed up against the faced seat, the latter being the more usual method.

The exciting coils are wound on formers of sheet iron of about No. 17 B.W.G. for medium sizes, or of some insulating material—vulcasbeston, presspahn, etc. When of iron, the former is lined with insulation, and wooden end flanges are often added. It is, in most cases, necessary to line the end flanges with wood, through which a channel may be cut to receive the entering and leaving ends of the wire; in this respect the other materials are more convenient, as the channel can be simply moulded in the flange. These formers are made an easy fit over the magnet cores, and when rectangular may well be curved on two sides, fig. 111, as the curved surface is more conducive to a tight and even winding, while the space between the coil and magnet permits a certain amount of ventilation. The winding, when the conductor is of small or moderate size, is accomplished by packing across the inside with wooden bars and spinning in a lathe slowly, the turns being well driven down by blows from a mallet. In large generators, the series winding is

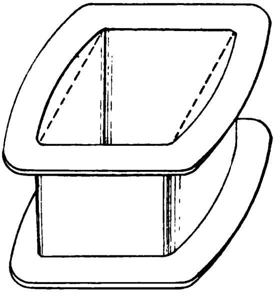

FIG. 111.

often of heavy copper strip (for example, in some recent machines, 1½ inches wide, by an eighth thick) wound on edgeways, or rather forged to shape in that position, and subsequently covered with insulation.

In some cases, to obtain greater cooling surface, the conductor in the field coils is wound in such a manner as to present a corrugated surface; while frequently in smooth coils the depth of winding is reduced at the ends, the surface sloping down to a narrow flange.

With compound-wound coils it is the usual practice to place the shunt internally and to wind the main turns outside, thus affording a protection to the finer and more easily injured shunt wire. In some large machines the main is wound upon a separate former, which slides over the shunt coil and is independently removable.

Calculation of the Field Winding.—The work of Bosanquet, Rowland, and Kapp has shown that a law analogous to Ohm's law applies to the magnetic circuit, with the important difference, however, that while the resistance of an electrical conductor at constant temperature is independent of the current density, that of an iron or steel magnetic circuit varies considerably with the induction.

The analogue of electro-motive force is the magneto-motive force H, which is equal to $\frac{4\pi}{10}$ times the ampere turns encircling the core of the magnet, while the induction corresponds to current density, and permeability to specific conductivity. Thus, for a conductor of sectional area A square centimetres, of length L centimetres, and specific conductivity σ, the current flowing under a voltage V is

$$\frac{V A}{\sigma L}.$$

For a magnetic conductor of the same dimensions, and of permeability μ, the total flux Z is

$$\frac{\mu H A}{L}$$

except in so far as affected by leakage, which, as previously pointed out, cannot be prevented in a magnetic circuit. The quantity $\frac{L}{\mu A}$ is the magnetic resistance or "reluctance," analogous to electric resistance.

If the magnetic circuit lie partly in air, partly in iron, as is the case in a dynamo, the total reluctance is the sum of the reluctances of the separate parts. Assuming for the present that no leakage exists, the ampere turns necessary to produce a flux F through a circuit of which the different parts have lengths, sectional areas, and permeabilities, $L_1 . A_1 . \mu_1$, $L_2 . A_2 . \mu_2$, etc., can be calculated by applying the foregoing expression to each part separately, and taking the sum of all for the total required. For

$$H = \frac{4\pi}{10} \text{ ampere turns} = Z \times \text{reluctance},$$

$$\text{ampere turns} = \frac{Z}{1{\cdot}26}\left(\frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} + \frac{L_3}{\mu_3 A_3} + \text{etc.}\right)$$

If B be reckoned per square inch, and lengths be taken in inches, this becomes—

$$\text{Ampere turns} = \frac{Z}{3{\cdot}2}\left(\frac{L_1}{\mu_1 A_1} + \frac{L_2}{\mu_2 A_2} + \frac{L_3}{\mu_3 A_3} + \text{etc.}\right)$$

For practical calculation, a curve giving the relation between the induction and the ampere turns necessary per inch length of the circuit is most convenient, and may be readily constructed from the B and H curve of the metal. Fig. 112 is such a curve for materials of the average quality.

As an illustration of the calculation of exciting power, let a flux of 9,000,000 lines be required through the circuit of fig. 113, for the present assuming no leakage to occur. The length of the armature path is 18 in.; its net iron section, reckoning both paths, is 97 square inches; in it B is then $\frac{9{,}000{,}000}{97}$, or 93,000, and from the curve for wrought iron the corresponding ampere turns per inch are seen to be 40. The

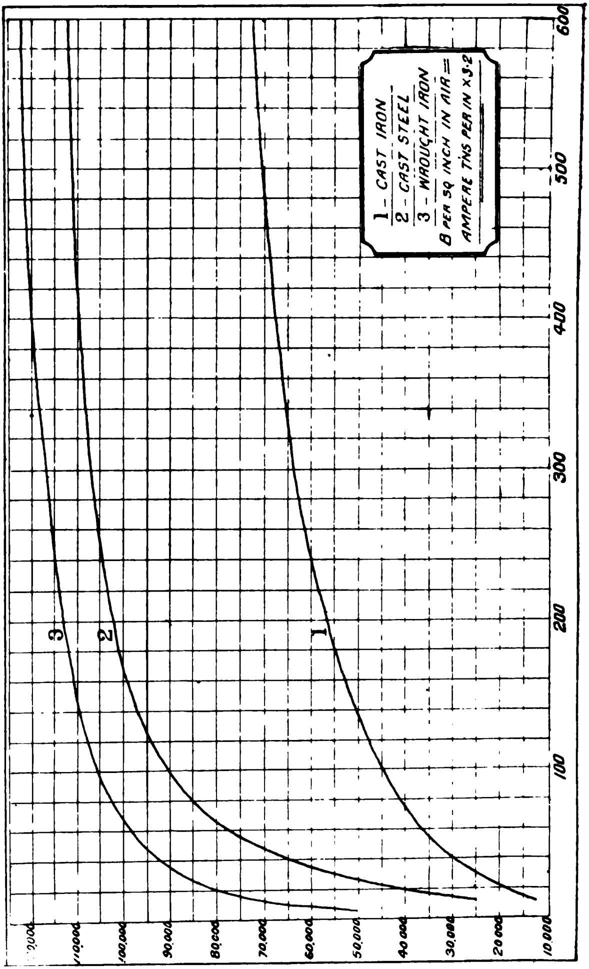
1 - CAST IRON
2 - CAST STEEL
3 - WROUGHT IRON
B PER SQ INCH IN AIR = AMPERE TNS PER IN × 3.2
1
2
3
110,000
100,000
90,000
80,000
70,000
60,000
50,000
40,000
30,000
20,000
10,000
100
200
300
400
500
600

FIG. 112.

total required for this part of the circuit is therefore $40 \times 18 = 720$.

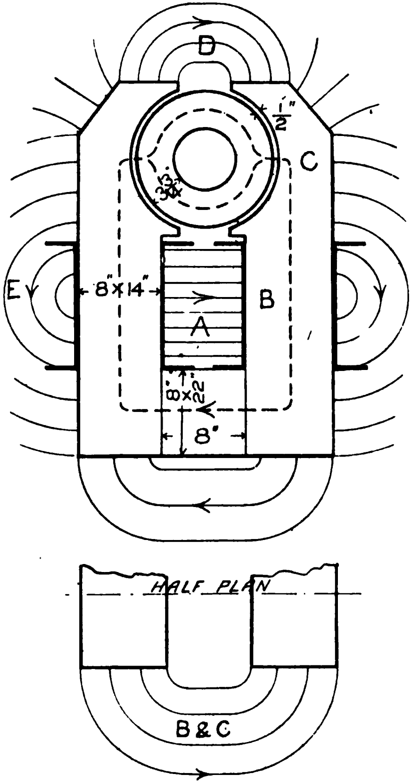

FIG. 113.

The length of the air gaps is $2 \times \frac{1}{2}$, or 1 inch; the area of the pole faces is $\pi \times 7 \times 14 \times \frac{130}{180}$, but, since the lines fringe out into the core, the length of the arc may be increased at each end by about four-fifths the length of one air-gap. The air section thus becomes 214 square inches, and B then is $\frac{9,000.000}{214}$, or 42,000. Since the permeability of air is unity, the ampere turns per inch length must be 13,100.

The two wrought-iron limbs have a total length of 54 in., and their cross-section is 112 square inches; B is then 81,000, and the ampere turns per inch 22·2, or in all, 1,200.

The length of cast-iron yoke is 8 in., its cross-section 176 square inches; B therefore is 51,000, and the corresponding ampere turns are 147 per inch, giving 1,176 for the whole yoke.

The ampere turns required for the various parts of the circuit are thus—

Core	720
Air gaps	13,100
Limbs	1,200
Yoke	1,176

The total ampere turns must, then. be the sum of these, or 16,196, about 16,200 in round numbers. To these would be added, in designing a machine, a number compensating for armature back turns and reaction.

In this example the armature is smooth, and the method of reckoning the air-gap length and area simple. With a slotted core, the question arises as to what dimensions are to be taken as representing these quantities, and, while approximate values may be obtained by taking the distance from the pole face to the faces of the teeth as the length, and the mean of the polar area and that of the tooth faces beneath it as the cross-sectional area, a more exact method is desirable. The preferable way is to sketch a tooth and gap, fig. 114; the lines may then be assumed to follow the paths shown, and the reluctance of the gap for each tooth is per unit length of core $\frac{L\,L'}{L\,W' + L'\,W}$, the conductivity being $\frac{W}{L} + \frac{W'}{L'}$, L' denoting the average length of the lines entering the sides of the teeth. If there be N teeth under a pole, the total conductivity may be taken as $(N+1)\left(\frac{W}{L}+\frac{W'}{L'}\right)$, to allow for the fringing at the pole tips.

So far magnetic leakage has been left out of consideration for the sake of simplicity. In reality, however, it is far from negligible, and must be taken into account for even an approximate determination of the exciting power required. That the leakage does occur all around the magnets of a machine is sufficiently evident by the attraction on iron bodies held near, and by the baneful effect on ordinary watches, while the general direction of its lines is shown in fig. 113.

The disadvantageous effects resulting from this leakage are—(*a*) that a number of lines, which do not enter the armature, travel for a part of their length in some portion of the iron circuit, thereby lowering its permeability; and (*b*) eddy currents are produced in all moving masses within the influence of the leakage lines.

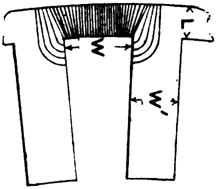

FIG. 114.

It is obvious that the greatest number of combined useful and leaking lines pass through the magnet underneath the exciting coils, and that the induction in the metal there is due to the sum of the useful lines Z, and that of the leakage Z_l. To ascertain the permeability—a point which is necessary for the calculation of the requisite ampere turns—it is therefore necessary to know some relation between Z and Z_l. The most convenient expression for use is

$$\frac{Z + Z_l}{Z},$$

and this has been published by Wiener for a number of different types and sizes of machine. A few of the more ordinary cases are presented from his paper, but in general his values give results which are slightly excessive.

Values of $\frac{Z + Z_l}{Z}$.

Output.	Over-type bipolar.	Under-type bipolar.	Manchester field.	Four-pole (fig. 103).	Multi-polar (fig. 107).	Multi-polar (fig. 108).
Kilowatts.						
1	1·45	1·65	1·8	..	..	..
5	1·35	1·55	1·65	1·65	1·4	1·8
10	1·28	1·45	1·55	1 55	1·32	1·65
25	1·25	1·4	1·5	1·5	1·3	1·6
50	1·22	1·35	1·45	1·45	1·28	1·55
100	1·2	1·3	1·4	1·4	1·25	1·50
200	..	1·25	1·38	1·38	1·22	1·45
300	..	1·2	1·35	1·35	1·2	1·4
500	..	..	..	..	1·18	1·35
1,000	..	..	..	..	1·16	1·3
2,000	..	..	..	..	1·15	1·25

Regarding the matter from a simple point of view, and considering its analogy to an electric circuit, the leakage is evidently proportional to the magnetising force of the coils, and inversely as the resistance of the air paths, or

$$Z_l = \frac{\text{total magnetising force}}{\text{air resistance}}.$$

For a given type of machine the air resistance varies inversely as the linear dimensions, for it is equal to

$$\frac{\text{length between surfaces}}{\text{area of leaking surfaces}}.$$

Hence for the same magnetising force, Z_l increases with the linear dimensions; but for the same inductions the magnetising force itself must be proportional to the linear dimensions, and consequently Z_l increases as their square. Since the useful flux, with constant densities, also increases on the square of the dimensions, the ratio

$$\frac{Z + Z_l}{Z}$$

should be constant for similar machines of different sizes.

The apparent discrepancy between this theoretical relation and the values given in the above table arises from the fact that as the size of the machines increases the length of the air gaps becomes relatively smaller, and hence the magnetising force necessary for the same induction increases less rapidly than the linear dimensions.

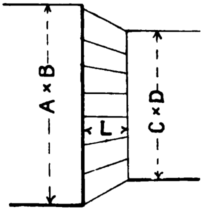

FIG. 115.

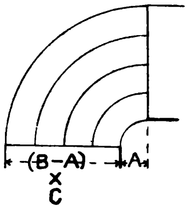

FIG. 116.

The air resistance may be roughly calculated for a circuit of given dimensions by means of formulæ suggested by Prof. Forbes. The quantity actually given by these is the air conductivity—*i.e.*, the reciprocal of the reluctance. Four different cases of leakage arise for consideration :—

(*a*) **Between two opposite and parallel planes, fig. 115, in which case**

$$\frac{1}{R_m} = \frac{A \times B + C \times D}{2 L}$$

or, using inches,

$$\frac{1}{R_m} = 1{\cdot}27 \frac{A \times B + C \times D}{L}.$$

(*b*) **Between two planes at right angles, fig. 116, when**

$$\frac{1}{R_m} = \frac{2 C}{\pi} \log_\epsilon \frac{B}{A} = 1{\cdot}466 \log_{10} \frac{B}{A} \times C$$

or, using inches,

$$\frac{1}{R_m} = 3{\cdot}73 \log_{10} \frac{B}{A} \times C.$$

(*c*) **Between two flat surfaces lying in the same plane, fig. 117, in which case, if not far apart,**

$$\frac{1}{R_m} = \frac{C}{\pi} \log_\epsilon \frac{B}{A} = {\cdot}733\, C \log_{10} \frac{B}{A}$$

or, using inches,

$$\frac{1}{R_m} = 1{\cdot}86\, C \log_{10} \frac{B}{A}.$$

(*d*) **Between surfaces similar to** (*c*) **when farther apart, fig. 118, when**

$$\frac{1}{R_m} = \frac{C}{\pi} \log_\epsilon \frac{\pi (B - A) + 2 A}{2 A} = {\cdot}733\, C \log_{10} \frac{\pi B - (\pi - 2) A}{2 A}$$

or, using inches,

$$\frac{1}{R_m} = 1{\cdot}86\, C \log_{10} \frac{1{\cdot}57 B - {\cdot}57 A}{A}$$

Even with these formulæ cases occur to which they do not directly apply, but approximate combinations and modifications may be made; on the whole, however, a considerable amount of judgment is required in their application.

In an existing machine the ratio $\frac{Z + Z_l}{Z}$ may readily be determined by looping a turn of wire round that part of the magnet which carries both Z and Z_l —this is really in the middle of a coil, but may be taken at its end—and another turn round the armature. If the ends of these turns be connected in succession to a ballistic galvanometer through a suitable resistance, the galvanometer readings on making or breaking the exciting circuit are proportional to the lines passing through the loops—that is, to $Z + Z_l$ and Z respectively.

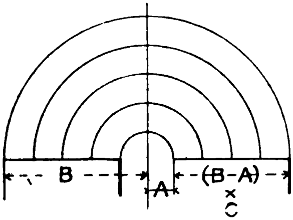

Fig. 117.

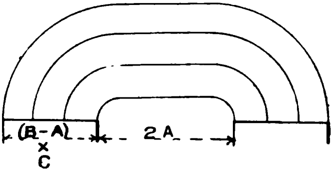

Fig. 118.

As an illustration of the application of these formulæ, the magnetic conductivity, or "permeance" of the air about the magnets, fig. 113, may be roughly calculated.

The conductivity of the space A between the vertical limbs is by (*a*)

$$1{\cdot}27 \, \frac{2 \times 12 \times 14}{8} = 53{\cdot}4,$$

the length of the limbs from the lower edge of the pole pieces to the upper surface of the yoke being 12 in.

That in the regions B at each side of the magnets is by (*d*)

$$2 \times 1{\cdot}86 \times 12 \times \log_{10} \frac{1{\cdot}57 \times 12 - {\cdot}57 \times 4}{4} = 28.$$

As the magnetic pressure at the top of the coils between limb and limb is the entire force produced by the coils, while beneath them it falls to almost nothing, the leakage is on an average produced by half the number of ampere turns. This may be allowed for by halving the conductivity, so taking the above as 26 7 and 14 respectively.

In the spaces C across the limbs above the coils, taking approximate dimensions for the equivalent rectangular areas, the conductivity is given by (*d*) as

$$2 \times 1{\cdot}86 \times 14 \times \log_{10} \frac{1{\cdot}57 \times 12 - {\cdot}57 \times 5{\cdot}5}{5{\cdot}5} = 23{\cdot}7.$$

Across the flat tops of the poles, also by (*d*),

$$1{\cdot}86 \times 14 \times \log_{10} \frac{1{\cdot}57 \times 12 - {\cdot}57 \times 3}{3} = 21{\cdot}3\,;$$

and between the outer sides of the magnet and the yoke, by (*c*),

$$1{\cdot}86 \times 14 \times \log_{10} 14 = 30.$$

Since, though there are two such regions, but half the ampere turns act in each. Leaving out of consideration other paths, the total air conductivity is thus the sum of the above, or, roughly, 116.

The calculation of ampere turns for the useful flux gave 16,200, the magnetising force of which is 16,200 × $\cdot 4\,\pi$ = 20,400, and the leakage is therefore 20,400 × 116 = 2,370,000 lines.

$$\frac{Z + Z_l}{Z} = \frac{11370000}{9000000} = 1{\cdot}26.$$

The induction in the magnets under the coils is increased by the leakage to 11,370,000 ÷ 112 × 101,000, and the ampere turns for this induction are seen from the curve to be 73 per inch. Hence the 1,200 allowed on page 242 must be increased in the ratio of 22·2 to 73.

With this addition to the exciting power the leakage is also further increased proportionally, so that it is well to be liberal in allowance of the ampere turns.

The total number of ampere turns having been found, the size of the wire in a shunt winding is determined by the permissible loss of watts and by the heating limit. Thus, if a shunt current C, with a terminal voltage V, is pre-arranged, the resistance R of the shunt must be equal to $\frac{V}{C}$, and the number of turns N must be equal to

$$\frac{\text{ampere turns}}{C},$$

if all the turns are in series; but if they be split into M groups, among which the current C is equally divided, the resistance of each group becomes $M\frac{V}{C}$, and the number of turns in each remains equal to

$$\frac{\text{ampere turns}}{C}$$

The rise of temperature may usually be kept within the limits already mentioned by an allowance of from 1·5 to 2 square inches of cooling surface on the external surface of the coils per watt lost. According to Esson, the rise of temperature in a coil is given in degrees Centigrade by

$$\frac{100 \times \text{watts lost}}{\text{external area in square inches}}.$$

Since with a given value of V and the same size of wire N C is constant, or diminishes but slightly owing to the outer turns being a little longer than the inner ones, it is evident that by reducing C and putting on more copper the heating conditions can always be satisfied—the more readily so as a small variation in C produces a considerable alteration in the wasted heat, which is proportional to C^2.

The size of the conductor for a series winding is determined by the loss of watts and by the heating limit, as before; but the loss cannot in this case be reduced by winding on more turns, as with a shunt, since the whole current must flow through them, and increase of length involves increased energy loss and drop of voltage. Magnets wound with both shunt and series coils may be evenly compounded or over-compounded. In the former case, the series ampere turns increase the magnetic flux as the current increases, so augmenting the voltage induced in the armature sufficiently to keep that at the terminals the same as when running on open circuit with the shunt alone. The series turns thus compensate for voltage loss over armature and field, including reaction losses. In over-compounding they do more than this, and increase the terminal voltage as the current becomes greater, the object being to maintain an even voltage at the end of mains or feeders supplying distant points. The rise of terminal volts should evidently be equal to the drop in the cable. The series coils may, of course, be grouped in series or in parallel with one another, but the calculation of the lost watts in any arrangement will be understood from what has been mentioned in connection with shunt coils; it may, however, be pointed out that by making provision for altering the arrangement of connections two or more definite voltages may be given by a generator—a fact which is sometimes an advantage.

The size of the wire in the coils may quickly be determined by the method of trial and error; the mean length per turn is estimated, and the number of turns required with a given current known; hence the total length of wire and its resistance for any section can at once be found. If the C^2R loss be then too great for the cooling surface, a larger section must be adopted for a series winding, or a greater number of turns for a shunt. It may be shown that if V be the voltage at the ends, d the diameter in inches of the wire, M the ampere turns required, l the mean length of one turn in inches, r the resistance per inch length per square inch section of the wire,

$$d = 1{\cdot}13 \sqrt{\frac{M\,r\,l}{V}}$$

for a shunt coil.

As an example, let a shunt coil be required for a magnet of which the section is 12 by 14 inches, the ampere turns required being 8,000, with a current of 4 amperes. The number of turns is evidently

$$\frac{8000}{4} = 2000.$$

The diameter of the wire must be

$$1{\cdot}13 \left(\frac{8000 \times {\cdot}66 \times 56}{120 \times 12^6}\right)^{\frac{1}{2}} = 055 \text{ in.,}$$

the average length of one turn being taken as 56 in. The resistance of the coil is evidently

$$\frac{120}{4} = 30 \text{ ohms.,}$$

and the number of watts converted into heat is

$$4^2 \times 30 = 480.$$

If two square inches be allowed per watt for cooling surface, the surface of the coil must be 960 square inches, and since the length round the periphery is assumed to be about 56 in., its length must be

$$\frac{960}{56} = 17 \text{ in.}$$

If possible, however, it is desirable to arrange the length of coil so as to permit of the turns being distributed in a whole number of layers.

Now, allowing 15 mils. for insulation, the diameter of the covered wire is ·07 in.; and if 3 per cent be allowed for play in winding, a length of 17 in. gives

$$\frac{17}{1{\cdot}03 \times {\cdot}07} = 236 \text{ turns per layer,}$$

and thus there would be

$$\frac{2000}{236} = 8\tfrac{1}{2} \text{ layers.}$$

By taking 1,998 turns and winding in 9 layers, each layer contains 222 turns, and the length of the coil becomes

$$222 \times 1{\cdot}03 \times {\cdot}07 = 16 \text{ in.},$$

and the cooling ratio is

$$\frac{16 \times 56}{480} = 1{\cdot}87 \text{ square inches per watt.}$$

In the above case, the current being originally given, the requisite length and surface has been found for the coil.

Let, now, the length of coil be originally given as 14 in., and the requisite cooling ratio obtained by adjustment of the current. Assuming the same perimeter as before, the cooling surface is

$$14 \times 56 = 784 \text{ square inches.}$$

The watts which may be dissipated are, then, 392, allowing 2 square inches per watt. The current must therefore be

$$\frac{392}{120} = 3{\cdot}27 \text{ amperes,}$$

and the number of turns is

$$\frac{8000}{3{\cdot}27} = 2450.$$

The number of turns in one layer is

$$\frac{14}{1{\cdot}03 \times {\cdot}07} = 195,$$

and the nearest number to 2,450 which gives a whole number of layers is 2,535, which may be divided into exactly seven.

Either of these calculations having been made, allowance can be added for insulation, and the dimensions of the former for the coil determined.

Allusion has already been made to the force of magnetic attraction between the pole faces and the core, though so far no account has been given of its magnitude. It may, however, be shown that the force of attraction, expressed in dynes, between a surface of area A square centimetres, and a parallel surface of equal or greater area, is equal to

$$\frac{A B^2}{8 \pi}$$

where B is the induction in C.G.S. lines per square centimetre between them. It will be noted that as the lines are taken as perpendicular to the surfaces and parallel to one another, the force is independent of the distance between the surfaces. This condition is, nevertheless, only fulfilled when the distance they are apart is small in comparison with their area. Expressed in pounds, the force becomes

$$9 A B^2 \times 10^{-8};$$

or, with A in square inches and B per square inch,

$$1{\cdot}4 A B^2 \times 10^{-8}.$$

Now, since the air gap of a dynamo presents a far greater resistance than any other part of the magnetic circuit, a slight eccentricity of the core will produce a large excess of induction in the narrower parts of the gap, and a considerable force pulling it in that direction. As an example, let a four-pole machine be considered, the area of the pole faces being each 1,000 square centimetres. Let the air gap originally have been 1 centimetre wide, and have presented 50 per cent of the total magnetic resistance, but have now become ·8 centimetre wide at the lower poles, and 1·2 wide at the upper. If the induction was at first uniformly 6,000 in the air gaps, it is now approximately

$$\frac{100}{50 + 50 \times 1{\cdot}2} \times 6000 = 5440 \text{ in the upper gaps,}$$

and

$$\frac{100}{50 + 50 \times {\cdot}8} \times 6000 = 6700 \text{ in the lower ones.}$$

The total radial pull in the two upper poles is $2 \times 9 \times 1000 \times 5440^2 \times 10^{-8} = 5{,}300$ lb. Each pole, however, pulls at an angle of 45 deg. with the vertical, so that the resolved upward pull is

$$\frac{5300}{\sqrt{2}} = 3750 \text{ lb.}$$

In exactly the same way, the downward pull of the lower poles is found to be 5,700 lb., and the pressure on the bearings is increased by 1,950 lb., the spindle also having its bending moment proportionately added to.

In a vertical bipolar machine the pull of each upper portion of the poles acts at about 30 deg. with the horizontal, and the resolved pull is found by dividing by 2. The case of a four-pole machine with diagonal poles is treated above, but when the poles are vertical and horizontal the resultant pull is simply the difference of the attractions of the two vertical poles. With multipolar machines the resolved pull is $\frac{2}{\pi}$ times the total pull of all the poles in the upper or lower half of the field.

If the iron core of an armature is not truly between the pole faces in a lengthwise direction, a similar force is produced drawing it toward the centre. This action may often be observed to produce a knock when a motor whose shaft has much play, and which is in an inclined position, is started or stopped.

In "umbrella" type generators—*i.e.*, those in which the spindle is vertical and the plane of rotation horizontal, such as are frequently adopted in America and on the Continent in conjunction with turbines driving from below—the collar or footstep bearings are sometimes relieved from some portion of the weight by means of a disc upon the shaft, which is attracted magnetically upwards. A drawback, however, to most devices of the kind is the waste of power in hysteresis and eddy currents.

CHAPTER X.

SHAFTS, BEARINGS, AND PULLEYS.

LIKE most other shafts, that of a dynamo or motor is subjected to both a twisting and a bending stress. The latter stress is, however, peculiar in that it is augmented by any yielding on the part of the shaft, owing to the inequality of magnetic attraction thereby produced between the core and poles.

On this account, and also by reason of the small clearance between the armature and pole faces, it is desirable to secure great rigidity, and to allow a liberal margin over the minimum calculated size.

The greatest bending moment exists at about the centre of the armature hub, and is mainly due to the weight of the armature, though it may be increased by both magnetic and gyrostatic forces. The twisting moment exerted by the pulley or coupling also produces a stress extending to the armature hub, but becoming almost nil under the commutator. Of these two stresses the bending is usually the greater, and when a spindle is fractured the break is most commonly between the armature hub and the commutator.

The resultant of the binding and twisting moments, regarded as a twisting moment, is given by

$$T_e = B + \sqrt{B^2 + T^2},$$

where T is the twisting and B the bending moment, T_e being the equivalent twisting moment, all in inch-pounds.*

If D denote the diameter of the shaft and S be the stress per square inch on the metal, then it may be shown that

$$S = \frac{T_e}{\cdot 196\ D^3},$$

and for steel shafts S should not exceed 7,000 lb.

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* Unwin, "Machine Design."

For approximate determination of the size of shaft the diameter of the part inside the armature, reckoned in inches, may be taken as equal to—

$$\sqrt[3]{\frac{\text{watts output}}{\text{revolutions per minute}}};$$

or, in the case of a motor—

$$\sqrt[3]{\frac{746 \times \text{horse power}}{\text{revolutions per minute}}}.$$

Though this approximation should be ultimately checked by the preceding method, its agreement with practice is shown by the following instances of manufactured shafts, chosen at random :—

Output in Watts.	Speed.	Diameter of Shaft as made.	Calculated Diameter of Shaft.
3,650	1,450	$1\frac{3}{8}$	1·36
4,160	330	$2\frac{5}{8}$	2·33
12,000	680	$2\frac{3}{4}$	2·6
9,000	250	3	3·3
12,000	290	$3\frac{1}{4}$	3·45
17,500	275	$3\frac{3}{4}$	3·95
80,000	450	$5\frac{1}{2}$	5·7
330,000	300	$8\frac{3}{4}$	10
550,000	270	$11\frac{1}{2}$	12·6

From this table it will be observed that for the machines of larger size the diameter given by the formula is slightly too large.

The spindle is usually of less diameter in the commutator, and may be yet further reduced at the end bearing. The details of construction depend largely upon the method employed for clamping up the armature hub; but figs. 113 to 115 are typical, fig. 113 being a belt-driven, figs. 114 and

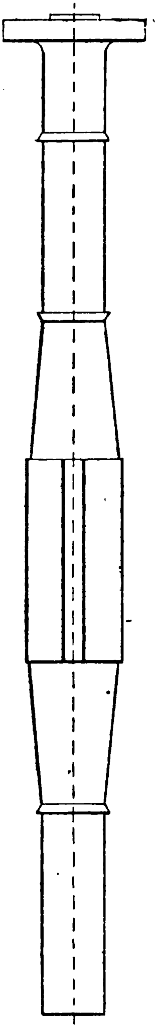

Fig. 121.

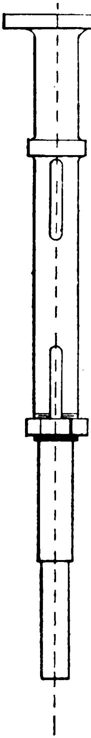

Fig. 120.

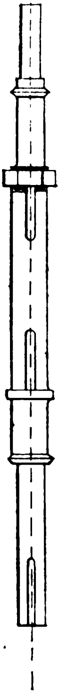

Fig. 119.

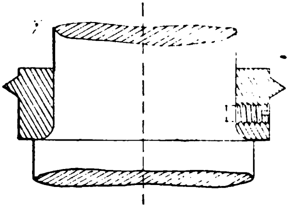

Fig. 122.

121 a small and a large direct-driven shaft. In very many cases a flywheel is carried on the coupling between the engine and dynamo shafts, and in large modern generators the torque is often transmitted directly from the flywheel to the armature framing, as shown in fig. 71.

To prevent oil, which would destroy the insulation, creeping along the shaft to the armature or commutator, it is necessary to turn V-shaped rings on the shaft just outside the bearings near these parts. The oil escaping from the end of the bearing is then thrown off the sharp edge by centrifugal force. It is often, however, more convenient to fit separate collars, fig. 122, furnished with these Vs than to turn them on the shaft itself.

The minimum bearing surfaces for a shaft are determined by the conditions that the wearing down of the brasses must not be excessive, and that the surface must present sufficient area to conduct away the heat generated by friction without rising to an unduly high temperature. Of these, it is the last that really sets a limit to reduction of the surfaces.

It may at once be seen that the energy converted into heat in a bearing is equal to

$$\mu P \times \frac{\pi}{12} D N$$

foot-pounds per minute, where P is the load on the bearing; D the diameter of the shaft in inches; N the number of revolutions per minute, and μ is the coefficient of friction, which, between steel and brass, with good lubrication, may be taken as, on an average, equal to ·05.

If K denote the energy, expressed in foot-pounds, which the metal of the bearing is capable of conducting away per square inch in one minute without an excessive rise of temperature internally, then the relation

$$\mu P \times \frac{\pi}{12} D N \leqq \pi D K \times \text{length of bearing}$$

must be satisfied, and K may be taken as about 15. A calculation of this nature is, however, of little use in practical design, and the ratio of length to diameter is commonly dependent on the size of the shaft, being about 4·5 in small machines and falling as low as 2½ in those of the largest size.

It is comparatively rare to find the rigid type of bearing, employed in reciprocating steam engines, adopted in modern dynamos, where the conditions permit of and demand a freer structure and abundant lubrication.

The old plummer block with the brass fitted rigidly within it, and the lubrication effected by an ordinary oil box, does indeed survive, most commonly in small machines of low cost; but as a rule this has given place to a bearing of the type introduced by Sellers, in which a spherical seating permits of the brass setting itself in alignment with the other bearings. A simple example of this type is shown in fig. 123, one end of the brass being modified to catch the oil thrown off the collar. For small machines the turning of the spherical portion may be avoided by thinning down the metal toward the ends as shown in fig. 124, where the inner end of the block is turned outside to form a seating for the brush crossbar.

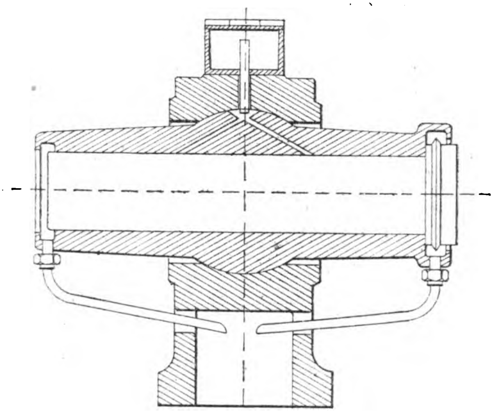

FIG. 123.

A bearing in which labour is reduced to the minimum is shown in fig. 125, where the only machined surfaces are those between the cap and block and the inside of the brass, all others being cleaned up and packed. Other examples are given by figs. 126 and 127, the latter showing a method of bushing with alloy.

The lubrication of the bearings is now usually effected continuously and automatically by means of rings or chains running upon the spindle, and driven by their friction with it. These dip below into a reservoir of oil, and so carry up a continual supply to the top of the spindle. When, as is more frequently the case, rings are employed, they are nearly always of brass, though displaying considerable variations in their sections—circular, semi-circular, channelled, etc. Although the oil carried up by one such ring would

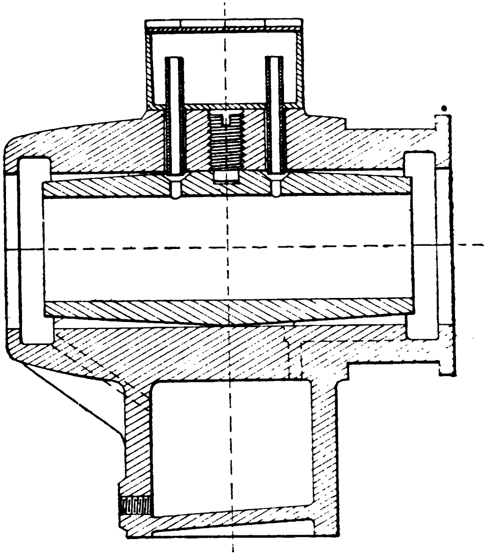

Fig. 124.

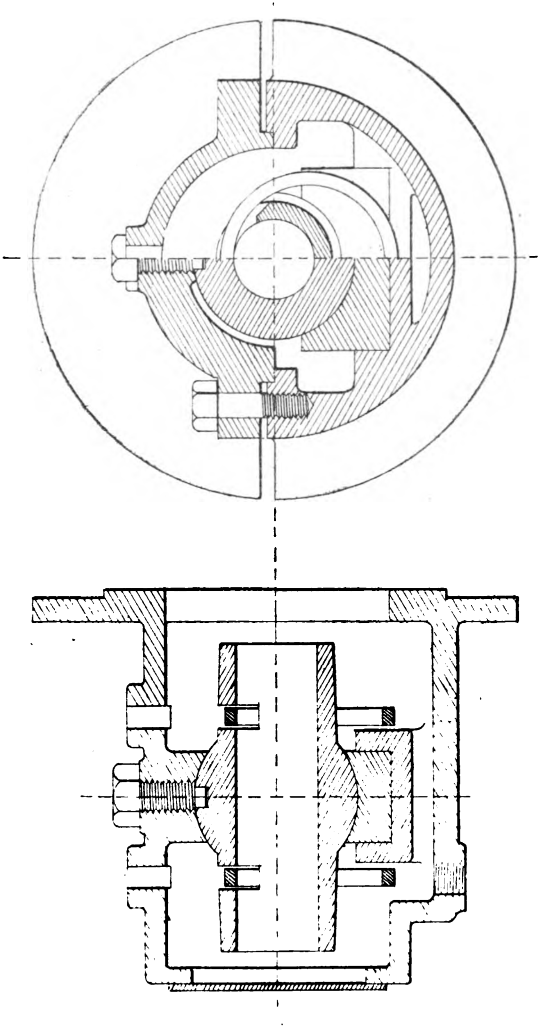

FIG. 125.

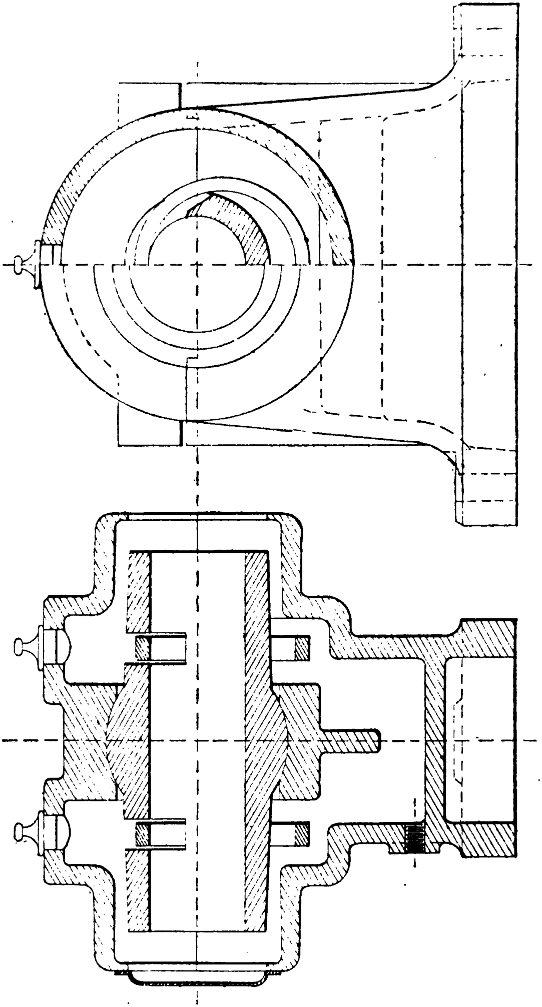

FIG. 126.

probably suffice for the efficient lubrication of a bearing of as much as a foot in length, it is advisable, and also in most cases more convenient, to supply even short bearings with two, this number being sufficient for any ordinary conditions.

The reservoir in which the oil collects should be of sufficient depth to allow of the metallic matter settling to the bottom, and should be fitted with a cock for drawing off.

With the object of saving the cost of the labour involved in boring the bearings to fit the shaft, it is the practice of some American manufacturers to cast the bush round a core of the same dimensions as the shaft. The rotation of this core in the half-set metal produces a fine surface of the dimensions required, so that machining is unnecessary.

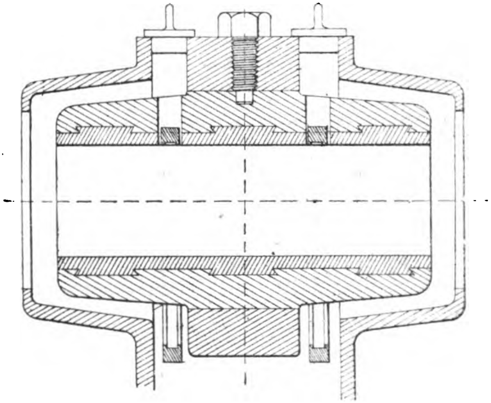

FIG. 127.

With regard to the means of indirectly driving machines, though rope gearing has been widely adopted for electrical plant, the employment of leather belting is by far more general, and while for a full discussion of such gearing works more specially dealing with the subject must be referred to, the following brief treatment will suffice for the purpose of determining the dimensions of the pulley in a given case.

As the size of the driving pulley is, in general, unknown, the belt velocity V comes firstly into consideration. This is usually somewhat greater with large than with small machines, but may be taken as about 3,000 ft. per minute, though in American practice a velocity of about 6,000 ft. per minute is sometimes adopted. If, then, the pulley is to run at N revolutions per minute, its diameter is evidently given in inches by

Angle, in degrees, embraced by belt.	$\frac{P_1}{P}$	$\frac{P_1}{P_2}$
30	6·88	1·17
45	4·75	1·27
60	3·70	1·37
75	3·08	1·48
90	2·67	1·60
105	2·37	1·73
120	2·14	1·87
135	1·98	2·03
150	1·83	2·19
165	1·73	2·37
180	1·64	2·57
195	1·56	2·78
210	1·50	3·00

$$D = \frac{12\,V}{\pi\,N} = 3{\cdot}82\,\frac{V}{N}$$

The horse power absorbed or given out by the pulley being W, and the belt velocity being V, the resultant tangential pull on the pulley rim, in pounds, is

$$P = \frac{W \times 33000}{V}$$

and this resultant pull is the difference of the pulls on the tight and slack sides of the belt. The final quantity required is thus not the value of P so calculated, but the pulls P_1 on the taut, and P_2 on the loose side of the belt.

The ratios which these bear to each other and to P depend upon the coefficient of friction between the belt and pulley, and upon the arc of the pulley's circumference embraced by the belt. For leather and cast iron the coefficient of friction may be taken as about ·3, and with this value the ratios are as given in table on page 264.

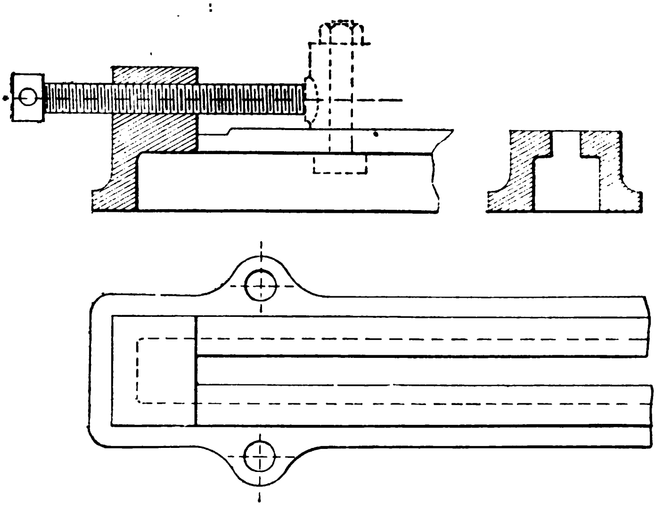

FIG. 128.

By the second column of this table the pull which the belt must be capable of withstanding is found, and in the third column the ratio of the two pulls, from which may be calculated the bending moment produced on the shaft.

The weakest part of the ordinary belt is the laced joint, and the strength there may be taken as about 320 lb. per square inch of the section of the belt. The thickness of belt being decided upon, its width can at once be determined, for

$$\text{width of belt in inches} = \frac{P_1}{320 \times \text{thickness}},$$

and an allowance of from half an inch upwards, according to size, gives the width of pulley.

To prevent the belt running off, a camber of about half an inch per foot width should be turned on the pulley, while for additional safety the pulleys of small machines usually carry a flange on their inner side, and to minimise variation of speed this flange is sometimes deepened and furnished with a thickened rim, so developing into a flywheel.

To facilitate the taking up of slackness in the belt, most machines so driven are not directly bolted down to the foundations, but are mounted on slide rails which permit them to be moved back, so increasing the distance between the centres of the pulleys. The tightening up is effected by screws passing through bosses at the ends of the rails, which

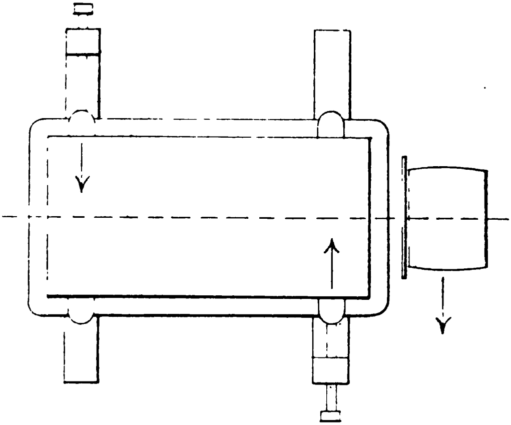

FIG. 129.

are themselves secured to the foundations by coach screws or dowel bolts. Fig. 128 represents such a rail, while fig. 129 shows that as the pull on the pulley produces a couple, one tightening screw should, where possible, be in front of, the other at the back of the machine, to avoid shear strain in the bolts clamping it down.

CHAPTER XI.

Brushes and Brush Gear.—The conditions which must be satisfied by the appliances for collecting the current from the commutator of a generator or conveying it to that of a motor are both electrical and mechanical. So far as the former are concerned, however, it is sufficient to provide the collectors with adequate cross-sectional area, and to ensure an ample area of contact with the commutator for the maximum current to be collected. Where, as is invariably the case, the current passes from piece to piece of the gear across what are often working surfaces (as, for example, from the brush holders to the pin supporting them), it is especially important to provide abundant contact surface. A last essential, of paramount importance, is that all parts of the gear should be adequately insulated from others at a different potential and from the body of the machine.

Mechanically, arrangement must be made for retaining the brushes in close contact with the commutator when in use, for withdrawing them when repair or renewal is necessary, and for adjusting them at any point along the commutator's length, in order that scoring of its surface may be prevented. It is also usual to so construct the gear that the entire set of brushes may be rotated through a small angle round the axis of the commutator, so permitting the position of the diameter of commutation to be slightly varied. In traction motors, and other enclosed types of machine, where access is difficult and the space confined, this last provision is usually omitted, the brushes being in many instances carried from the casing. Occasionally means are afforded of raising any single brush from the commutator and retaining it in that position, but this is comparatively unusual.

The brushes usually employed are of copper in the form of sheet, wire, or gauze, though carbon is very widely used, particularly in the case of motors; indeed, where the direction of rotation is reversible their use is compulsory. A common form of copper brush consists of a number of thin sheet-copper strips, cut to the requisite length and width, soldered together for about half their length. A brush of this kind is usually somewhat lacking in flexibility, and to remedy this defect as far as possible the unsoldered wearing end is divided by saw cuts in a lengthwise direction, fig. 130. This type of brush has also been employed with that part which is grasped by the holder, reduced in width, fig. 131, so permitting a smaller holder to be used. The impossibility of withdrawing the brush in an outwards direction is, nevertheless, an inconvenience. A better, and now more generally used, form of brush is obtained by folding copper gauze to the required dimensions, whereby great flexibility is secured, while the slight reduction of actual copper in the section may be omitted from consideration.

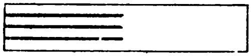

FIG. 130.

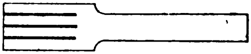

FIG. 131.

With regard to the dimensions, the length, though determined to some extent by the design of the holder, is mainly a simple matter of convenience; while the thickness commonly bears some rough relation to the width, varying from $\frac{1}{8}$ in. in those $\frac{1}{2}$ in. or so wide, to $\frac{3}{8}$ in. in those of over 1 in. in width. Carbon collectors are frequently of squarer section and less length than those of copper, being often blocks in which the length is little greater than the width or thickness.

The cross-section—*i.e.*, the product of width and thickness—of copper brushes should be such that the current collected by them does not exceed 200 amperes per square inch of their area of contact with the commutator, and in most cases 170 amperes is about the density allowed across this area. For carbon collectors the maximum permissible is considerably less, and may be taken as from 30 to 40 amperes. For both metal and carbon, however, when the total current is small, the densities are often very much less than those mentioned.

In a few instances—chiefly in machines for arc lighting—the brush consists of a single strip of sheet copper making contact tangentially with the commutator; but usually the end is cut obliquely and the brush is inclined at an angle of from 45 deg. to 60 deg. with the tangent at the point of contact. The area of contact is thus from 1·16 to 1·4 times the cross-section of the brush. Carbon collectors are, however, in most cases set radially, or nearly so.

It is an economical practice where machines of various outputs are under construction to adopt a somewhat narrow brush as a standard, and to employ a greater or less number at each point of collection, according to the current to be dealt with, rather than to employ wider brushes for the heavier currents.

The entire set of holders for the brushes at one collecting point are usually supported by a gun-metal or brass rod, along which they may be shifted, and so fixed at any point along the length of the commutator. As will be seen from the examples figured, the holder commonly consists of two parts, the one rigidly secured to the supporting rod, the other, which holds the brush, rotatable about the axis of the rod.

A form of holder which, with various modifications, is widely used is as fig. 132. It has the advantage of simplicity, and the spring affording the necessary pressure for good contact, also carries the brush and conveys the current to the rod. To secure both good conductivity and flexibility this spring is built up of a number of thin brass sheets. The adjustment of the pressure is effected by the handle, which also clamps the holder in position when screwed up.

A pattern in which the force is furnished by the tension of a spiral spring is illustrated in fig. 133. The contact pressure can, with this arrangement, be more nicely adjusted than is the case with some others. The portion of the holder carrying the nut by which this adjustment is made is secured to the brush bar by a screw, and flanking it on each side are extensions of that portion holding the brush. To distribute the pressure of the screw fixing the brush itself a brass plate is usually interposed, as shown in the figure.

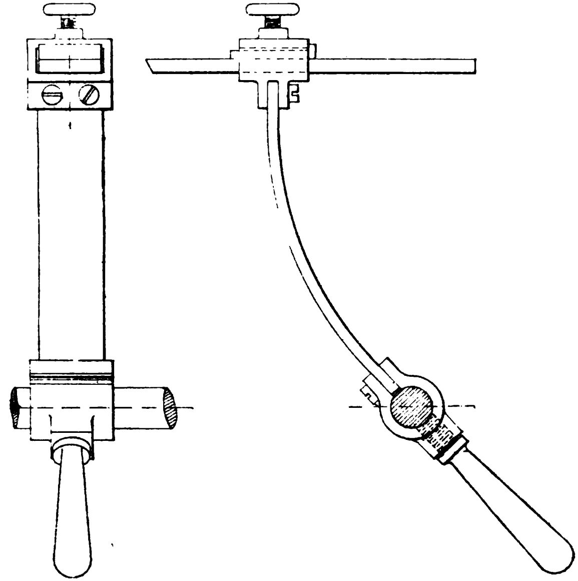

FIG. 132.

In very many brush holders the parts fixed to the brush bar and carrying the brush, respectively, are arranged much

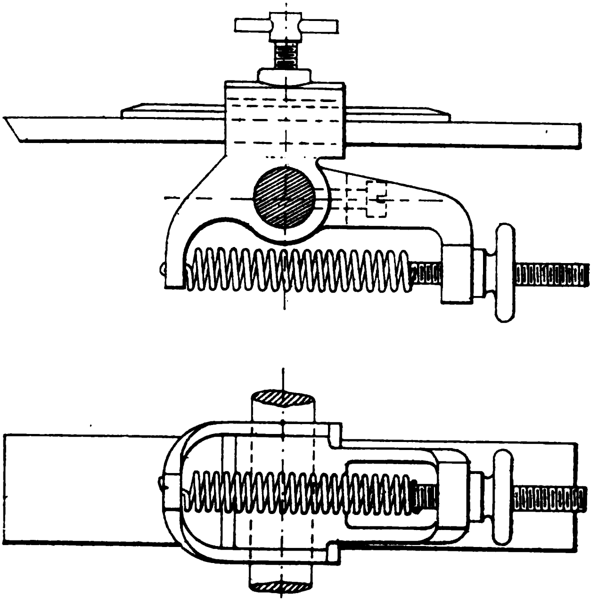

FIG. 133.

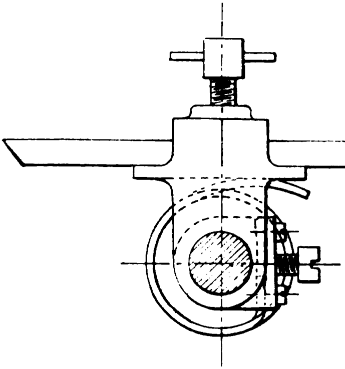

FIG. 134.

as in fig. 133, but the pressure is obtained by a wire or strip spring tending to uncoil itself from round the former part, to which one of its ends is fixed, and so forcing down the brush. The general principle of this class of holder will be understood from fig. 134.

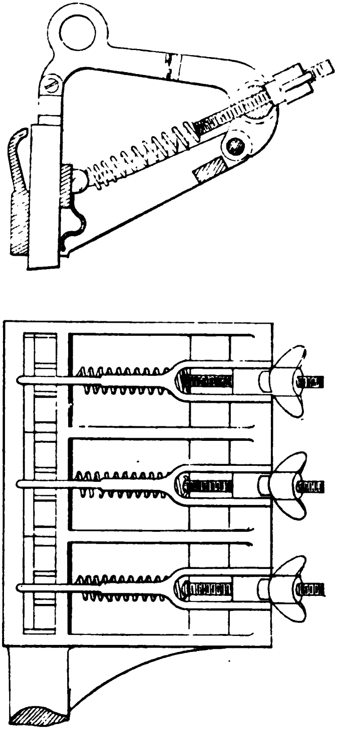

FIG. 135.

While the examples described above are applicable to either copper or carbon brushes, a number of others are only adapted to the latter. Among these may be noted the collective holder employed by the Westinghouse Company. In this, which is similar to fig. 135, separate holders have disappeared, and a single casting embraces the entire set of collectors, though due provision for the adjustment of the pressure on each singly is adequately made. Another point of difference from most other patterns is the absence of a brush bar, the holder being supported by an extension of the main casting.

Fig. 136 shows a holder in which the flat spring not only produces the contact pressure, but also keeps the carbon firm within the holder.

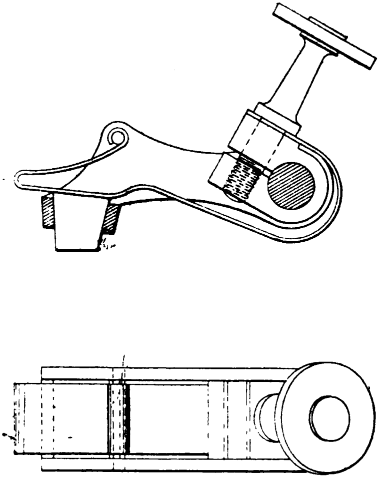

FIG. 136.

In designing brush holders the weight should be reduced as far as possible consistent with due strength, both for economy and for the minimising of inertia, since the less the inertia of brush and holder the less will be the jumping produced by any irregularity in the commutator surface, or by vibration of the whole machine. The magnitude of these disturbances really determines the contact pressure which is necessary between the brush and commutator; these, however, being incalculable, a pressure of about 2 lb. per square inch of contact area may be allowed in stationary machines, and from two to three times that amount in traction motors or machines subject to exceptional vibration. The upper limit of pressure is that at which the wear becomes excessive. Where possible the holder should be so constructed that as the brush end is worn down it is fed forward in line with its length, so maintaining a constant position of commutation; this desideratum, however, is seldom obtained. To prevent entire stoppage should the removal of a brush be necessary, two should be fitted at each collecting point, even in cases where one alone would give sufficient surface.

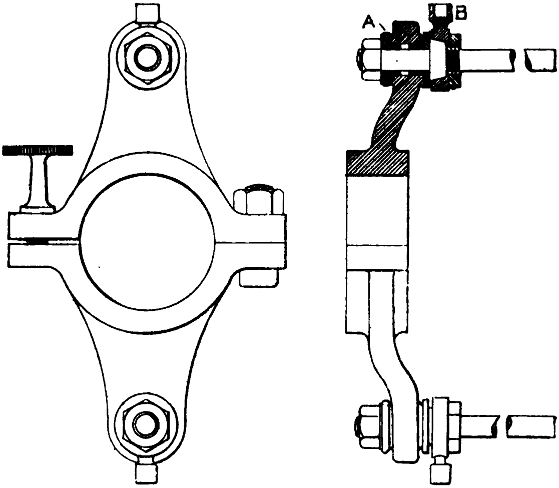

Fig. 137.

It may be mentioned that the coefficient of friction of either copper or carbon brushes with a slightly greasy commutator is probably about ·4.

The rods carrying the brush sets are all attached to but insulated from an iron casting, which in bipolar and four-pole machines is carried upon the inner end of the bearing, or upon a ring screwed to its foot. Fig. 137 shows such a crossbar for two sets of brushes, the insulation of the brush rods being effected by fibre, ebonite, or boxwood rings A. The electrical connection to the terminals is made by sweating the cable into the thimbles B, which make a cone contact with the rod. A less satisfactory connection, often used, is shown in fig. 138. The curvature of the arms of the crossbar is such that on the one side the nut clears the bearing, and on the other the nearest brush can be brought to bear on the edge of the commutator.

For large multipolar generators the carriage for the brush bars is usually a ring supported in guides attached to the side of the magnet yoke ring, as shown in figs. 139 and 140, the latter representing the brush gear of a large traction generator by the British Westinghouse Company. These guides are sometimes fitted with rollers to reduce the friction, or in other cases lined with white metal. The same figures illustrate the worm and rack gear employed in large machines for adjusting simultaneously the position of the brushes; in small machines, however, this adjustment is made directly by a handle on the crossbar or ring. Most generators being parallel wound, the entire sets of positive and negative brushes are respectively connected to rings, which may be enclosed, as in fig. 139, where they are contained in the ring encircling the commutator, or exposed, as shown in fig. 140.

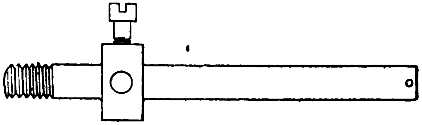

FIG. 138.

Occasionally provision is made for raising simultaneously

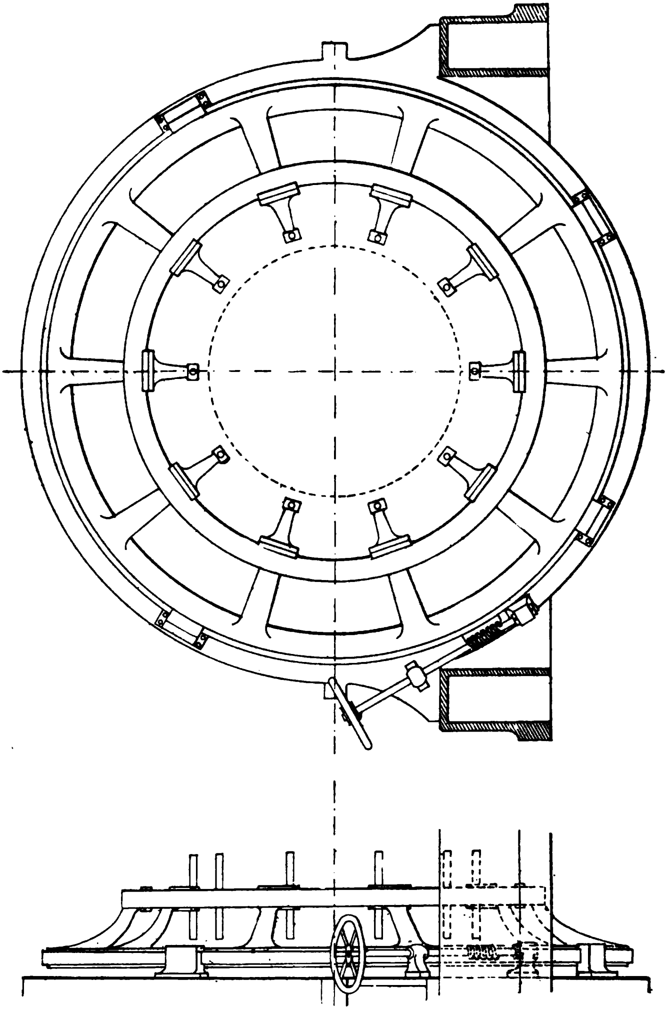

FIG. 189.

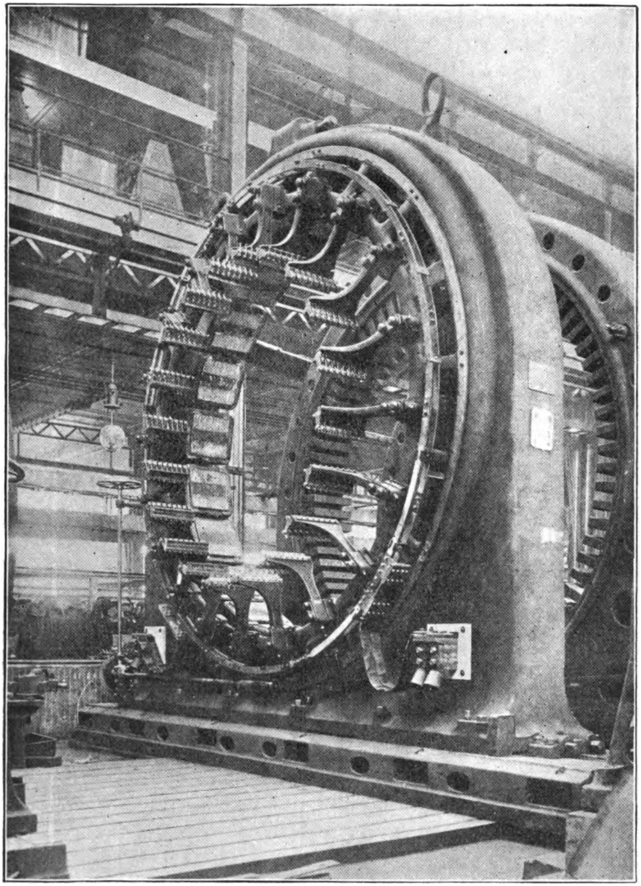

FIG. 140.

all the brushes from the commutator, but this is comparatively unusual.

As regards connections, one cable from the brushes is directly connected to a main terminal of the machine; the other, except in the case of a shunt winding, is led to the field coils, which are connected to the other main terminal.

In small machines the usual form of terminal is merely a block provided with a hole, into which the wire is clamped by a screw. A far superior pattern, which is nearly always employed in the case of large generators, has coned holes into which plugs, carrying thimbles into which the cables are soldered, are drawn by nuts. Fig. 141 represents a simple example of these terminals. Whatever type of terminal be employed, the insulation is commonly effected by mounting them on teak blocks screwed to the magnets or framing.

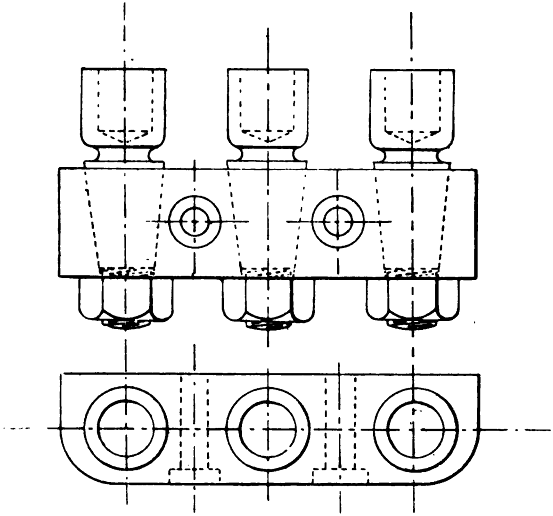

Fig. 141.

CHAPTER XII.

Commutation.—Since, as previously pointed out, the maximum current which an armature can carry is dependent upon the temperature rise and the possibility of sparkless collection—the former circumstance having already been discussed—it now becomes necessary that the conditions which must obtain for satisfactory commutation should be considered.

From the preceding chapter on the windings of armatures it will have become evident that in a winding of n parallel circuits any coil, previously to its hinder commutator segment coming under a brush, is carrying $\frac{1}{n}$th of the total current. As soon as the hinder segment passes under the brush the coil is short-circuited through it, and this state continues until the leading segment passes from under the brush toe. By this time a current equal to that originally flowing must again pass through the coil, but in the reverse direction.

The first step toward a clear understanding of the process of commutation is to consider the effect produced in a conductor by any alteration of the current flowing in it. Two facts already dealt with bear upon this question. Firstly, that any current-bearing conductor is linked with a number of magnetic lines, the number being directly proportional to the current if no magnetic material be near, and in any case increasing with an increase of current. Secondly, that a current is produced in any closed conductor when the number of magnetic lines with which it is interlinked varies. (Chapters III. and IV.) Thus, if the current flowing in a circuit be increased, the accompanying magnetic flux is also augmented, and this produces an electro-motive force in opposition to the current producing it, and thus the latter only gradually assumes its final value. Similarly in the case of a diminishing current, the resulting reduction of the magnetic lines retards its fall. The tendency of this self-inductance of a conductor is thus to oppose any change in its electric or magnetic conditions.

The coefficient of self-inductance of a circuit is the number of C.G.S. magnetic lines linked with that circuit when unit current (10 amperes) passes through it. If, as is usually the case, it be denoted by L, then when a current of I units flows the number of lines is L I. With regard to this coefficient, it is to be noted that if the circuit consists of a number of convolutions, so that any magnetic line threads itself a number of times through the circuit in the same direction, then that line is to be counted as equivalent to the same number of separate lines.

A change of one C.G.S. line per second in the flux, reckoned in this way, produces unit C.G.S. electro-motive force, or 10^{-8} volts, and so, whatever the rate of alteration in the magnetic lines per second, the induced voltage is obtained by multiplying that rate by 10^{-8}. If, however, the current be expressed in amperes, and the self-inductance in sechoms, the factor becomes 10^{-9}.

Now, in the number L I, the term L is constant and invariable in all cases where the magnetic lines and current are directly proportional to each other, and in such circumstances the rate of change of the flux, L I, becomes L multiplied by the rate of current change, or

$$L \frac{dI}{dt},$$

where

$$\frac{dI}{dt}$$

simply represents a number, viz., the rate per second in C.G.S. units at which the current is increasing at any instant, a minus sign prefixed denoting diminution.

Thus, the ordinary expression of Ohm's law—

$$E = R I,$$

becomes with an increasing current—for instance, when the E.M.F. is first applied—

$$E = Ri + L\frac{di}{dt}$$

where E denotes the E.M.F., and i the current at any instant.

In a somewhat similar way the current in a circuit from which the E.M.F. has been removed is given by

$$0 = Ri + L\frac{di}{dt}$$

$$i = \frac{L}{R}\,\frac{di}{dt};$$

while in a circuit carrying an increasing current

$$\frac{E}{R} - i = \frac{L}{R}\,\frac{di}{dt};$$

or since $\frac{E}{R}$ is the final value which the current reaches when steady, representing this by I,

$$I - i = \frac{L}{R}\,\frac{di}{dt}.$$

These equations when solved enable the value i of the current to be found at any instant. If the electro-motive force applied to the circuit be instantaneously increased from 0 to E, the value of the current t seconds later is given by

$$i = \frac{E}{R}\left(1 - \epsilon^{-\frac{R}{L}t}\right)$$

and if E be again suddenly reduced to 0, then t seconds later

$$i = \frac{E}{R}\epsilon^{-\frac{R}{L}t}$$

From these last equations it will be noted that the inductive effects of a circuit depend not merely upon its coefficient of inductance, but on the ratio of the resistance to that quantity; and that in various circuits the currents produced by the same electro-motive force increase at an equal rate if this ratio be the same for each.

In fig. 142 is shown the curve of current rise for a circuit of small inductance. Had the inductance in this case been greater, the current growth would have been slower, the curve sloping up less steeply. Fig. 143 is the curve of current decay when the electro-motive force is removed from the same circuit. Each of these curves brings out prominently the fact that no abrupt change in the current is produced, and this is generally true for any conductor whatsoever possessing inductance.

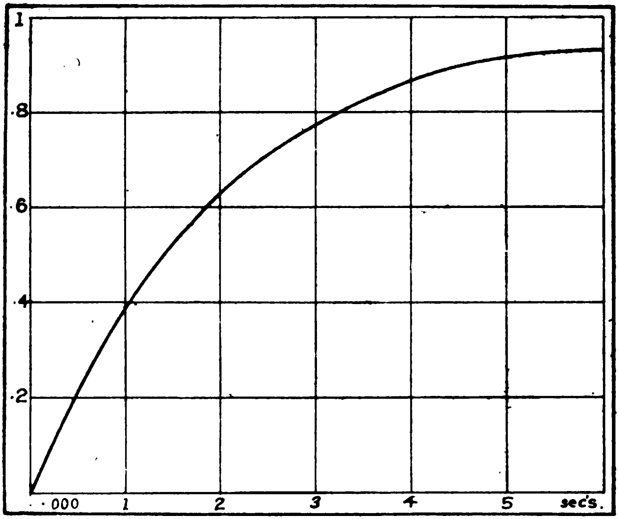

FIG. 142.

From these considerations it is evident that at the first instant of short circuit through a brush, an armature coil is carrying its previous current, and that this only gradually dies out. Since at the end of the short circuit it must carry the full reversed current, the conditions must be such that during the period of short circuit—*i.e.*, the time occupied by a point on the commutator surface moving across the thickness of the brush—the current may die out and grow to full strength in the reverse direction. Unless this be so, since the leading armature coils up to the segment A, fig. 144, are carrying a greater current than the commutated one B, and the current in neither can instantaneously alter its value, it follows that the current is compelled to jump as a spark to the brush from the commutator segment A, which has just emerged from under it. The extent of this sparking thus depends upon the difference between the current passing round the coil at the termination of its short circuiting, and that which normally passes through it in other positions when it is one of the series of more or less active inductors. Such production of

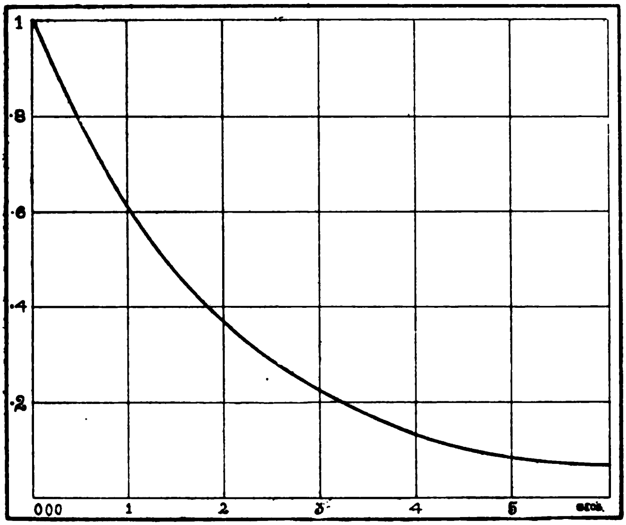

FIG. 143.

sparks is inadmissible, as it leads to rapid pitting of the commutator surface, whereby the sparking is rapidly increased and the commutator itself becomes ruined.

The fact of a reverse current having to be established during commutation implies, of course, the production of a reversed electro-motive force in the coil during that period, and this is effected by rotating the brushes so that commutation takes place at a time when the coil is under the influence of the magnetic flux from the magnets. In this way, by giving "lead" to the brushes of a generator—that is, moving them round somewhat in the direction of rotation—commutation occurs when the coil is cutting across magnetic lines from the horn of the pole-piece it is approaching, and so an electro-motive force is induced in it which both hastens the decay of the previous current and establishes another in the required direction. With a motor, on the other hand, the current in the armature coils is driven through them against the electro-motive force they themselves induce by the greater external pressure of supply, and thus the brushes are moved backward against

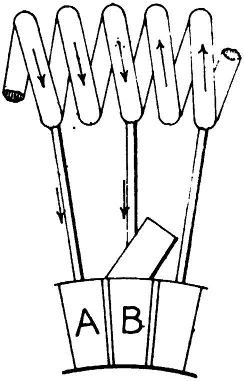

Fig. 144.

the direction of rotation—or given "lag"—to obtain the requisite electro-motive force.

Since the magnitude of this electro-motive force must be proportional to the currents to be reversed, a greater lead or lag is necessary as the current in the armature increases. The continual adjustment of the brushes is, however, usually avoided in large modern machines by appropriate reduction of the coefficient of self-inductance in the coils, etc., and by the adoption of carbon brushes. Indeed, with traction generators, where the output is subject to large and frequent variations, such adjustment would be impracticable, and arrangement for constant lead at all loads is usually a requisition of the specification.

Assuming that at the beginning of short circuit the coil enters a magnetic field which is uniform, and of such strength that the induced electro-motive force is E, the normal current through the coil may be denoted by I, its resistance, including that of connector lugs, by R, and the coefficient of self-inductance, in secohms, by L.

Further, the original current, which dies out, may be regarded as negative, the reversed increasing one as positive. The value of the former at any instant t seconds after the commencement of short circuit is then

$$-I\epsilon^{-\frac{R}{L}t}$$

and that of the latter,

$$\frac{E}{R}\left(1-\epsilon^{-\frac{R}{L}t}\right)$$

The resultant current in the positive direction is thus

$$\frac{E}{R}\left(1-\epsilon^{-\frac{R}{L}t}\right)-I\epsilon^{-\frac{R}{L}t}$$

Now, if the current is to be of value I at the conclusion of short circuit, and T denote the total time of this short-circuiting,

$$I=\frac{E}{R}\left(1-\epsilon^{-\frac{RT}{L}}\right)-I\epsilon^{-\frac{RT}{L}}$$

and

$$E = I R \left(\frac{1 + \epsilon^{-\frac{R T}{L}}}{1 - \epsilon^{-\frac{R T}{L}}} \right)$$

The total time T, however, is equal to

$$\frac{\text{width of brush contact}}{\text{linear velocity of commutator surface}},$$

and E is equal to the product of B, the field strength; V, the linear velocity of the inductors of the coil; m, the number of external inductors per coil; l, their length—or, rather, the length of the magnetic field in which they lie—parallel to the axis of rotation, and $10^{-8.}$

Thus the lead must be such that commutation occurs when the coil is in a field of strength, such that

$$B = \frac{I R}{m\, l\, V} \left(\frac{1 + \epsilon^{-\frac{R T}{L}}}{1 - \epsilon^{-\frac{R T}{L}}} \right) \times 10^{8.}$$

Although this case is of service in facilitating comprehension of the practical conditions, it nevertheless differs from the practical one in several important points. The fringing field in reality increases more or less rapidly in strength as the pole horn is approached, so that not only is there a want of uniformity, but the coil passes through a reversing field for some short distance before reaching its position of commutation. Since, however, the ratio of the angle of lead to the angle subtended by the length between the pole horns affords some criterion of the efficiency of a machine, these points may be neglected in forming a rough estimate of this ratio.

Taking them into consideration, it will be seen from fig. 153 that the magnetic field somewhat rapidly increases in strength as the pole piece is approached; but if the angular width of the coil is small—as is usually the case—it may be regarded as increasing at a uniform rate along this small portion of the interpolar gap. The value of the induced electro-motive force acting on the coil during commutation may thus be taken as

$$E_o + E\,t$$

where E_o is the electro-motive force induced at the beginning of short circuit; t, the time reckoned from that instant; and E, a number such that when multiplied by the whole time, T, and added to E_o, it is equal to the electro-motive force induced in the coil at the end of the short circuit. As before, the dying current is given by

$$I\,\epsilon^{-\frac{R}{L}t}$$

but the increasing current now satisfies the equation

$$E_o + E\,t = L\,\frac{d\,i}{d\,t} + R\,i,$$

from which it may be shown that, as it is the resultant of both these, the actual current equals

$$\left(\frac{E_o}{R} - \frac{E\,L}{R^2}\right)\left(1 - \epsilon^{-\frac{R}{L}t}\right) + \frac{E\,t}{R} + I\,\epsilon^{-\frac{R\,t}{L}}$$

and, so, if commutation is effected in such a way that at the end of the short circuit i is equal to $-$ I.

$$I\left(1 + \epsilon^{\frac{R\,T}{L}}\right) + \left(\frac{E_o}{R} - \frac{E\,L}{R^2}\right)\left(1 - {}^{-\frac{R\,T}{L}}\right) + \frac{E\,T}{R} = 0,$$

and so the values of $\frac{R\,T}{L}$ and of the reversing field which will permit of the necessary reversal may be found.

So far, the effect of the contact resistance between the brushes and commutator has been entirely left out of consideration. As a matter of fact, the influence of this resistance affects the commutation in the highest degree, and may, indeed, be said to govern the question of sparkless collection. Let I, as before, denote the full current per parallel section of the armature; i, the current in the coil at any instant; and R′, the contact resistance between the brushes and a single commutator segment. Assuming that a brush is equal in width to a segment, the resistance between it and the leading segment uniformly increases, while that between it and the following one becomes correspondingly less. The former resistance is thus at any time t during the period T of short-circuit given by

$$R' \frac{T}{T - t}$$

and that of the latter by

$$R' \frac{T}{t}$$

Applying Kirchoff's equation to the coil, and noting that the connecting strips between the commutator segments and the coil carry currents $I + i$ and $I - i$ in the leading and following segments respectively,

$$R' \frac{T}{t}(I - i) + R_c (I - i) - r i - R_c (I + i) - R' \frac{T}{T - t}$$

$$(I + i) + E_0 + E t - L \frac{d i}{d t} = 0$$

where R_c is the resistance of a single connector, and r that of the coil itself. Thus putting $R = r + 2 R_c$

$$E_0 + E t - L \frac{d i}{d t} - R i - \frac{R' T}{T - t}(I + i) + \frac{R' T}{t}(I - i) = 0.$$

From this it follows that if the current density across the brush contact were uniform, the last two terms would vanish, and the resistance there would be of no effect. This relation of current density, however, is only possible at one particular speed and angle of lead, and is usually quite unobtainable.

This equation is not adapted for solution by the ordinary methods, but Drs. Arnold and Mie* have attacked it successfully, developing i as the sum of a series of powers of $\frac{t}{T}$.

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* "Electrotechnische Zeitschrift," February, 1899; also embodied in "Theory of Commutation," C. C. Hawkins. (Tucker.) 1901.

It is shown that the current i in the coil is given by the expression

$$i = \mathrm{I}\,(1 + a_1 k + a_2 k^2 + a_3 k^3 + \text{etc.}) + \mathrm{E}_0 \frac{\mathrm{T}}{\mathrm{L}} (b_1 k + b_2 k^2 + b_3 k^3 + \text{etc.}) + \frac{\mathrm{E\,T}^2}{\mathrm{L}} (c_1 k^2 + c_2 k^3 + c_3 k^3 + \text{etc.})$$

where a, b, and c, etc., are functions of the quantities having values as given below, and $k = \frac{t}{\mathrm{T}}$.

Let $\frac{\mathrm{R'T}}{\mathrm{L}}$ be denoted by r, and $\frac{\mathrm{RT}}{\mathrm{L}}$ by r';

then,

$$a_1 = -\frac{2r + r'}{1 + r},\ a_2 = \frac{a_1 (1 - r') + r'}{2 + r},\ a_3 = \frac{a_2 (2 - r') + a_1 r'}{3 + \mathrm{R}},$$

and $$a_n = \frac{a_{n-1} (n - 1 - r') + a_{n-2}\, r'}{n + r};$$

$$b_1 = \frac{1}{1 + r},\ b_2 = -\frac{b_1 (r + r')}{2 + r},\ b_3 = \frac{b_2 (2 - r') + b_1 r'}{3 + r},$$

and $$b_n = \frac{b_{n-1} (n - 1 - r') + b_{n-2}\, r'}{n + r};$$

$$c_1 = 0,\ c_2 = \frac{1}{2 + r},\ c_3 = -\frac{c_2 (r + r')}{3 + r},\ c_4 = \frac{c_3 (3 - r') + c_2 r'}{4 + r}$$

and $$c_n = \frac{c_{n-1} (n - 1 - r') + c_{n-2}\, r'}{n + r}.$$

For a full discussion of the matter, accompanied by curves corresponding to actual cases, the papers referred to must be consulted, but the points of chief practical interest are here mentioned.

The quantity $\frac{\mathrm{R'T}}{\mathrm{L}}$ must, if collection is to proceed sparklessly, be greater than 1. If by reason of high self-inductance, high speed, or the brush contact-resistance being low, this condition is not complied with, then the current density at the end of short circuit is excessive, the commutator and brush are heated, and sparks are produced. It will be noted that it is in the brush contact that resistance is necessary. To increase the total resistance of the short-circuited coil, as is sometimes done by making the strips connecting them to the commutator of high-resistance metal, is seldom beneficial, and may often be disadvantageous. The difference in action

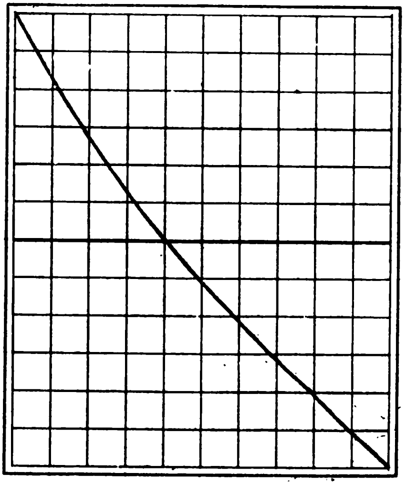

Fig. 145.

between the two resistances is, of course, due to the contact resistance varying proportionally to the time, while that of the connectors is constant.

If the period of short circuit be taken as a base horizontally, the current in the coil at various instants may be plotted as ordinates, and a curve obtained. These curves of current change may be either concave or convex upwards, figs. 145 and 146, but in each case the effect of the brush contact resistance is to render them straighter than they would be were it absent. Figs. 147 and 148 represent the cases of

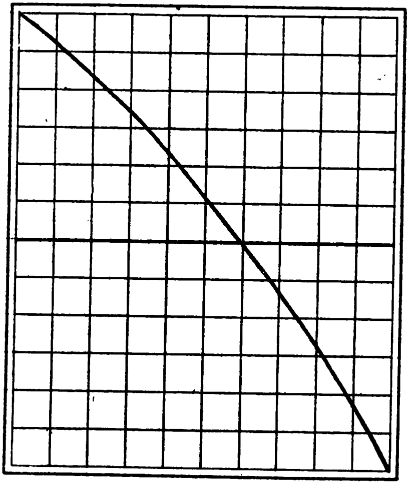

FIG. 146.

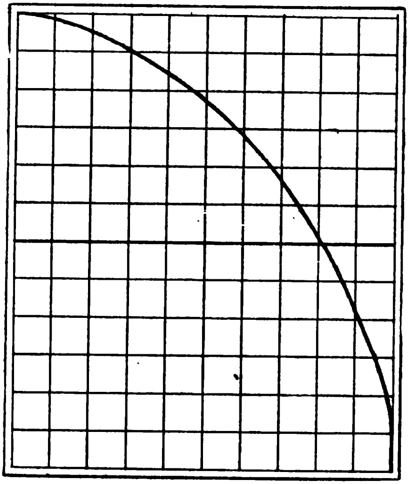

FIG. 147.

under and over reversal of current. The rate of current change at the end of the period T must not be too rapid. It is desirable, indeed, that by that time the rate, $\frac{d\,i}{d\,t}$, should have gradually become equal to nought; or that, in other words, the current should have become steady and equal to its full value before the end of the short circuit, remaining at that value. Though it is often impossible to arrange for this condition without making undue sacrifices, yet the rate of change should be made small, particularly in the case of high-speed machines. In cases where $\frac{R'\,T}{L}$ is less than unity the rate of change in the current at the end of the time T, as given by the previous equation, is infinite, except in one

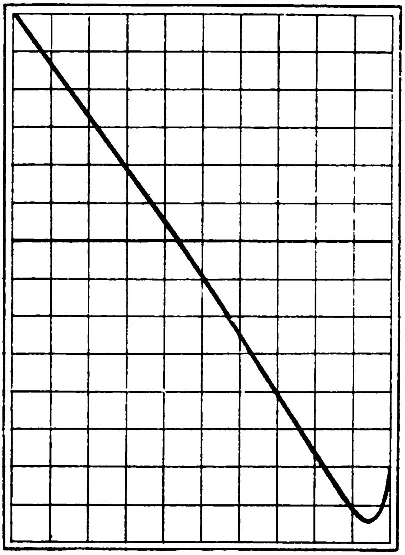

FIG. 148.

especial and practically-unobtainable case. Except, too, in this case, the rate, when $\frac{R' T}{L}$ is greater than 1, is given for $t = T$ by

$$\frac{I (R + 2 R') + E'}{R' T - L}$$

where E′ is the electro-motive force induced in the coil by its motion in the magnetic field at the end of the period T.

The quantities, then, affecting commutation, viz., R′, T, L, and the magnetic field which determines the values of E_o, E, and E′, may now be considered.

The contact resistance between copper brushes and copper is given as varying between ·00078 and ·00233 ohms per square inch of surface, according to various conditions. In the case of carbon collectors the average resistance is about ·03 ohms per square inch. Owing to the uncertainty of this resistance, and the probability of its varying from time to time with changes in the brushes and in the commutator surface, it is not advisable to adopt too high a value in estimating the probability of sparking, though the chances are that it would increase rather than diminish.

It is owing in part to this contact resistance with carbon being so much—nearly thirteen times—greater than that with copper, that brushes of this material are now so generally employed. Nevertheless, as the maximum current density permissible with carbon is only about one-fifth of that with copper, a larger surface is necessary, leaving the advantage in this respect of carbon over copper as about three to one. A further point in favour of carbon is that nothing in the nature of fusion with the commutator can occur even when sparking does take place.

The period T is evidently given by

$$T = \frac{60 W}{\pi D N}$$

where N denotes revolutions per minute, W the width of brush contact, and D the diameter of the commutator.

The estimation of the self-inductance, L, of a coil is a somewhat difficult matter, but may be facilitated by a consideration of certain simple cases. Thus, taking firstly the winding of a smooth armature, fig. 149, let A denote the circumferential width of the coil, l its length parallel to the shaft, M the number of turns per section. It may be assumed that the mutual inductance between the coil under consideration, and those on either side of it, prevents any of the magnetic lines passing through the latter. Thus (pp. 246-7), the intensity at the distance a from its centre is equal to

$$\frac{4\pi}{10} \times \frac{2\,M\,a\,l}{\pi\,a\,A} = \frac{\cdot 8\,M\,l}{A}$$

it being assumed that the whole resistance to the flux is in the air outside the core. The intensity is thus uniform along the whole width, the mean number cut by each turn is $\cdot 4\,M\,l$, and the value of L becomes $\cdot 4\,M^2\,l \times 10^{-9}$ secohms. It must be understood that l here represents the length in centimetres of that portion of a complete single turn which lies upon the iron core. The inductance of that part of a turn which forms the end connections is far smaller, but some small estimated addition may be made to the calculated value of L on its account.

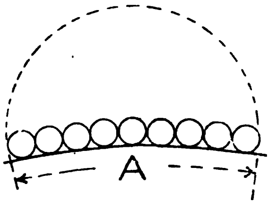

FIG. 149.

Turning to the case of a slotted armature, and assuming, as before, that mutual inductance suppresses magnetic lines in adjacent coils, the magnetic circuit of the inductance flux consists of the teeth on each side, the core below, and the air space above. The flux is thus, fig. 150, given by

$$\frac{4\pi l}{10}\left(\cdot 733 \log_{10}\frac{B}{A}\right)$$

if the reluctance of the iron be neglected (see page 246). The value of L is thus

$$\cdot 4\pi M^2 l\left(\cdot 733 \log_{10}\frac{B}{A}\right) \times 10^{-9}.$$

The increase in L, due to the reluctance of the teeth being neglected, compensates to some extent for the omission of the ends of the coil from consideration.

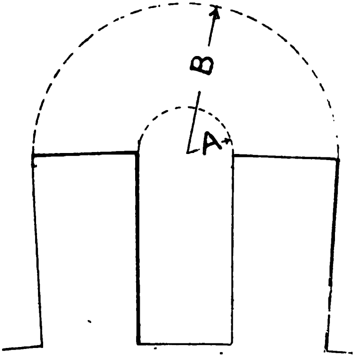

FIG. 150.

Though calculations of the nature indicated above serve to furnish some idea of the magnitude of L, its value is influenced by so many circumstances, including the method of winding, that no very near approach to accuracy can be expected.

The inductance of armature coils has lately been discussed by Mr. Hobart,* and average values of L per unit length of conductor have been given. Considering the case of a coil such as is employed in modern multipolar machines, fig. 151, with relatively lengthy end connections, the portions A, A, which lie in the slots of the core, constitute the "embedded length," while the end connections and such parts of A, A as pass across insulation, air discs, etc., are alluded to as the "free length."

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* "Journal of Institute Electrical Engineers." No. 153.

According to the author of the paper, the average induction per ampere-turn per centimetre length of the free part

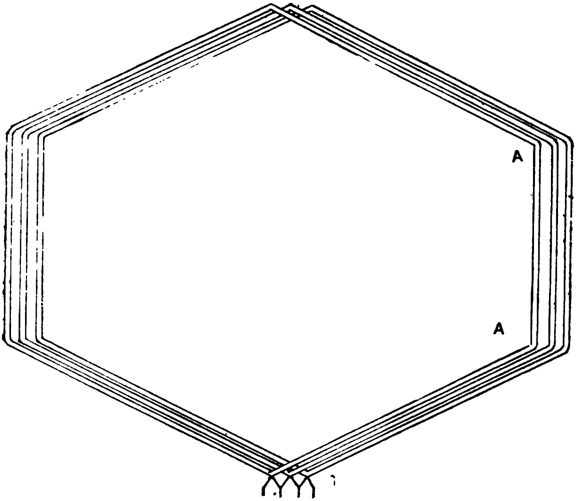


Fig. 151.

of a coil is about ·8 C.G.S. lines, while in the embedded length its value is about four such lines.

As an example of the application of these values, let in fig. 151 the length of the embedded lengths be 30 centimetres, and the length of the free portion be 100 centimetres per turn. Let each coil contain six turns, and let three coils be simultaneously short-circuited under the positive, and three under the negative, brush. Then :

Lines per ampere-turn for free length.................. = ·8 × 100 = 80
Lines per ampere-turn for embedded length .... = 4 × 30 = 120
Lines per ampere for free length of one coil .... = 80 × 6 = 480
Lines per ampere for embedded length of one coil...... = 120 × 6 = 720
Lines per ampere for total group of three coils ...... = 3 (720 + 480) = 3,600

The total inductance of a single coil is thus

$$3600 \times 6 \times 10^{-9} \text{ secohms.}$$

The experience of Mr. Hobart led him to the conclusion that in a carefully-designed machine satisfactory commutation might be expected if

$$\frac{4\pi \times \text{total flux} \times \text{current to be reversed}}{\text{period of short circuit}} \times 10^{-8} \leqq 3.$$

This statement, so far as its theoretical deduction is concerned, seems, however, to be based on the assumption that the curve of current during short circuit follows a sine law, a condition of things which seems to be scarcely borne out by the work of Drs. Arnold and Mie.

Applying the expression to the above case, taking the period during which the coil is short-circuited as ·023 seconds, and the current as 120 amperes—

$$\frac{4\pi \times 3600 \times 120}{\cdot 023} \times 10^{-8} = 2{\cdot}36,$$

which is well within the limit.

For the purpose of determining the values of E_0 and E, it is necessary that the strength of the magnetic field at all points of the core surface situated between the pole horns should be known. Considering, therefore, firstly, that part of the field which is due to the magnets, and leaving all armature reactions out of consideration, the induction in the air gaps under the pole faces may be regarded as uniform and equal to B except in the neighbourhood of the extremities of the horns. Under, and from the faces of, the pole horns the lines spread outwards, producing a fringe and giving a weakened field under the horn. If, now, the reluctance of the core in the region between the pole horns be negligibly small, the magneto-motive force between the magnets and the core may be considered the same at all points; and if L denote the length of the path followed by magnetic lines entering the core at any point, l denoting the length of a single air gap, the induction of the lines entering at that point is

$$B\frac{l}{L}.$$

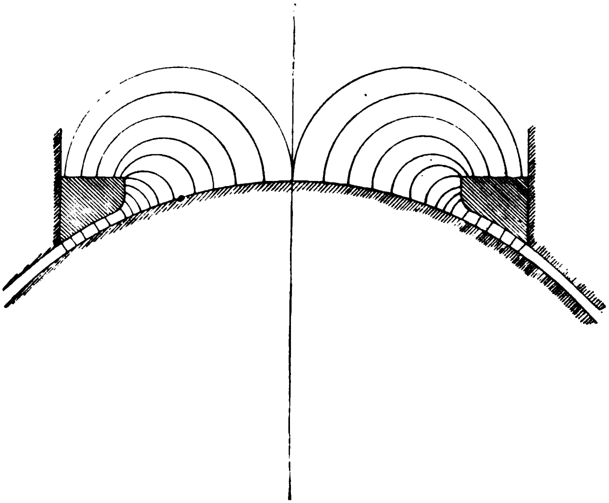


Fig. 152.

The paths followed by the lines can be estimated by the exercise of some judgment and experience of constructed machinery, and the values of L for various points on the core measured from a sketch, fig. 152.

While lines enter from the one pole, some, nevertheless, start out for the other at any point; but an approximate estimate of the induction of the issuing lines may be made by noting that midway between the horns the entrant and emergent inductions must be equal. If the path of the lines to this median point be L′, each induction may be taken as

$$B\frac{l}{L'},$$

and the induction of the issuing lines may be assumed to diminish uniformly from that value at the centre to nothing under the pole from which the entering lines set out.

Taking the values of l from fig. 152, and developing out the armature circumference into a straight line, as in fig. 153, the values of the induction due to the field alone may be set out as ordinates, giving the curve A A. It is evident that the areas enclosed above and below the straight line are equal to each other, since each measures the total number of lines entering and leaving the core.

This symmetrical distribution of flux is, however, considerably altered when the armature carries any current, owing to the armature reaction. In this case, as already shown, the brush requires lead, so giving rise to back ampere turns and also to a cross flux (page 211).

The back ampere turns, since they produce a magnetomotive force directly opposed to that of the magnetic coils, may be regarded as reducing the ampere turns of the field by their own number. The exact reduction of the flux produced by them can only be ascertained from the static characteristic of the machine—that is, from a curve showing the relation between exciting ampere turns and flux through the armature. A rough and excessive estimate of the effect of the back turns may be made by neglecting variation of permeability in the iron. In these circumstances, if X be the ampere turns of the field, X′ the back ampere turns, B the normal flux due to X, the reduction of flux becomes—

$$B\frac{X'}{X}$$

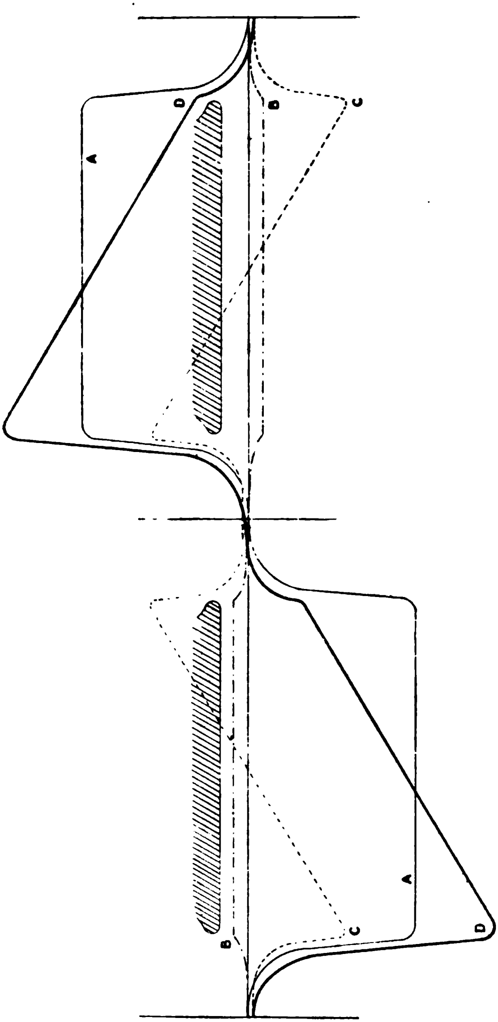


FIG. 153.

for regions under the pole faces, while for positions between the pole horns it is approximately

$$B \frac{X' l}{X L}.$$

This reduction of flux is set out in the curve B B of fig. 153.

The cross-turns yet remain to be taken into account. To trace the nature and magnitude of their effects, let fig. 154 represent the armature conductors under one pole face. If it is assumed that the entire reluctance of the paths followed by lines produced by currents in these conductors lies in the air gaps, then the induction at any point under the pole face produced by a single conductor is given by

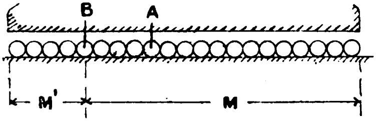


FIG. 154.

$$B' = \frac{4\pi}{10} \frac{C}{2l},$$

where C is the current in amperes and l the length of air gap.

It thus appears that the induction due to a given current in a conductor lying at any point A, fig. 154, is uniform and constant over the region between A and the pole horn, that it is quite independent of the position of A under the pole face, and that its direction is of opposite sign on each side of A.

Such being the effect produced by a single conductor, that of the total group may be deduced. At any point B under the pole-face lines in one direction are produced by all conductors lying to the right; if these be M in number, the induction produced by them is equal to

$$\frac{4\pi}{10} \frac{MC}{2l}.$$

On the other hand, all conductors to the left produce an induction in the inverse direction ; if these be M′ in number, the value of this is

$$\frac{4\,\pi}{10}\,\frac{M'C}{2\,l},$$

and thus the resultant induction becomes

$$\frac{4\,\pi}{10}\,\frac{C}{2\,l}\,(M - M'.$$

This expression shows that when M is equal to M′—*i.e.*, at the centre of the pole, the induction due to the cross turns in zero, increasing to a maximum under the pole tips. If ϕ denote the angle subtended by the pole face, N the total number of turns on the armature, n the number of parallel circuits in the armature, the maximum induction at the pole horn is

$$\frac{4\,\pi}{10}\,\frac{N\,C\,\phi}{2\,l\,n\,360} = \frac{1}{573}\,\frac{N\,C\,\phi}{l\,n},$$

the C in this expression being taken as the total current in the armature.

For points on the armature between the pole horns the induction, of course, rapidly becomes less than the above expressions, owing to the increased length of air path. The value of l for such points must be increased in proportion to the length of the estimated air path.

While the induction at any point under a pole face is dependent only on the conductors lying under that face, the region between the pole horns must be regarded as affected by the conductors under each of the poles on either side. Thus at the point midway between the pole horns the effect is double that due to the single group under one pole. The induction due to the cross turns calculated in this manner is set out by curve C C of fig. 153. It is to be noted that the total flux is not reduced by the cross turns, but only distorted.

The resultant of the three curves A, B, C, evidently is the curve of actual resultant induction. From this, shown as curve D D, in fig. 153, the general nature of the distortion produced by armature reaction may be noted, and the field strength at any angular distance from the axis of symmetry ascertained.

The magnetic field having been investigated generally in this manner, the original question of commutation may be reverted to. The necessity for determining the field strength arose from the fact that a knowledge of E_0 and E was required, these quantities being such that $E_0 + E\,t$ equals the electro-motive force induced in the coil at any time t from the beginning of short circuit. It is advantageous for this determination to enlarge that part of the resultant induction curve in which commutation occurs. In fig. 155 the interpolar part is thus enlarged, the horizontal figures indicating distances in degrees from the axis of symmetry. Now $E_0 + E\,t$ can be written in the form $2\,\pi \times$ radius of core × revolutions per second × conductors per segment × $(H_0 + H\,t)$ × length of core × 10^{-8}, where H_0 is the mean field strength in which the conductors move at the beginning of short circuit, and H is a number such that when multiplied by T the total period of short circuit, and added to H_0, it gives the strength of field in which they move at the end of that period.

Thus, in the curve of fig. 155, if the lead of the brushes be such that short circuit begins at a position 20 deg. from the axis of symmetry, the value of H_0 is seen to be 260. If the period of short circuit is such that it terminates when the coil has rotated 3 deg. further—*i.e.*, when it is 23 deg. from the axis—the increment of the field strength during the time is 258, and H is equal to $\frac{258}{T}$.

It is to be remarked in connection with the above investigation of the magnetic field that, as is the case with the various other methods of treatment published by different authors, at the best only an approximation to the actual conditions can be obtained. The several productions of magneto-motive force and the numerous alternative paths for the lines are so complex around the armature as to baffle any attempt at exact calculation.

In the case of an actually existing machine the circumferential distribution of flux may be practically investigated by a modification of the method described on page 103. Two pointed wires are mounted in a non-conducting block, as shown in fig. 156, the distance between their points being exactly equal to the pitch of the commutator segments.

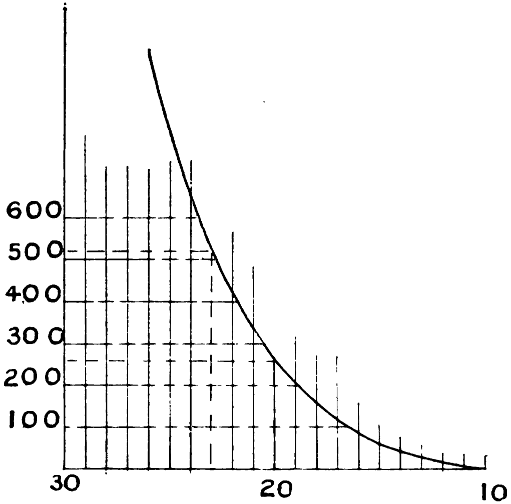


FIG. 155.

The outer ends of the two wires are connected to a sensitive voltmeter, and the readings of this are proportional to the strength of field in which the coil is moving when short-circuited through it. The block supporting the wires is mounted on an arm, which, as it is rotated, permits of readings being taken at all points round the commutator. Where a condenser and ballistic galvanometer are available, their use, connected as shown in fig. 156, is preferable. The condenser becomes charged when the key is depressed to the left, and is discharged through the galvanometer, giving a throw which is proportional to the induction when the key is depressed to the right. It is, however, to be noted that when the armature is carrying current the electromotive force generated in the coil (which is proportional to the field strength) is greater than the measured quantity by an amount equal to the drop over the coil's resistance.

It is apparent from fig. 153 that the cross turns have the disadvantageous effect of weakening the induction in just that region where it is requisite for successful commutation. The necessity for an adequate reversing field under the leading pole horn thus, independently of other considerations, sets a limit to the current which the armature can carry; and, so far as this point is concerned, any means of reducing the cross flux produced by a given armature is a means of increasing its output.

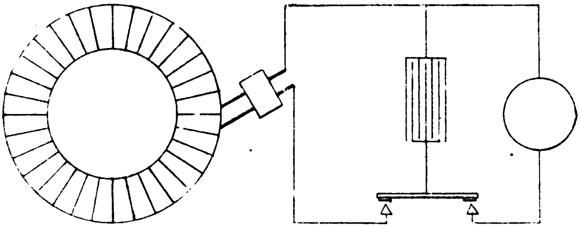


FIG. 156.

The most obvious method of effecting the desired result is to increase the number of poles. By doubling, for example, the number of poles, the number of active conductors under each is halved, and consequently the cross induction is at once reduced by 50 per cent. In reality the reduction is a trifle less, owing to the iron portion of their path having some reluctance; but this may, as already remarked, be regarded as negligible in view of the high reluctance of the air gaps. Multiplication of the poles is, nevertheless, not always feasible or advisable, and several other devices with the same object have at various times been proposed.

One sufficiently simple means of minimising the cross flux is to employ a normal flux of such strength that the pole horns are well saturated, in which case the effect of both the cross and back turns is small. For instance, turning to the permeability curve for wrought iron, fig. 112, it may be seen that at high inductions quite a large reduction of the magnetising force produces but a small diminution in the induction. Where laminated poles are employed the condition of saturation may be obtained by cutting off the horns of alternate plates, as shown in fig. 157.

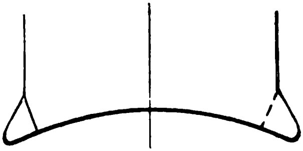


Fig. 157.

Another method of securing saturation of the pole horns is to bridge the interpolar gap by a thin cast-iron sheet, as in fig. 158. This has the disadvantage of augmenting useless leakage of lines from pole to pole, though ensuring high saturation.

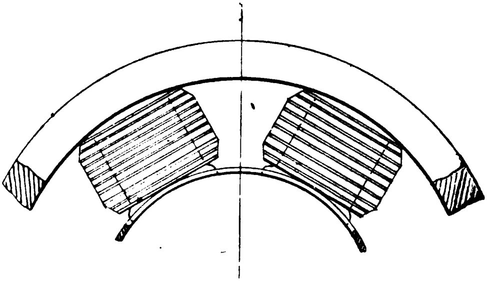


Fig. 158.

A weakening of the cross flux may be effected by interposing deep grooves in the pole face, as in fig. 159, across the path of its lines; but unless the magnet can be entirely divided by such groove the weakening is simply due to the reluctance of the lengthened path in the iron.

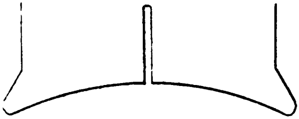


FIG. 159.

Several methods depending upon the production of an opposing magneto-motive force have been suggested or employed. Among these may be noted that of winding a coil round the magnet across the middle of the pole face, as in fig. 160. This, being in series with the armature, produces a flux opposite and proportional to that of the cross turns.

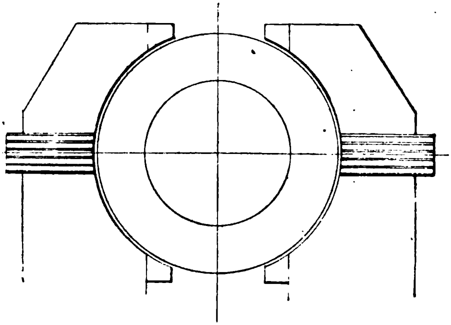


FIG. 160.

A modification of this method is to lay the turns in a number of grooves over the pole face, instead of in one single slot. Professor Ryan has suggested the employment of conductors laid parallel to the axis of the machine in holes through the magnets, fig. 161. The device would be applicable in the case of machines carrying heavy currents—such as those employed for electrolytic purposes—the conductors being in series with the armature. Yet another

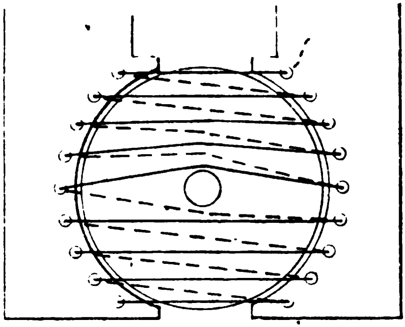


FIG. 161.

means of attaining a constant reversing field is to place a small auxiliary pole over the centre of the pole gap proper, and to excite it by a series coil in such a way that its lines oppose those of the cross flux. In actual practice it is, nevertheless, quite rarely that any of these latter devices are employed, the usual course being to limit the number of armature ampere turns per pole.

Apart from these devices for maintaining a strong field under the leading horn, is the method of armature winding introduced by Sayers. In this arrangement the winding which normally carries the current is not connected to the commutator by the usual strips, but through compensation coils which are wound on the core in positions to the rear of the main coils with which they are in series during commutation. At a period when the main coil has passed from under the pole face, its compensating coil is still under the trailing horn, and being wound in the reverse direction, produces the electromotive force necessary for the reversal of the current. Owing to their position in a field of full intensity the compensating coils require but a single or few turns, while as they are out of the circuit altogether for the greater part of the time, the section of their wire may be small. One advantage of the device is that the current may be commutated before the coils have reached the centre of the interpolar gap, and in this way the brushes may have lag instead of lead. As a result of this, what are usually the back turns become forward turns, strengthening the field.

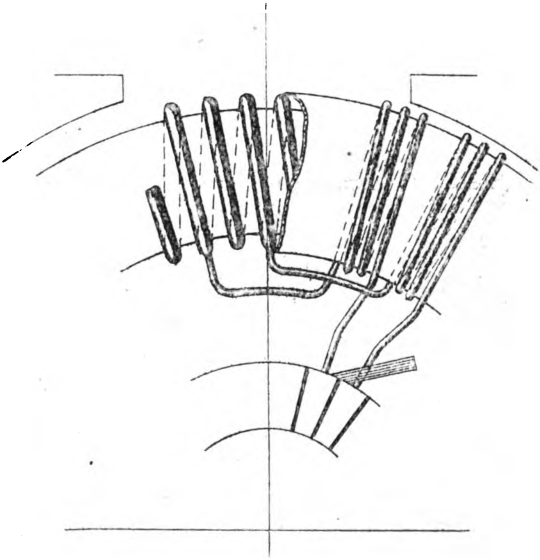


FIG. 162.

Another device is also employed by Sayers in which the compensating coils are not only under the influence of the trailing horn, but also in the field of a small auxiliary pole fitted midway between the pole horns.

In the diagram, fig. 162, of the first method of winding it is to be understood that the separation of inductors and compensating coils into two layers, as shown, is purely diagrammatic.

CHAPTER XIII.

Motors for the Conversion of Electrical Energy into Mechanical Work.—The fundamental principle of the electric motor is that a conductor situated in a magnetic field and carrying a current is acted upon by a mechanical force tending to drive it parallel to itself and at right angles to the magnetic lines. Thus, if in fig. 163 AB is carrying a current, and the lines of the field enter the paper perpendicularly, the mechanical reaction impels AB either to the right or toward the left in the plane of the paper, according to the direction in which the current flows. The direction of motion may be inferred from the facts that—(*a*) the magnetic field of a conductor due to its own current has a right-hand screw relation to the current—*i.e.*, if a screw advance in the direction in which the current flows, it rotates in the same direction as that of the encircling magnetic lines; (*b*) the motions produced by electro-magnetic forces are always such as to produce conditions reducing those forces. It may now be seen that in fig. 163, if the current flow from A to B, the magnetic lines produced by it rise upwards on the right-hand side and

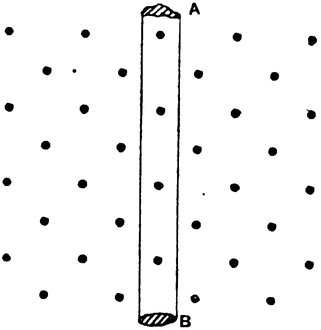


FIG. 163.

pass down into the paper on the left. The external field producing the motion is thus reduced in strength on the right-hand side of the conductor, and, by (*b*), the conductor moves to the left hand, bringing as many of its right-hand lines as possible to oppose those creating the motion. These actions are rendered more intelligible by fig. 164, in which the conductor would evidently move upwards.

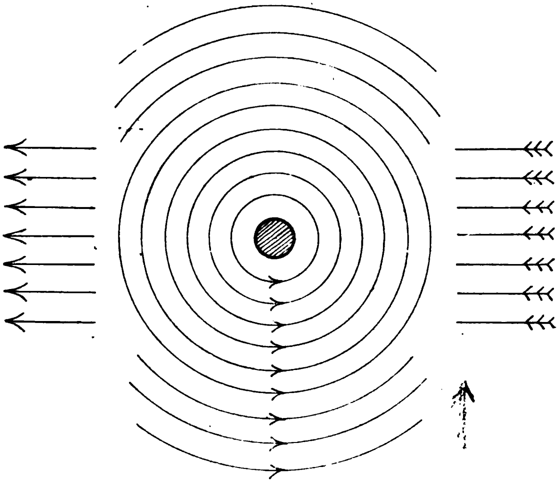


FIG. 164.

As regards the magnitude of the force, one centimetre length of a conductor carrying unit C.G.S. current in a magnetic field of one C.G.S. line per square centimetre is acted upon by a force of one dyne. Hence the force on any conductor is given by

Force in dynes = induction × current × length of conductor, when the conductor lies at right angles to the direction of the field. If the conductor be curved, or if it lie at some other angle with the direction of the magnetic lines, for the last term in the above equation must be substituted the length between its ends when these are projected on a plane at right angles to the field, as shown in fig. 165.

It is evident that the direction of the force will be reversed if either the direction of the field or of the current are singly reversed, but that if both be simultaneously altered the force remains unchanged. If the conductors A and B represent the sections of a coil encircling the core in fig. 166, the current will pass upwards in the one and downwards in the other. If the poles be as indicated, and A be carrying the upward current, the force acting upon A is directed upwards; at the same time the direction of the current is downward in B, while that of the magnetic lines is unaltered, and the force is consequently at B directed downwards. The two forces thus combine to produce a twisting moment about the axis. In a similar manner, if the entire surface of the core be covered with coils, the force produced on the coils carrying the current in one direction is in the opposite sense to that exerted on those carrying the current in the reverse direction, and a couple tending to rotate the armature results.

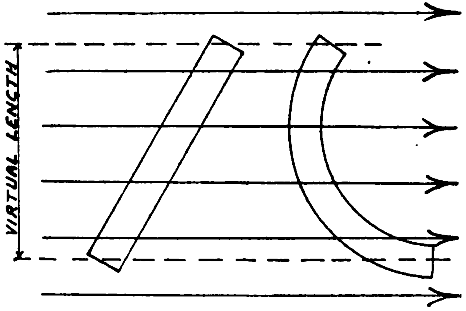


FIG. 165.

In general, then, a dynamo and motor have no essential difference in construction, the one absorbing mechanical energy and converting it into electrical energy, the other performing the reverse operation. If in a dynamo working as a generator the direction of the armature current be reversed, but the polarity of the field remain unchanged, then it commences to run backwards as a motor. This will be the case with a shunt-wound machine; for the positive terminal will be of the same sign when the current is reversed. On the other hand, if the current of a series-wound generator be reversed, the polarity of the field is also changed, and the armature consequently rotates in its previous direction. What will occur in the case of a compound-wound machine depends upon the relative strengths of the shunt and series coils, as these will evidently be acting in opposition to each other.

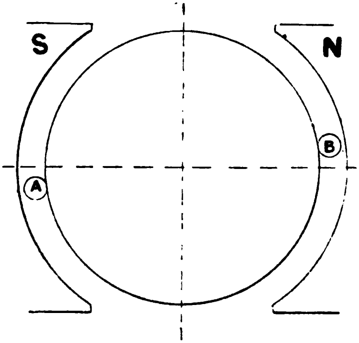


FIG. 166.

The total pull round the armature of a motor will be the sum of the pulls on all the conductors round its circumference, and these will obviously depend upon the direction which the magnetic lines follow in the gap between the pole faces and the core. The resultant effect of the whole number may be shown to be independent of this; for let

Z = the flux from a single pole,
T = the total number of external conductors,
l = the active length of each conductor,
D = the radius to centre of conductors,
ϕ = the angle subtended by a pole face,
c = current in each conductor,
C' = total armature current;

then B, the average induction, is given by

$$B = \frac{Z}{\pi D \frac{\phi}{2\pi} l}$$

and under each pole face there lie $T \frac{\phi}{2\pi}$ conductors.

The total tangential pull round the circumference is thus

$$\frac{Z}{\pi D \frac{\phi}{2\pi} l} . 2T \frac{\phi}{2\pi} l . \frac{c}{10} \text{ dynes,}$$

which simplifies to

$$\frac{2ZTc}{\pi D} \times 10^{-1} \text{ dynes}$$

in the case of a bipolar machine. If, however, there be p pairs of poles, and n pairs of parallel circuits in the armature winding, this expression becomes

$$\frac{ZTC'p}{\pi D n} \times 10^{-1}$$

This force acts at a distance $\frac{D}{2}$ from the axis, and the torque is thus

$$\frac{ZTC'p}{2\pi n} \times 10^{-1}$$

which, expressed in inch-pounds, becomes

$$\frac{1{\cdot}41\, ZTC'p}{n} \times 10^{-8}$$

If with this torque the motor is running at N revolutions per minute, the work per minute is $2\pi N$ times the torque, or

$$2\pi \left(\frac{1{\cdot}41\, ZTC'Np\, 10^{-8}}{12n}\right) \text{ foot-pounds per minute,}$$

or

$$\frac{{\cdot}735}{33000} \left(\frac{ZTC'Np}{n} 10^{-8}\right) \text{ horse power.}$$

As an example, consider an eight-pole motor having a flux of 25×10^6 C.G.S. lines per pole, and 1,100 conductors on the armature, the current being 500 amperes, and the armature winding consisting of eight parallel circuits. Here

$$\begin{aligned} Z &= 25 \times 10^6, \\ T &= 1100, \\ C' &= 500, \\ p &= 4, \\ n &= 4, \end{aligned}$$

and the torque is therefore

$$\frac{1{\cdot}41 \times 25 \times 10^6 \times 1100 \times 500 \times 4 \times 10^{-8}}{4}$$

or 193,875 inch-pounds.

If this motor were running at 300 revolutions per minute, the horse power developed would be

$$\frac{{\cdot}735}{33000}\left(\frac{193875 \times 300}{12}\right) = 108.$$

The horse power so calculated would be that exerted on the armature, and would be greater than that delivered from the shaft by the amount of friction, windage, eddy current, and hysteresis losses.

It will be evident that, while the external conductors of a motor armature are being driven across the field lines by the current sent through them, they must at the same time have an electro-motive force induced in them in exactly the same way as if they were driven across the field mechanically from the shaft. A consideration of the relation between the direction in which the lines are cut and that of the induced electro-motive force shows that the latter is in opposition to the electro-motive force applied to the motor. The magnitude of this "back electro-motive force" is, exactly as if the machine were running as a generator, proportional to the speed and to the total magnetic flux.

As a consequence of this the current passing through the armature circuits of a motor is less than the quotient of the armature resistance into the electro-motive force of supply. If E_b be the pressure applied at the brushes, e the back electro-motive force, and R_a the armature resistance, the current flowing in the armature is equal to

$$\frac{E_b - e}{R_a},$$

and (page 216)

$$e = \frac{T Z N p}{60 n} \times 10^{-8}$$

A motor to which an electro-motive force is applied speeds up until the power absorbed is equal to the sum of its useful work and internal losses. The latter increase with the speed, and thus, if a motor be run light, its speed increases until the energy taken in is the equivalent of the frictional, hysteresis, eddy current, and other losses. Thus, if an electro-motive force of 200 volts be applied to the brushes of a motor, the resistance of the armature of which is ·4, and the armature current as measured be six amperes when running light,

$$200 - e = \cdot 4 \times 6 = 2 \cdot 4$$
$$e = 200 - 2 \cdot 4 = 197 \cdot 6.$$

The total energy absorbed in the armature is

$$200 \times 6 = 1200 \text{ watts};$$

the power spent in heating the conductors is

$$6^2 \times \cdot 4 = 14 \cdot 4 \text{ watts};$$

and the remainder, 1185·6 watts, is spent in overcoming the frictional, windage, and eddy resistances to motion.

It will be seen that the smaller the armature resistance is, or the less the resistances to motion are, the nearer does the value of the back electro-motive force approach that of supply, and thus the idea is suggested that in an ideally perfect machine of 100 per cent efficiency they would attain equality, and that in practical machines the ratio of e to E_b measures the armature efficiency. That this is so may, indeed, be shown, for, from the expressions given on pages 314-15, the rate of work at N revolutions per minute is

$$\frac{2\pi N}{60} \times \frac{Z T C' p}{2\pi n} \times 10^{-1} = \frac{Z N T p}{60 n} \times 10^{-1} \times C',$$

and

$$e = \frac{Z N T p}{60 n} \times 10^{-8},$$

whence the rate of work may also be written

$$e C' \times 10^{-7},$$

and dividing this by 10^{-7} to bring ergs to watts, the rate of work becomes $e C'$.

The electrical efficiency of a motor is thus seen to be

$$\frac{e C'}{E C}$$

where E denotes the electro-motive force supplied at the terminals, and C the total current, including any supplied to shunt coils on the magnets. From this it may appear that a machine is most efficient when $e = E$—that is, when the impressed and back electro-motive forces are equal; but in this case no current at all could flow, and the work done would consequently be nothing. The practical value of a machine, however, depends on the commercial efficiency, which is the ratio of the energy given out mechanically from the shaft to the energy absorbed electrically, so that in the case mentioned the commercial efficiency would be a minimum. If the current which is required to run the machine light, and is consequently necessary for overcoming losses, be denoted by c', if R be the combined resistance of armature and series coils, and if the resistances to motion be assumed constant at all speeds, then the current doing useful work is

$$\frac{E - e}{R} - c',$$

and the commercial efficiency has the value

$$\frac{e\left(\frac{E - e}{R} - c'\right)}{E\,C}$$

If for C in the denominator its equivalent $(E - e) \div R$ be substituted, and the whole expression be differentiated with respect to e and equated to zero, it is seen that the efficiency of a series-wound machine is a maximum when

$$(E - e)^2 = E\,R\,c'.$$

In the cases of compound and shunt machines, however, the substitution for C made above is not possible, and the expression for the value of e at maximum efficiency becomes more cumbrous.

As a result of the current in the armature coils of a motor flowing in the opposite direction to that which would result from the electro-motive force induced by themselves, it follows that the cross-ampere turns distort the flux in an opposite direction also. Thus in a motor the field under the leading horn is strengthened, and that under the trailing horn is weakened. But when commutation takes place, a current has to be started in a coil in the reverse direction to that which would be required in a generator; and thus it becomes necessary to shift the position of commutation back under the trailing horn, giving the brushes lag. The shifting of the brushes in this way evidently will produce back turns, opposing the main flux exactly as with a generator.

Series Motors.—The most important respect in which a series winding is advantageous compared with a shunt is that it enables a very large torque to be exerted when the armature is at rest or rotating slowly. Such a winding is, therefore, applicable in cases where it is necessary for the motor to start under load or where the statical resistance is greater than the normal; consequently series-wound motors are employed for traction work, cranes, hoists, &c., as well as in factories and mills where their reversibility, as well as the starting power, is a useful feature. As regards reversal, this may, of course, be effected by reversing the direction of the current in either the field coils or the armature, but not in both. Owing to the high inductance of the field, the whole advantage lies with the method, usually followed, of reversing the armature current.

The large initial torque on starting is due to the fact that the total flux of the machine, to which, as already shown, the torque is proportional, increases with the current. If the flux produced were directly proportional to the exciting current, then it is evident that the torque, being itself proportional to the product of current and flux, would vary as the square of the current in a series machine, but directly as the current in one with constant excitation. Now, though the flux does not increase with the current at this rate, yet it approximates to it along one portion of the magnetisation curve, and if the normal induction in the iron lie on this part, the torque increases in some ratio less than the square of, but greater than, the current.

To take full advantage of this property of giving a large increase of torque with a relatively small current variation, it is, then, essential that the machine should be so designed that the working induction lies below the knee of the magnetisation curve, otherwise the extra current drawn from the supply mains is nearly proportional to the increased torque.

The speed regulation of a series motor is very imperfect, inasmuch as any variation in speed alters the field flux in such a way as to augment that variation. If the load be reduced, the speed and back electro-motive force rise, cutting down the current and weakening the field. The immediate result of this is that the speed still further increases to maintain the previous back pressure, and the reactions continue until a stable condition is reached at a greatly increased speed. In short, a series motor can furnish a wide range of torque with but a moderate alteration in the current, the speed, however, varying considerably.

Shunt Motors.—Though incapable of exerting such great differences of torque with a given difference of current as series motors, shunt machines have the entire advantage with respect to speed regulation under varying loads. If the electro-motive force of supply at the terminals be constant, the magnetic flux may also be considered constant, and the effect of an increase of load is primarily to reduce the speed. This reduces the back electro-motive force, and a larger current flows through the armature. The augmented torque, which is proportional to the current, then accelerates the rotation, diminishing the speed variation. In many cases the increase of armature current will reduce the magnetic flux by reason of the strengthened armature reaction, and under these circumstances the speed variation is still smaller. An additional advantage of shunt-wound motors is that they are not particularly sensitive to fluctuations in the supply pressure, the speed falling but slightly for a moderate drop in the impressed electro-motive force. On these accounts this type of motor is very generally employed for ordinary purposes, particularly where uniformity of speed is desirable, as for driving machine tools, &c.

A shunt motor cannot satisfactorily be driven from a single series or compound-wound generator unless exceptional precautions are taken to ensure the electro-motive force of the generator being very steady. The difficulty arises from the fact that if the speed of the motor become too great, or the pressure from the generator fall, the motor may send a reverse current through its leads to the generator, the polarity of which must necessarily become reversed if series wound, and would possibly become so if compound wound, while the speed would rise to a dangerous extent.

Compound-wound Motors.—For certain purposes a compromise between the large starting torque of the series motor and the speed regulation of a shunt-wound machine is most advantageous, and under these circumstances a "cumulative" or ordinary compound winding may be employed, in which both shunt and series turns act in the same direction. In motors operating electrical lifts and elevators, where after the initial acceleration the velocity is uniform, the large torque at starting may be obtained by adding a number of series turns equal to about 10 per cent of the shunt. When once full speed is attained, and the current has fallen to its normal value, the effect of these is very small, and the motor runs to all intents and purposes as a shunt machine.

For special purposes, where a uniform speed is essential—as, for example, in driving textile machinery—the motors have differential compound windings, in which the series turns more or less weaken the main flux produced by the shunt coils. If the load on such a machine be reduced, the tendency to accelerate is checked by the fact of the back electro-motive force increasing, and, the current becoming smaller, the series coils are rendered less powerful and the flux due to the shunt is strengthened. As a result of the increased field strength, the armature current and torque are further reduced, and so the reaction results in a weakening of the torque to suit the remaining load, and an absence of, or an exceedingly small, speed variation. Indeed, by putting on a sufficient number of series turns the speed can be made to rise as the load is made greater. As against these advantages, the differentially-wound motor has no great starting torque, is unable to carry much overload, and is heavier than a shunt motor of the same power.

The differences in character between motors furnished with these various means of excitation may be rendered clearer by considering the action under various conditions of the same armature and magnets successively fitted with series, shunt, and compound windings.

For example, a certain machine has 204 conductors and an armature resistance of ·05 ohms when at its running temperature, and the armature has sufficient surface area to get rid of the heat equivalent of 500 watts. The magnets are such that, with a reasonable induction, the total flux through the core may be 5·9 million C.G.S. lines, with 12,400 ampere turns. If the resistance of the series coils be taken as ·05, the total resistance of the machine is ·1 ohm.

When series wound let the values of the torque corresponding to different currents be as follow :—

Current in amperes.	Flux in C.G.S. lines.	Torque in inch-pounds.
0	$\cdot 5 \times 10^6$	0
10	$1\cdot 1 \times 10^6$	32
20	$2\cdot 6 \times 10^6$	150
30	$3\cdot 5 \times 10^6$	325
40	$4\cdot 25 \times 10^6$	490
50	$4\cdot 8 \times 10^6$	691
60	$5\cdot 2 \times 10^6$	900
70	$5\cdot 5 \times 10^6$	1,120
80	$5\cdot 7 \times 10^6$	1,312
90	$5\cdot 8 \times 10^6$	1,500
100	$5\cdot 9 \times 10^6$	1,700
110	$5\cdot 9 \times 10^6$	1,870

A relation between the speed and current must now be obtained. This may be got from the expression

$$\text{current} = \frac{E - e}{R}$$

by substituting for *e* its equivalent

$$\frac{Z\,T\,N}{60} \times 10^{-8}.$$

For any particular value of the current, then,

$$N = \frac{(E - C\,R)\,60}{Z\,T} \times 10^8.$$

The pressure of the supply mains being 100 volts, in this case

$$N = \frac{100 - \cdot 1\, C}{3{\cdot}4\, Z} \times 10^8,$$

and corresponding values of C and Z may be taken from the preceding table. Tabulating thus C and N—

Current.	Revolutions per minute.
0	0
10	2,650
20	1,100
30	813
40	660
50	580
60	530
70	497
80	474
90	456
100	450
110	445

And from these two tables a third can at once be drawn up showing the corresponding values of speed and torque.

Torque.	Revolutions per minute.
0	0
32	2,650
150	1,100
325	813
490	660
691	580
900	530
1,120	497
1,312	474
1,500	456
1,700	450
1,870	445

CASE II.—The magnets excited by shunt coils of such a number and resistance that when the motor runs at 450 revolutions per minute the torque is the same as in Case I., viz., 1,700 inch-pounds.

The first point to be considered is what the values of the current and magnetic flux must be to satisfy this condition. These will not be the same as in Case I., because there the difference between the impressed and back electro-motive forces drove the current through the combined resistance of armature and series turns, while in the present case it drives the current through the resistance of the armature alone. The difference between the electro-motive forces must therefore be less, and since the supply pressure is the same as before, the back electro-motive force must be increased; but the only factor of this which can change is the flux, which must consequently be greater.

To determine the current and flux, then, for a torque of 1,700 inch-pounds at a speed of 450 revolutions per minute, it should be noted that the product of current and flux must be the same as in Case I., for to this the torque is proportional. Thus—

$$1{\cdot}41\,Z \times 204 \times C \times 10^{-8} = 1700$$
$$ZC = 5{\cdot}91 \times 10^{8}$$

Again, applying the expression

$$C = \frac{(E - e)}{R}$$

it is found that

$$C = \frac{100 - \dfrac{Z \times 204 \times 450 \times 10^{-8}}{60}}{{\cdot}05}$$

or $$C + {\cdot}0003065\,Z = 2000 \quad . \; . \; . \; . \; . \quad \text{(i.)}$$

and squaring,

$$(C + {\cdot}0003065\,Z)^2 = 4000000.$$

Then

$$(C + {\cdot}0003065\,Z)^2 - 4\,(0003065\,Z\,C) = 4000000 - {\cdot}001227\,(5{\cdot}91 \times 10^{8})$$

or
$$(C - \cdot 0003065\,Z)^2 = 3276616$$
$$C - \cdot 0003065\,Z = -1810 \quad \ldots\ldots \quad \text{(ii.)}$$

and from (i.) and (ii.)
$$C = 95,$$

the other value, 1905, being impracticable, and consequently
$$Z = \frac{5{\cdot}91 \times 10^8}{95} = 6{\cdot}22 \times 10^6$$

(*a*) Neglecting armature reactions, the flux is unaffected by the magnitude of the armature current, and under these circumstances the torque is simply proportional to that current. The values are thus :—

Current in amperes.	Flux in C.G.S. lines.	Torque in inch-pounds.
0	$6{\cdot}22 \times 10^6$	0
10	$6{\cdot}22 \times 10^6$	179
20	$6{\cdot}22 \times 10^6$	358
30	$6{\cdot}22 \times 10^6$	538
40	$6{\cdot}22 \times 10^6$	716
50	$6{\cdot}22 \times 10^6$	895
60	$6{\cdot}22 \times 10^6$	1,065
70	$6{\cdot}22 \times 10^6$	1,250
80	$6{\cdot}22 \times 10^6$	1,430
90	$6{\cdot}22 \times 10^6$	1,610
95	$6{\cdot}22 \times 10^6$	1,700
100	$6{\cdot}22 \times 10^6$	1,790
110	$6{\cdot}22 \times 10^6$	1,970

The relation between speed and current is given by
$$N = \frac{(100 - \cdot 05\,C)\,10^8}{3{\cdot}4 \times 6{\cdot}22 \times 10^6}$$

and thus every 10 amperes increase of current is accompanied by a drop of 2·36 revolutions per minute in the speed.

Since the speed with 95 amperes is 450 revolutions per minute, the last table supplies the data for the following :—

Torque.	Revolutions per minute.
0	0
179	470
358	468
538	465
716	463
895	460
1,065	458
1,250	456
1,430	453
1,610	451
1,700	450
1,790	449
1,970	447

in which the speeds are given to the nearest whole number

(*b*) When the back ampere turns of the armature are taken into account, the speed is found to fall to a less extent as the torque increases; while, if of sufficient power, they may produce uniform or even rising speed for increase in torque. Thus, assuming the back armature turns to be 9 per cent of the exciting turns, when carrying the full 95 amperes, the flux may be regarded as the resultant of $6{\cdot}84 \times 10^6$ lines due to the shunt coils, and $\cdot 62 \times 10^6$ negative lines due to the back turns.* When any other current flows in the armature the total flux will then be given by

$$\left(6{\cdot}84 - \frac{\text{current}}{95} \cdot 62\right) 10^6.$$

---

* This assumption is made here to bring out prominently the action of the back turns; it is really only justifiable when the induction lies on the lower nearly straight portion of the magnetisation curve.

The values of current, flux, and torque are—

Current in amperes.	Flux in C.G.S. lines.	Torque in inch-pounds.
0	$(6{\cdot}84 - 0) \times 10^6 = 6{\cdot}84 \times 10^6$	0
10	$(6{\cdot}84 - {\cdot}06) \times 10^6 = 6{\cdot}78 \times 10^6$	226
20	$(6{\cdot}84 - {\cdot}13) \times 10^6 = 6{\cdot}71 \times 10^6$	385
30	$(6{\cdot}84 - {\cdot}20) \times 10^6 = 6{\cdot}65 \times 10^6$	573
40	$(6{\cdot}84 - {\cdot}26) \times 10^6 = 6{\cdot}58 \times 10^6$	790
50	$(6{\cdot}84 - {\cdot}32) \times 10^6 = 6{\cdot}52 \times 10^6$	1,090
60	$(6{\cdot}84 - {\cdot}39) \times 10^6 = 6{\cdot}45 \times 10^6$	1,115
70	$(6{\cdot}84 - {\cdot}46) \times 10^6 = 6{\cdot}38 \times 10^6$	1,285
80	$(6{\cdot}84 - {\cdot}52) \times 10^6 = 6{\cdot}32 \times 10^6$	1,455
90	$(6{\cdot}84 - {\cdot}59) \times 10^6 = 6{\cdot}25 \times 10^6$	1,630
95	$(6{\cdot}84 - {\cdot}62) \times 10^6 = 6{\cdot}22 \times 10^6$	1,700
100	$(6{\cdot}84 - {\cdot}65) \times 10^6 = 6{\cdot}19 \times 10^6$	1,780
110	$(6{\cdot}84 - {\cdot}72) \times 10^6 = 6{\cdot}12 \times 10^6$	1,940

and determining speeds and currents, as in Case I., but substituting ·05 in place of ·1 for the resistance—

Current.	Revolutions per minute.
0	0
10	432
20	434
30	436
40	438
50	440
60	442
70	444
80	446
90	449
95	450
100	451
110	453

the speeds being in whole numbers.

Hence the values of torque and speed are—

Torque.	Speed.
0	0
226	432
385	434
578	436
790	438
1,090	440
1,115	442
1,285	444
1,455	446
1,630	449
1,700	450
1,780	451
1 940	453

and it will be seen that speed and torque, with this number of back turns, increase together. With an appropriate number of back turns it is evident the speed would be constant. In practice, small and smooth-wound shunt motors usually slow down as the torque increases; on the other hand, large machines, and those of moderate size with slotted cores, commonly speed up.

Case III.—The magnets excited by shunt coils, the effect of which is reduced by a number of oppositely-wound series turns. This arrangement is virtually the same as that of a shunt motor with back ampere turns, and the calculations and results are similar. Though back turns serve for regulation, they are not a desirable feature of a machine, and it is obviously preferable to apply the demagnetising coils to the magnets.

The speed variations, with alterations in the load, can now be compared for the three cases.

Torque.	Speed.		
	Case I.	Case II. (*a*).	Case III. and Case II. (*b*).
400	735	468	435
600	613	465	437
800	552	462	440
1,000	515	459	442
1,200	487	456	445
1,700	450	450	450
2,000	442	447	454

The extent of the speed variation in the first and third cases will depend upon the saturation of the magnetic circuit at normal load; in a differentially-wound compound machine the effect of the series turns would be obviously but slight if the shunt produced a high saturation. A fuller and more exact discussion of this and the following points may, however, be found in a later chapter dealing with graphical methods and characteristic curves.

As regards sensitiveness to variations in the supply pressure, it is evident that in this also a shunt motor is superior to one that is series wound. For since in a series motor the product of flux and current must at constant load remain the same, and they both increase or diminish together, the action of the motor must keep the current constant. Thus the speed is proportional to $E - CR$; or, since the resistance is small, it may be considered as having the same percentage variation as the supply pressure.

In a shunt motor the exciting current increases exactly in proportion to the pressure, and the effect on the flux depends upon the degree of saturation of the iron, but is in any case a strengthening. Hence, with a constant torque the speed increase is less than if the flux were unaltered, as with a series winding.

Rheostats for Starting and Regulating Motors.—It has been shown that when a motor is at rest, and consequently the back electro-motive force is nil, the current flowing through the armature conductors would be equal to the electro-motive force of the supply leads divided by the armature resistance plus that of any series winding. As an economically built machine will naturally, to keep its dimensions within reasonable limits, be designed for carrying a current only moderately greater than its normal working value, such a current as would flow if the full supply voltage were suddenly switched on to the terminals would burn it out. Consequently, except in the case of motors so small that the moment of inertia of their armatures allows them to speed up before the excessive current has had sufficient time to do any harm, a resistance must be inserted in circuit with a motor armature before switching on the current.

As speed rises and the back electro-motive force increases, the resistance inserted may be reduced until it is all cut out and the motor runs directly from the supply leads. With shunt-wound and compound motors it is necessary, as a first step, to close the field circuit, as otherwise there would be no accelerating torque beyond that produced by the residual magnetism. The starting gear, indeed, should always be so constructed that it is impossible to cut out the resistance from the circuit without first closing the field, as, unless this is so, there is always danger of burning out the armature from accident or negligence.

The resistances employed for this purpose may be either wire-wound or of liquid, but are preferably and more commonly of the former type, the chief advantages of the electrolyte resistance being its continuity of variation and its smaller initial expense. The general character and method of coupling of an ordinary wire-starting resistance is as shown in fig. 167. Constructionally, such a resistance in its simplest form consists of two parts—the switch panel and the resistance coils. The former carries a multiple contact switch, to the arm of which one of the supply leads is connected; the first contact closes either the shunt circuit alone, or simultaneously closes the shunt and armature circuits. The last contact is connected directly to the terminal of the motor, and when this is reached all resistance is out.

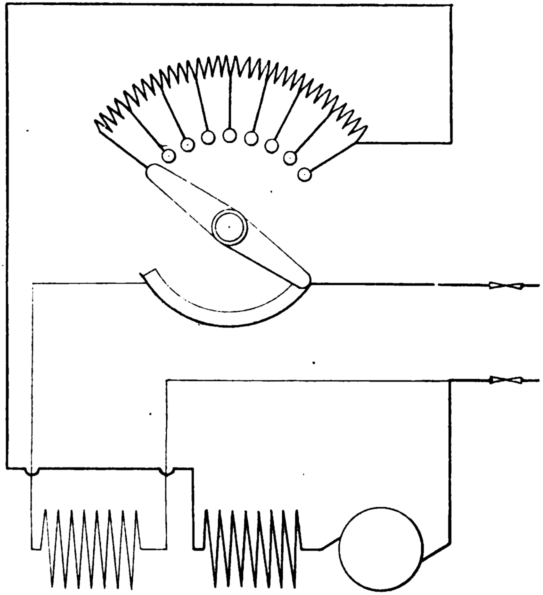


Fig. 167.

To guard against the possibility of the supply current falling, or being altogether cut off temporarily, and again coming on after the armature has slowed down or stopped, many starting resistances are furnished with a no-current release. This consists of an electro-magnet beside the last contact, which is excited by the current, and holds the switch arm over so long as a current of sufficient strength passes. Should this become too weak, the arm is released and pulled over into the off position by a spring. The arrangement will be understood from fig. 168. A further improvement is the addition of a maximum-current release, which, acting in a similar way, releases the arm if the current becomes excessive. The switch shown in fig. 168 is provided with this also. Where the motor current is large, sparking is apt to occur at the switch contacts, especially at that contact which closes the shunt winding, by reason of the latter's large self-inductance; and to extinguish the arc produced on this account a magnetic blow-out is sometimes fitted to the first contact.

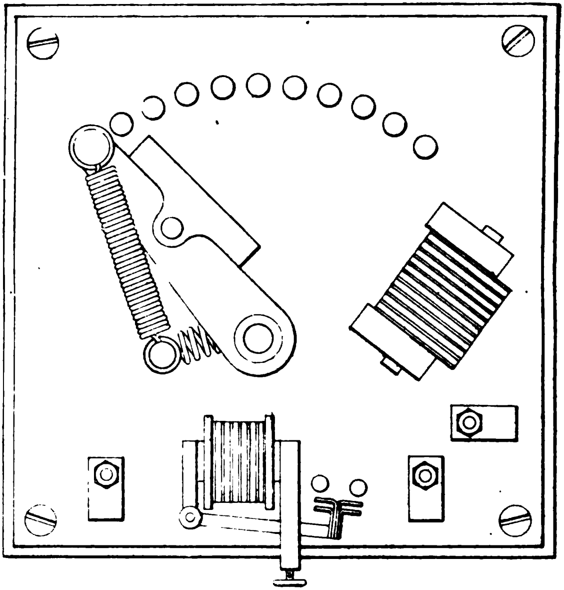


Fig. 168.

The resistance coils themselves are usually of German silver or another of the many high-resistance alloys, and may be simply loose spirals stretched across a frame, or they may be wound upon tubes of asbestos or some other non-conductor. In cases where the resistance is subject to vibration, the loose coils are liable to short-circuiting, and are less satisfactory than the last.

To minimise the space required by large resistances the coils are sometimes wound on asbestos tubes, which are afterwards pressed flat, fig. 169, and packed closely together, with strips of asbestos between them. This type of resistance has the advantage of requiring no combustible material in its construction, of having a large radiating surface for its weight and size, and of precluding all possibility of short-circuit.

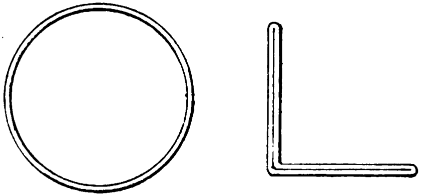


FIG. 169.

In another type of resistance the wire is embedded in a cement covering one side of a metal plate, the other side of which is ribbed, so as to form a large radiating surface. Rheostats of considerable capacity, for carrying the heavy currents of large motors, are not uncommonly furnished with resistances in the form of cast-iron grids carried on non-conducting supports.

The connections of a rheostat supplied with the Thomson-Houston pattern of no-voltage and overload releases, and with a magnetic blow-out at the first step, are as indicated in fig. 170, where the switch arm is shown in the off position. It will be seen that the spiral spring, having one end pinned to the panel, is extended as the arm is put over, and is kept in tension so long as the iron armature on the bar is attracted by the release magnet, but that as soon as the latter loses grip it pulls the arm round back to the off position. The overload release acts indirectly through this arrangement by opening the no-voltage magnet's circuit. The connections of a resistance for a larger machine may be as shown in fig. 171. Here a separate switch furnished with sparking contacts and a short-circuiting resistance closes the field.

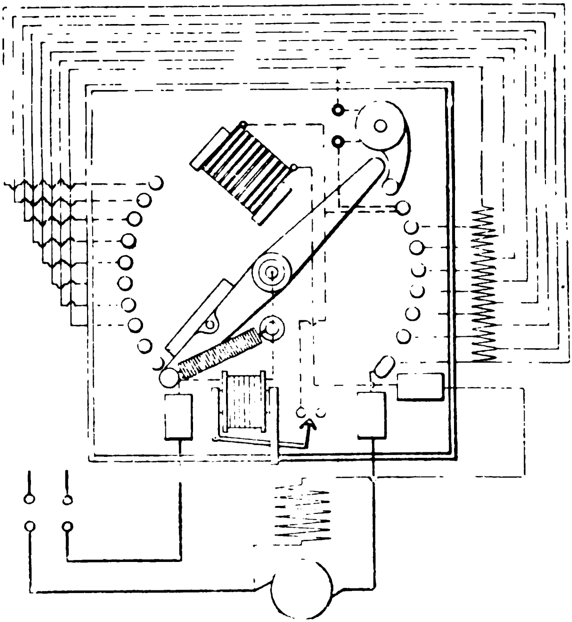


FIG. 170.

In designing a starting resistance the number of contacts will be determined by the total normal current of the machine, and by the maximum permissible rush of current from the supply leads as each section of the resistance is cut out. In many cases, to avoid too great disturbance to lamps fed from the same leads, this maximum is fixed, apart from any consideration of the motor's welfare. The operation of starting a motor with such a rheostat falls into two stages: the first few contacts close the field, and allow permanently increased currents to flow through the armature until the driving torque exceeds that of the load and acceleration begins; the subsequent contacts merely give temporary excesses of current, which are brought back to the normal full-load value as acceleration proceeds by the increasing back electro-motive force.

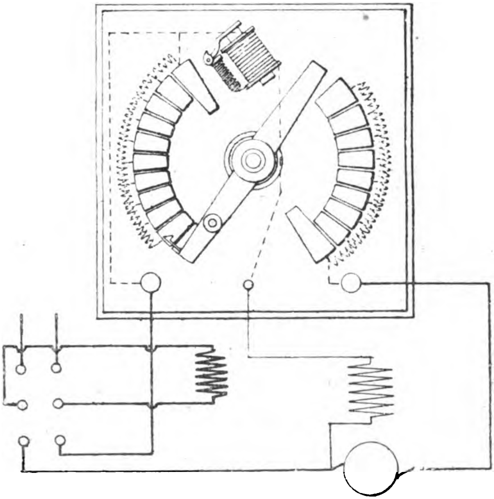


FIG. 171.

In designing, then, a starting resistance for a motor, of which the armature resistance is R_a, and of which the full load current is C, in a case where the line voltage is E and the maximum sudden rush of current is c, the resistances in circuit, including those in the motor itself, at the first few contacts will be determined by the necessity for the differences in the values of the ratio $\frac{E}{\text{resistance}}$ at successive contacts being less than c.

Starting onward from the contact at which rotation commences, and at which, starting up under constant load, the current is consequently C, let the resistances in circuit for this and the further contacts be denoted by R_1, R_2, R_3, etc. $\cdots R_a$. Then, at the following contact, the current rises momentarily to $C\frac{R_1}{R_2}$, falling gradually back to C; at the next it similarly rises to $C\frac{R_2}{R_3}$, etc. Thus, since the maximum current is $C + c$,

$$\frac{C+c}{C} = \frac{R_1}{R_2} = \frac{R_2}{R_3} = \frac{R_3}{R_4} = \text{etc.} = \frac{R_{n-1}}{R_a}$$

and it is evident that the resistances left in at successive contacts are in a geometrical progression.

Consequently,

$$\frac{R_1}{R_a} = \left(\frac{C+c}{C}\right)^{n-1}$$

where n denotes the number of contacts passed over beyond that one at which the resistance in circuit is equal to $\frac{E}{C}$.

Thus, since $R_1 = \frac{E}{C}$, the number of contacts necessary may be determined; for

$$n = \frac{\log \frac{E}{C} - \log R_a}{\log \frac{C+c}{C}} + 1.$$

In practice the contacts are usually spaced equi-distantly round the arc, and in these circumstances, for uniform starting, it may be seen that with a constant field strength the time interval in passing over contacts should increase in a geometrical progression; with a series-wound field the times should increase in a lower ratio, or be tolerably equal.

As an example of the calculations, let a starting rheostat be required for a shunt motor having a full-load current of 30 amperes from leads at 500 volts, the armature resistance being ·5 ohm, and the maximum allowable current rush 10 amperes. Then

$$\begin{aligned} C &= 30 \\ c &= 10 \\ R &= \cdot 5 \\ E &= 500 \end{aligned}$$

The shunt current for such a motor would be less than 10 amperes, and so can be put on entirely at the first contact. The first armature contact must have in circuit a resistance of $\frac{500}{10} = 50$ ohms. The full rheostat resistance must thus be $50 - \cdot 5 = 45\cdot 5$ ohms. It will be sufficiently accurate, however, to make the resistance 50 ohms. At the second armature contact the current becomes 20 amperes, and the resistance in circuit must be 50 ohms; while at the next, which is the first of the geometrical series, the resistance must be $\frac{500}{30} = 16\cdot 7$ ohms. The number of contacts in this series, including the first, is given by

$$n = \frac{\log 16\cdot 7 - \log \cdot 5}{\log 40 - \log 30} + 1 = 13\cdot 4$$

Since a fraction is, of course, impossible, the next higher integer, 14, is necessary.

The common ratio f of the successive resistances R_1, R_2, etc., is now given by

$$\cdot 5 = 16\cdot 7 f^{14}$$

whence

$$\log f = \frac{\log \cdot 5 - \log 16\cdot 7}{14}$$

$$f = \cdot 778$$

and the resistances in at the various later contacts are :

16·7 × ·778 = 13 ohms
13 × ·778 = 10·1 „
10·1 × ·778 = 7·8 „
7·8 × ·778 = 6·1 „
6·1 × ·778 = 4·73 „
4·73 × ·778 = 3·67 „
3·67 × ·778 = 2·85 „
2·85 × ·778 = 2·22 „
2·22 × ·778 = 1·73 „
1·73 × ·778 = 1·34 „
1·34 × ·778 = 1·04 „
1·04 × ·778 = ·81 „
·81 × ·778 = ·63 „
Armature resistance = ·5 „

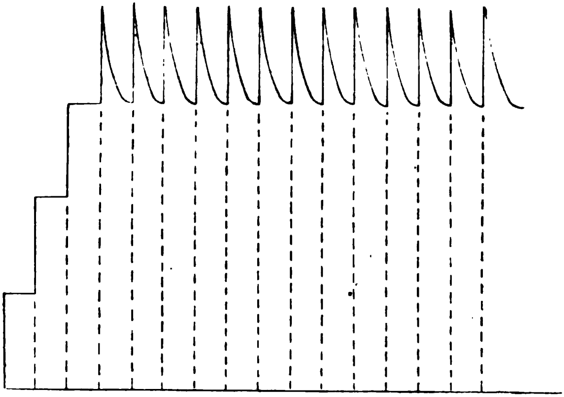


FIG. 172.

Unless the switch has been passed over the contacts too quickly, a definite speed of rotation corresponds to and is attained at each. These speeds, if the flux is regarded as constant, are proportional to the back electro-motive forces. But the back electro-motive force *e*, when a resistance R is in circuit, is obtained from the expression

$$C = \frac{E - e}{R}$$

$$e = E - CR.$$

If the full speed of the motor be 600 revolutions per minute, the back electro-motive force is then

$$500 - 30 \times \cdot 5 = 485,$$

and the speed N′ at any other value e' is given by

$$N' = \frac{e'}{485} \times 600.$$

Tabulating the various quantities :—

Contact.	Resistance.	Armature current.	Back electro-motive force.	Speed.
1	..	shunt only	..	..
2	50	10	0	0
3	25	20	0	0
4	16·7	30	0	0
5	10·1	30	197	244
6	7·8	30	266	330
7	6·1	30	317	393
8	4·73	30	358	443
9	3·67	30	390	482
10	2·85	30	414	511
11	2·22	30	433	535
12	1·73	30	448	558
13	1·34	30	460	569
14	1·04	30	469	580
15	·81	30	476	589
16	·63	30	481	595
17	·5	30	485	600

The values of the speed are calculated on the basis of no armature reaction.

The current curve during the starting up is as shown in fig. 172, from which it will be seen that the successive excesses of current during acceleration are brought down to the normal.

Liquid resistances commonly consist of a vessel of soda solution, into which are immersed cast-iron electrodes connected respectively with one pole of the supply mains and with one terminal of the motor. The current density may be about 8 or 10 amperes per square inch of surface on the electrode. As these are lowered into the liquid the resistance diminishes, until, when reduced as far as possible, it is cut out altogether. A resistance of this type permits of considerable adjustment, as it may be reduced or increased by addition of soda (common washing soda, Na_2CO_3) or water; but it is not easy to calculate the resistance in advance for a particular case, as the specific resistance of the electrolyte is not inversely as its strength, while its temperature coefficient is, moreover, considerable.

Starting resistances have also been made in which powdered graphite is employed in place of an electrolyte. One electrode being lowered into the powder the resistance becomes less the further it sinks in, until finally the moving electrode comes in contact with the other, and the graphite no longer carries the current between them.

It will be apparent from what has preceded that a resistance in series with the armature of a motor may be employed for the speed regulation when running, though inevitably an uneconomical device owing to the watts wasted by the relatively large current. The more satisfactory method of regulation is to put an adjustable resistance in the shunt circuit, and in this case an increase of resistance reduces the flux, so causing the speed to rise; while, the current being small, the loss in the resistance is unimportant. If, however, the magnets be well saturated, the resistance necessary is considerable.

While dealing with this subject mention may also be made of the shunt-regulating resistances employed for adjusting the electro-motive force of generators. These, as a rule, require a greater number of contacts than a starting resistance, and the resistance per contact is usually made uniform; but they are generally similar in other respects, except that the automatic releasing arrangements are unnecessary. As the resistance is inserted the magnetic flux is weakened, and the electro-motive force falls proportionally.

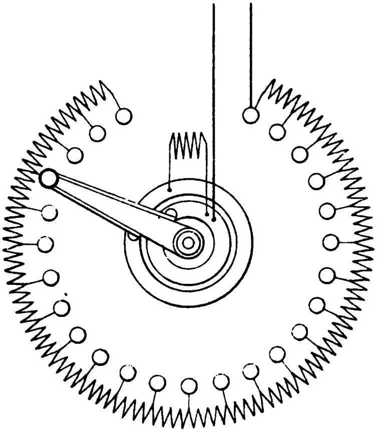


FIG. 173.

By the device of fitting a double contact piece on an insulating lever and allowing each piece to make contact with a metal ring at its inner end, the steps of a regulating switch may be made twice as numerous as the contacts. For a resistance equal to that between each contact may have its ends connected to the rings, fig. 173, and is thus thrown in parallel with each section of the resistance before the latter is cut out.

The Ward-Leonard System of Regulation.—In this method of control the field of the motor is excited directly from the supply leads and is maintained at its full strength, but the armature receives its current independently from an accessory generator. The latter, which may be engine or motor-driven, has its field excited to various extents by current from the supply leads, under the control of an adjustable resistance, the whole arrangement being as diagrammatically represented in fig. 174.

Upon starting up the motor its field is switched in, the generator is set rotating, and the generator field is feebly excited. The low generator voltage thus produced is sufficient to drive a moderate current through the slowly rotating motor, and as the latter speeds up, producing greater back electro-motive force, the pressure of the generator is increased by cutting resistance out of its field circuit and augmenting its flux. The loss of energy in controlling the speed of a motor by this method is evidently far smaller than would be the case with a regulating resistance in the armature circuit, as the current passing through the rheostat is so much less. What is really effected is that the advantages of a series motor's large starting

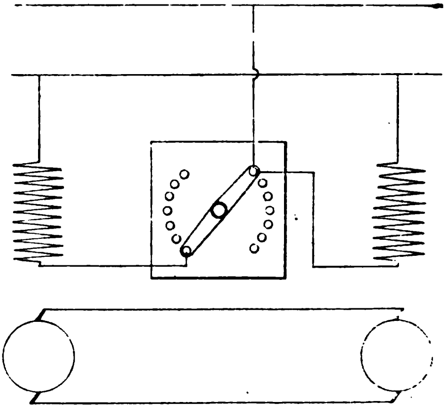


FIG 174.

torque and of a shunt motor's economical regulation are combined.

The price paid for this is a heavy plant and a considerably increased initial cost, but a certain amount is doubtless saved in wear and tear of the motor.

The Bullock "Teaser" System.—This is another arrangement, employed mainly in the States, the object being, as with other systems, to obtain a large starting torque without wasting an excessively large amount of energy in resistances.

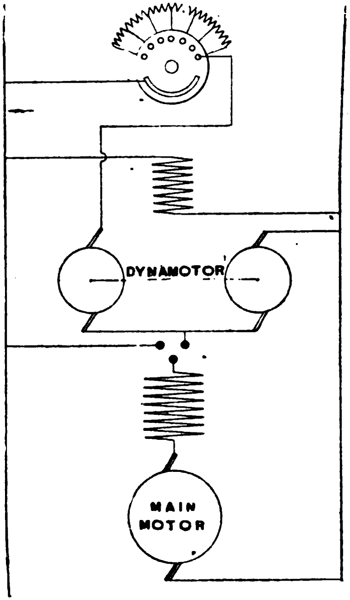


Fig. 175.

The characteristic feature of this device is a motor-generator (Chapter XIV.) having an armature wound with two closed windings. Of these the motor winding has the greater number of turns—suppose n times that of the other—and consequently produces an electro-motive force n times as great.

As shown diagrammatically in fig. 175, the positive brush of the motor side is connected through an adjustable resistance to the supply lead and its negative, with the positive brush of the generator side is, on starting, connected to the positive terminal of the main motor. The field of the accessory motor-generator is directly excited from the supply leads, while that of the main motor is commonly compound wound—not differentially. The electro-motive force across the main motor terminals is, when thus connected, something less than $\frac{1}{n}$th that between the supply leads, while the current is about n times that drawn from them by the motor side of the motor-generator. As the resistance in the circuit of the latter is cut out the speed of the large motor increases until it reaches a value at which the motor may be switched over directly on to the supply leads without drawing from them an undue rush of current. The various alterations in the resistances and connections in this, as in most other complicated starting arrangements, are effected by a controller, to which all the connections are brought. As the handle of such a controller is turned the various connections are consecutively made by contacts on its spindle.

Motors for Electric Traction.—Though the enormous development of electrical tramways during recent years has called into existence a variety of devices for conveyance of the current from the generators to the motors on the car, and a number of distinct systems of controlling speed and power, yet continuous currents have almost universally been adopted in conjunction with series wound motors. The line pressure is commonly between 400 and 500 volts, and the motors are of an enclosed four-pole type. In railway work a far greater divergence of both systems and details is exhibited, and no standard practice has at present been

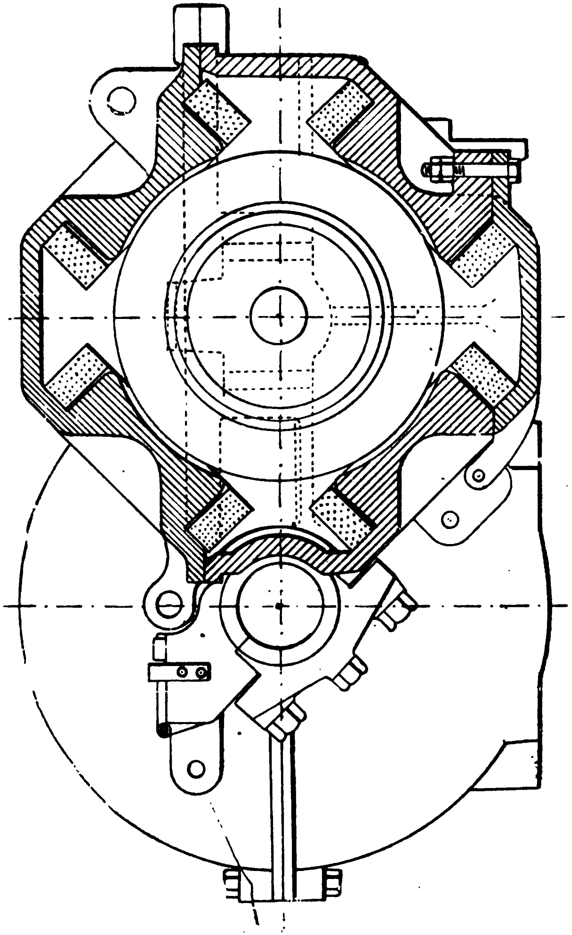


FIG. 176.

evolved. Tracks of considerable length afford scope for the particular advantages of three-phase working, such as have in various cases been adopted, more particularly in Transatlantic and Continental instances; but numerous continuous current schemes are in operation, or projected. In these cases the motors and their gear are often assembled together on a separate locomotive, the coaches being hauled, and the design of the motors themselves has, as yet, taken on no standard form.

The general character of a modern tramway motor may be gathered from fig. 176, which represents a section of the Oerlikon Company's case and field, a characteristic feature of this particular example being the oblique bearings gripping the car wheel axle. The four short poles are each embraced by an exciting coil, and arranged diagonally to permit of the upper and lower halves of the case being separated to allow access to the armature. Small doors are also commonly provided giving access to the brushes which are usually carried from the inner surface of the casing. The poles, which are in this instance cast solid with the case, are often laminated, and in all circumstances it is necessary to secure the field coils by specially strong fastenings on account of the shocks and rough usage to which these motors are liable.

Fig. 177 shows the general construction of a Thomson-Houston motor of somewhat greater power than usual, differing from the previous example in carrying its axle bearings vertical and in having a simpler arrangement for obtaining access to the brush-gear. The G. E. 67 motor of this type is capable of exerting 40 horse power on a 500 volt circuit, with a maximum rise of temperature of 75 deg. Cen. The entire lower half of the casing is hinged, and swings downward when unfastened. The bearings can be lubricated both by oil and grease, and consist of cast-iron shells lined with white metal; those on the car axle being 8 in. long, those on the armature spindle being 3 in. by 8 in., and $2\frac{5}{8}$ in. by $6\frac{1}{2}$ in. respectively. The pole pieces are in this case not cast with the casing, but are built up of laminations and bolted to it. There are 111 coils on the armature with one commutator segment per coil. The commutator itself is $9\frac{1}{8}$ in. in diameter, and has a wearing surface $3\frac{1}{8}$ in. long. The distance from the centre of shaft to the bottom of the casing is 12 in., and to the bottom of the gear case $12\frac{1}{2}$ in.

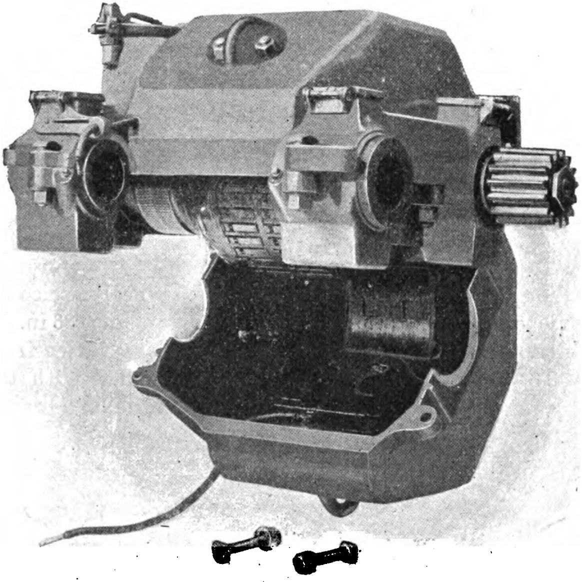


FIG. 177.

In some instances the casing of a motor differs from both the preceding types in having the axle bearing inside instead of projecting, in which cases the axle is, of course, enclosed from side to side.

The armature has usually twice as many coils as there are slots, and the number of commutator segments may be equal to, or double the number of, coils. The winding is in two circuits—*i.e.*, it is a four-pole series winding, and but two sets of brushes are necessary. These are necessarily of carbon, since the motor is reversible.

The ordinary tramway motor drives through a single reduction gear, a pinion at the end of its spindle farthest from the commutator driving on to a gear wheel carried on the axle of the wheels. The latter are usually 33 in. in diameter, and the velocity ratio of the gearing varies from about 2·8 to 5. The casing which is embodied with the field magnet is of high permeability cast steel, and carries two bearings by which one of its ends is supported from the axle, so keeping the pinion in engagement with the gear wheel, any relative motion of the motor case and car truck merely causing the pinion to run backwards or forwards round the gear wheel.

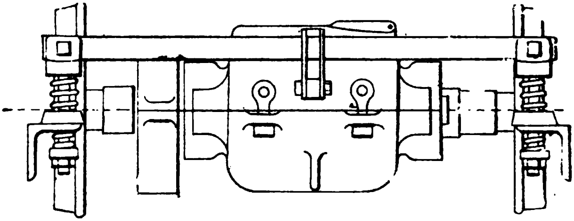


Fig. 178.

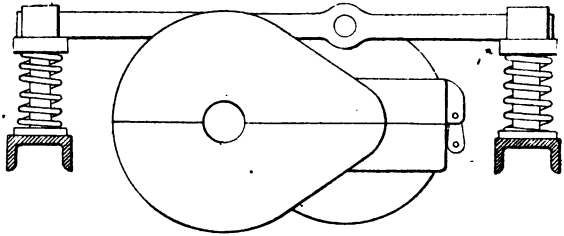


Fig. 179.

The motor may be suspended from the truck in either of four ways, viz., the nose, yoke, side bar, and cradle methods. By the first of these methods the motor is carried from a transverse portion of the framing of the truck by a link attached to a projection—the nose—at one end of its case, fig. 178, the cross transverse bar itself being supported by springs. From this, the yoke suspension differs only in being more rigid, tbe intermediate link between the nose and bar being dispensed with. With either of these suspensions a portion of the motor's weight is carried by the bearings on its case which encircle the axle. This is not so with the side bar suspension in which the motor is carried by bars running lengthwise at each side, fig. 179, the points of attachment being approximately in the same vertical plane as the centre of gravity of the motor. The carrying bars themselves are carried on springs at each end. The cradle differs from this only in the side bars forming a

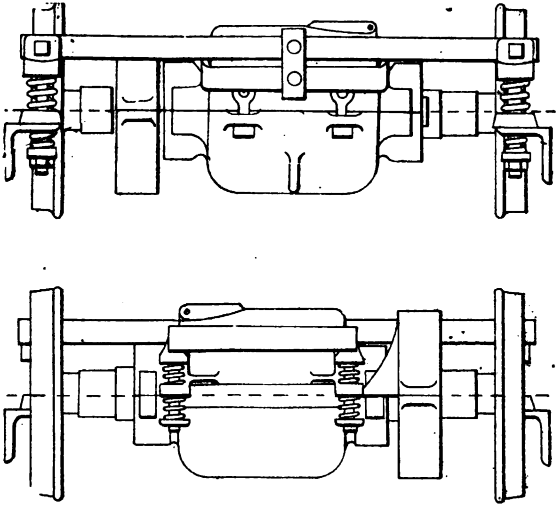


Fig. 180.

single U shaped bar carried at the bend by a transverse piece supported on springs. The ends of the U which are here shown, fig. 180, free are often connected by a bar and are generally carried by springs on the axle bearings.

Current and Speed.—To ascertain the relation between the speed of a car and the current absorbed by the motor on various gradients, let—

E = the line voltage.
C = current.
R = resistance in motor circuit.
N = revolutions of motor per minute.
T = number of conductors on motor armature.
Z = total magnetic flux per pole through armature.
p = number of pairs of poles in motor field.
n = number of pairs of parallel circuits in motor armature.
M = speed of car in miles per hour.
D = diameter of car wheels in feet.
v = velocity ratio of gearing.
W = total weight of car and load in tons.
a = tractive force in tons necessary per ton weight on level.
β = gradient of track.

Then the motor speed is given by

$$N = (E - C R) \frac{60 \times 10^8 \times n}{Z T p}$$

and

$$M = \frac{60 \pi D N}{5280 v} = \frac{60 \pi D}{5280 v} (E - C R) \frac{60 \times 10^8 \times n}{Z T p}.$$

The horse power required by the car on any gradient must now be obtained. The foot-pounds per minute spent on track resistance are

$$\frac{5280 M}{60} 2240 W a,$$

and those expended in raising the car are,

$$\frac{5280\,\mathrm{M}}{60}\,2240\,\mathrm{W}\,\beta,$$

the total horse power is thus

$$\frac{5280 \times 2240}{33000 \times 60}\,\mathrm{M\,W}\,(\alpha + \beta),$$

or

$$5{\cdot}97\,\mathrm{M\,W}\,(\alpha + \beta),$$

and if the efficiency of the motor and gear be η, the watts absorbed from the line must be

$$\frac{746}{\eta} \times 5{\cdot}97\,\mathrm{M\,W}\,(\alpha + \beta)$$

or,

$$\frac{4454\,\mathrm{M\,W}}{\eta}\,(\alpha + \beta).$$

The current per motor, assuming the car to be fitted with two, as is commonly the case, is then

$$\mathrm{C} = \frac{2227\,\mathrm{M\,W}}{\mathrm{E}\,\eta}\,(\alpha + \beta).$$

The relation between the speed and current of any given motor may be determined by calculation or experiment, and a curve, fig. 181, set out showing corresponding values of M and C. For any particular gradient β the current must satisfy the last equation and also lie upon the curve. A simple graphical method, first described by M. Fisher-Hinnen, may be applied to the solution. Let any value of M be selected and the corresponding value of C calculated, the point in the diagram, fig. 181, representing these values of C and M being o'. A line through the origin and o' then cuts the curve in a point which gives the actual values of M and the current in each motor.

If the variation in the efficiency η at different loads be taken into account it is necessary to proceed in a similar way for other speeds and gradients, but when approximate information only is required—one in which the variation of

η may be neglected—the actual values of the speed and current may be found in a simpler manner. Considering M as constant at the value previously selected, let the value of C be calculated for two values of β, and let $\beta_2 - \beta_1$ denote the difference between the two gradients, $C_2 - C_1$ that between the two currents. Then an increase of amount $\beta_2 - \beta_1$ in gradient will be accompanied by an increase $C_2 - C_1$ in current, whatever the actual magnitudes of β_2 β_1, C_2, and C_1 may be. If, therefore, a line be drawn through the original point o' parallel to $o\,x$, and, starting from o', points at the distance $C_2 - C_1$ apart are set off along it, the intersections with the curve of straight lines drawn through these points and the origin will give the values of speed and current corresponding to the various gradients β, β_1, β_2, β_3, &c. The current required to accelerate the car is, of course, considerably greater. If the car be on a gradient β the resistance to motion acting at the periphery of the driving wheels is in all

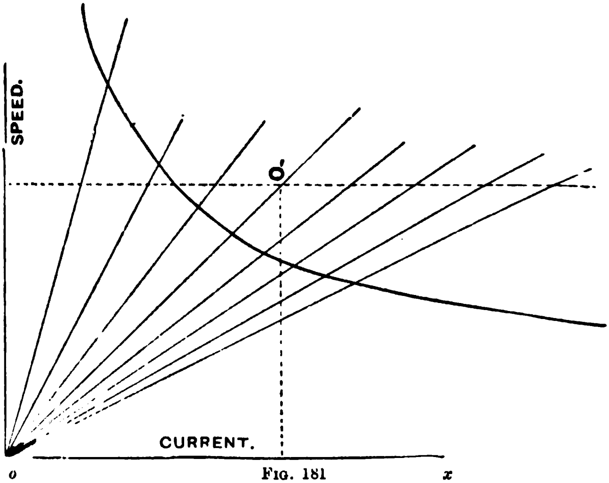


Fig. 181

$$2240\ W\ (a + \beta)$$

and the resistance torque at the wheel axle is for each of the two motors

$$\frac{1}{2}\left(\frac{12\,\mathrm{D}}{2} \times 2240\,\mathrm{W}\,(a + \beta)\right)$$

Let car speed now be taken in feet per second as m. Then $\frac{d\,m}{d\,t}$ is the acceleration in feet per second per second, and this multiplied by the mass of the car equals the accelerating force, which is thus

$$\frac{d\,m}{d\,t} \times \frac{2240\,\mathrm{W}}{32}\ \text{lbs.}$$

Since this force must be applied at the circumference of the driving wheels, the accelerating torque must be

$$\frac{d\,m}{d\,t} \times \frac{2240\,\mathrm{W}}{32} \times \frac{12\,\mathrm{D}}{2}\ \text{inch-pounds,}$$

and the accelerating torque on each motor spindle becomes

$$\frac{1}{2\,v\,\eta}\left(\frac{d\,m}{d\,t} \times \frac{2240\,\mathrm{W}}{32} \times \frac{12\,\mathrm{D}}{2}\right)\ \text{inch-pounds,}$$

the total torque per motor being

$$\frac{1}{2\,v\,\eta}\left\{\frac{12\,\mathrm{D}}{2}\,2240\,\mathrm{W}\left(\frac{1}{32}\,\frac{d\,m}{d\,t} + (a + \beta)\right)\right\}$$

But the torque of a motor is, as previously shown, equal to

$$1{\cdot}41\,\mathrm{Z\,T\,C}\,\frac{p}{n}\,10^{-8}$$

Hence, the current required for each motor when starting with a car acceleration $\frac{d\,m}{d\,t}$, is,

$$\mathrm{C} = \frac{10^8\,n}{2{\cdot}82\,\mathrm{Z\,T}\,v\,\eta\,p}\left\{13440\,\mathrm{W\,D}\left(\frac{1}{32}\,\frac{d\,m}{d\,t} + (a + \beta)\right)\right\}$$

The total current per motor may be regarded as consisting of two parts, the one overcoming resistance to uniform motion, the other producing acceleration. Since the resistance torque will not vary greatly with alteration of speed, it may be regarded as constant, and the corresponding current as constant during the period of acceleration. The equation between time and velocity during the period may now be obtained, for

$$\frac{d\,m}{d\,t} = \mathrm{J}\,(\mathrm{C} - \mathrm{C}')$$

where

$$\mathrm{J} = \frac{32\,(2{\cdot}82\,\mathrm{Z\,T}\,v\,\eta\,p)\,10^{-8}}{13440\,\mathrm{W\,D}\,n}$$

and C′ equal the final steady current. But, denoting the quantity

$$\mathrm{Z\,T}\frac{p}{n}\;\frac{v}{\pi\,\mathrm{D}}\,10^{-8} \text{ by K,}$$

the back electromotive force is at any instant equal to mK, and hence the current at the time is

$$\frac{\mathrm{E} - \mathrm{K}m}{\mathrm{R}}$$

where E and R may represent either the line voltage and the resistance between the line and earth, or the voltage at the terminals of the motor and its own resistance.

Substituting this value of C

$$\frac{d\,m}{d\,t} = \mathrm{J}\left(\frac{\mathrm{E} - \mathrm{K}\,m}{\mathrm{R}} - \mathrm{C}'\right)$$

and

$$\frac{d\,m}{d\,t} + \frac{\mathrm{J\,K}}{\mathrm{R}}\,m - \mathrm{J}\left(\frac{\mathrm{E}}{\mathrm{R}} - \mathrm{C}'\right) = 0$$

The solution of this differential equation is

$$m = \frac{\mathrm{E} - \mathrm{R\,C}'}{\mathrm{K}} + \mathrm{A}\,e^{-\frac{\mathrm{J\,K}}{\mathrm{R}}t}$$

where A is a constant, and t is reckoned in seconds from the commencement of acceleration.

The constant A may now be determined. Let

$$\frac{\mathrm{E} - \mathrm{R\,C}'}{\mathrm{K}},$$

the final speed attained, be denoted by m_f, and the original speed when acceleration began by m_o. Then, considering the speed originally when $t = 0$,

$$m_o = m_f + A;$$

and evidently

$$A = -(m_f - m_o)$$

so that the solution of the equation may be written as

$$m = m_f - (m_f - m_o)\,e^{-\frac{JK}{R}t}$$

or when the car starts from rest, as

$$m = m_f\left(1 - e^{-\frac{JK}{R}t}\right)$$

Whence, differentiating, the acceleration may be written

$$\frac{dm}{dt} = m_f\left(\frac{JK}{R}\,e^{-\frac{JK}{R}t}\right)$$

The acceleration thus diminishes logarithmically owing to the increase of the back electromotive with speed. If, however, the resistance be reduced proportionally to the speed so as to maintain C constant, then the acceleration is also constant and equal to $J\,(C - C')$.

If while the car is running the motors be entirely disconnected from the line, and the armatures be short-circuited through a resistance, they will run as generators with the flux supplied by the residual magnetism. Though the electromotive force induced may not be great, the resistance is also small and the current consequently is considerable, so that an effective brake action is produced.

If m be the car speed in feet per second, that of the armature is $\frac{m\,v}{\pi D}$ revolutions per second, and the electromotive force generated is

$$Z\,T\,\frac{p}{n}\;\frac{m\,v}{\pi D} \times 10^{-8}$$

or with the previous signification of K, it is $m\,K$. If, now, the total resistance of the two armatures and the rheostat in series be R, a current $\frac{m\,K}{R}$ circulates, and the rate at which work is being done is $\frac{(m\,K)^2}{R}$ watts, or $\frac{(m\,K)^2}{746\,R}$ horse power. If the speed of the car be M miles per hour

$$m = \frac{5280}{3600}\,M$$

or its speed in feet per minute is

$$60\ m = \frac{5280}{60}\,M = 88\ M.$$

Hence on the gradient β it does work on the motors equal to

$$88\ W\,M\,(\beta - a)\,\eta \text{ foot-pounds per minute,}$$

or

$$\frac{88}{33000}\,W\,M\,(\beta - a)\,\eta \text{ horse power.}$$

Thus

$$\frac{88}{33000}\,W\,M\,(\beta - a)\,\eta = \frac{(m\,K)^2}{746\,R}$$

or, substituting for m,

$$M = \frac{\cdot 924\,W\,R\,(\beta - a)\,\eta}{K^2}.$$

With shunt motors it would be possible to carry this braking action farther, and to return into the line the current generated when running down inclines. Attractive as this economy of energy appears, it has not been found to be practicable, owing to the difficulties in the way of economical starting and regulation.

It is unusual and inadvisable to connect together the brushes of two motors between the armature and the field coils, for it will rarely happen that each motor generates exactly the same back electromotive force when running at the same speed, and the discrepancy in this respect increases with the speed. If the brushes of both are connected in the way mentioned, the currents in the field coils of each will always have the same proportion, being equal if their resistances are equal, and quite independent of the armature currents, and consequently the armature of the machine having the stronger field will do less work than the other, since its current is smaller. At a sufficiently high speed, indeed, it would generate current circulating through the brush connection and the two armatures. When there is no such connection, the greater back electromotive force of that motor which has the naturally stronger field reduces its current as before, but in this case the exciting current is

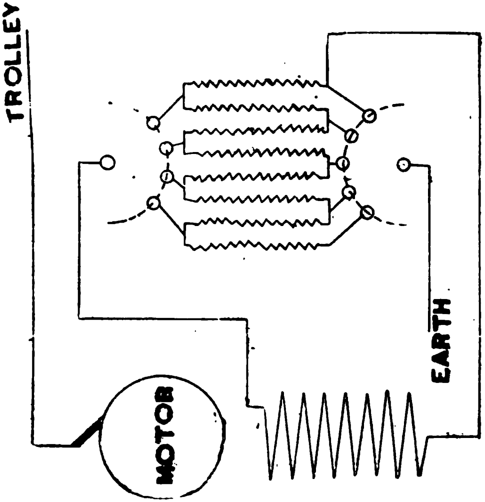


FIG. 182.

also reduced and the field weakened, the inequality to a great extent correcting itself automatically.

This remark applies only when the machines are running as motors; when braking, the connection is advantageous as equalising the field strengths.

Methods of Control.—The most obvious, and simplest method of regulating the speed of a tramway motor is to insert resistance in series with it. The employment of this means has the practical disadvantage of necessitating a very large resistance in circuit when the car is running down inclines not sufficiently steep for the motor to be altogether cut out. The best that can usually be done is to provide a resistance adequate for keeping the speed within due limits on the level, and to negotiate slight gradients by getting up a speed sufficient to carry the car down with the motors cut out.

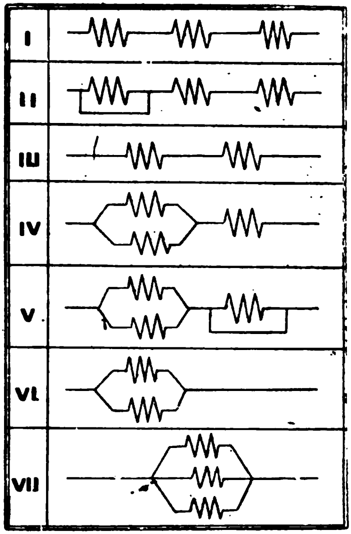


FIG. 183.

A method employed by the Oerlikon Company is to switch resistances in parallel with the field coils, so shunting a portion of the current, reducing the magnetic flux, and giving an increase of speed. The resistances used may, as will be seen from fig. 182, also serve as the starting rheostat.

In the Sprague system of control the field coils, of which there are two, are each divided into three parts which may be arranged into various series and parallel combinations by a controller. The successive groupings are as in fig. 183, the

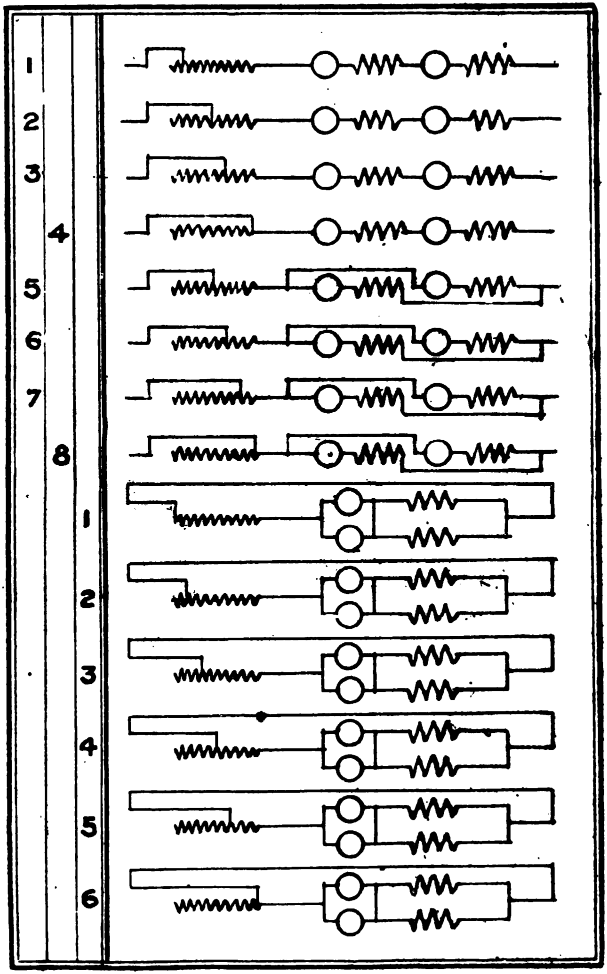


FIG. 184.

third and sixth not being stops, but merely transitional connections to prevent short-circuiting.

The system which is by far the most generally adopted is that of series-parallel control in one or other of its various forms. The characteristic feature of this method is the coupling of the two motors in series with resistance and one another at starting, and the final switching into parallel when speed is attained. The successive groupings in the Thomson-Houston and in the General Electric methods of control are as shown in figs. 184 and 185 respectively. In the latter it will be noted that at the higher speeds the magnet fields are shunted. The series-parallel controller, in which these successive couplings-up are effected by the rotation of a single handle to various points, consists essentially of a vertical cylinder bearing a number of contact sectors, or "wipes," at different positions along its length. These make contact with "fingers" carried on insulating supports attached to the inside of the casing, and connected to terminals to which the cables from the various points are brought. In addition to the large controlling cylinder a second smaller one is provided to effect the changes of connections necessary on reversing or braking. This reversing cylinder is interlocked with the controlling cylinder in such

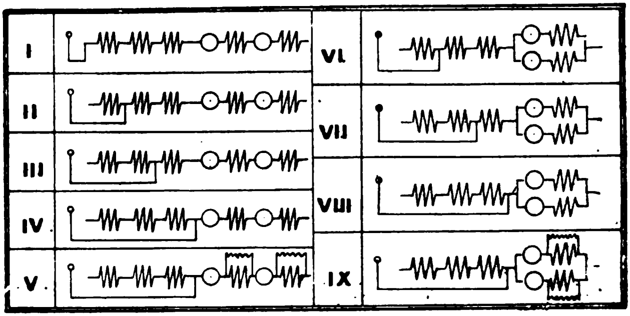


FIG. 185.

a way that the handle of the former can only be moved when connection with the trolley is opened by the controlling cylinder. Further, unless the reversing cylinder be set properly to one of its positions, for forward or backward running, it is impossible to turn the controlling handle at all; and, as a rule, the reversing handle is incapable of removal from its cylinder unless the latter is so set as to cut out both motors altogether.

In the Thomson-Houston and some other controllers an electro-magnet is fitted, and the magnetic circuit is so arranged that the arcing between the contacts is blown out.

The complete scheme of connections on a car fitted with two motors and a controller—one of Messrs. Witting Bros.—is shown in fig. 186, the developed controlling cylinder being on the left and the smaller reversing cylinder to the right. The dotted lines, which are numbered, indicate the successive contacts or "notches" which cut out the resistance, and throw the motors from series into parallel; the horizontal or stepped thin lines divide the wipes into groups, each group being insulated from the others, except when connected through the fingers, but all the wipes of any one group being directly connected together on the cylinder. The fingers are, with two exceptions, shown to the left of the cylinder, and, when the car starts up, may be regarded as moving to the right on to the vertical lines mentioned above; when braking they must be regarded as moving to the left from the right hand side. The two exceptions are, in the actual controller, not in the same vertical row as the others, but occupy a position at another point of the circumference of the cylinder. The upper of these is connected to the end of the regulating resistance, which is in five sections, of which all may be thrown in when the motors are running as brakes, but of which only four are employed in driving. The lower is in connection with the ends of the field coils, and couples the ends which when braking are positive, so short-circuiting the armatures. The reversing cylinder has two positions, for motoring or braking in either direction,

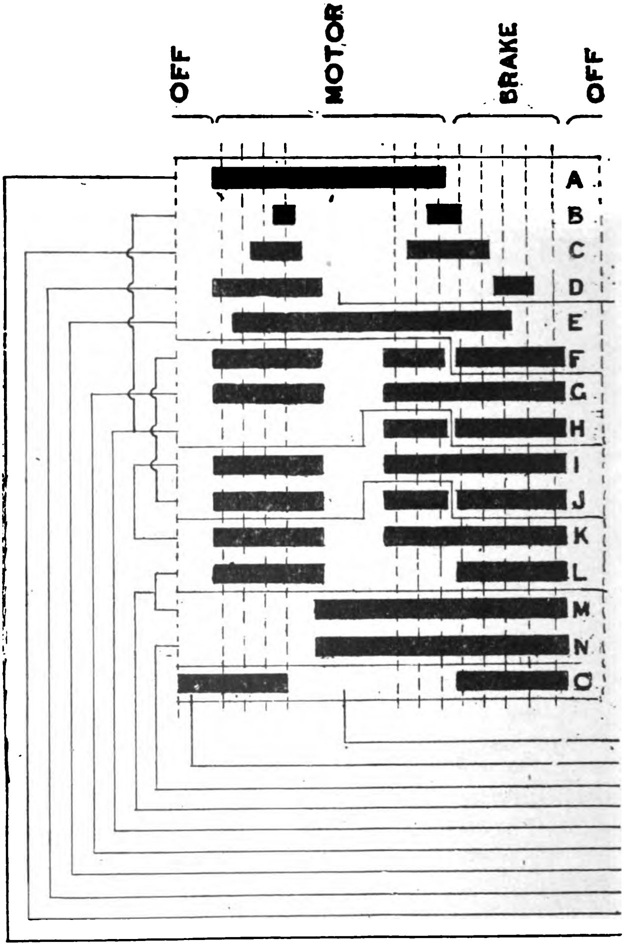


FIG. 186.

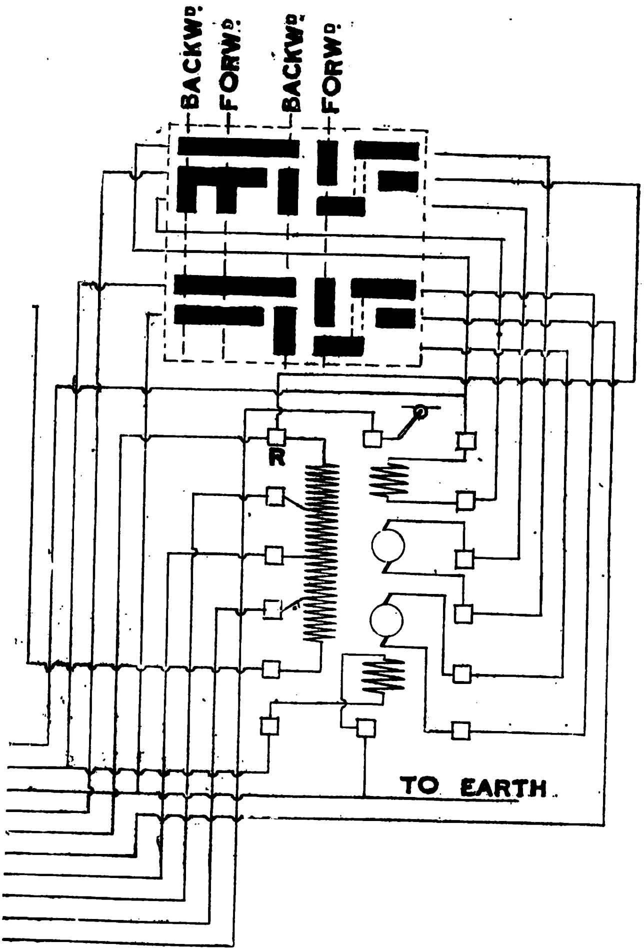
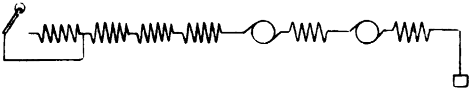


FIG. 186.

and the upper wipes of each set of contacts are connected, as shown by the broken lines, with one of the lower ones.

Upon starting under power the regulating cylinder passes from the "off" position, connecting the trolley to the uppermost wipe, whence the current passes through the finger of R_2 to the resistance. From the point R at the end of the latter it flows to the upper set of contacts on the reversing cylinder, and so to the armature of the first motor, and from this to the field by the contacts of the upper set to which also it returns and is then led away to the regulating set. The current entering at the row L now passes through the looped cable connections to the row G and so back through the reversing·contacts to the armature of the second motor. Again passing through the reversing contacts (the extreme ends of the developed diagram being continuous so that in reality there is a single horizontal wipe at the top) it passes through the field coils and so to earth. The coupling is thus—

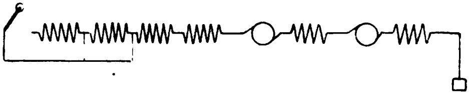
Position 1.



(a)

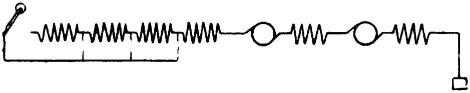
Positions 2, 3, and 4 are similar but with reduced resistance.

Position 2.



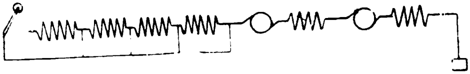
(b)

Position 3.



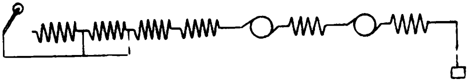
(c)

Position 4.

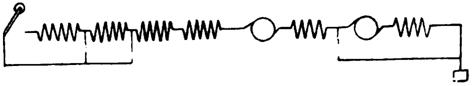


(d)

Two un-numbered and transitional sets of connections are made between positions 4 and 5, thus—



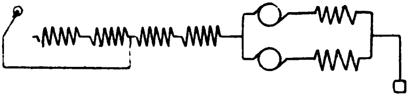
(e)



(f)

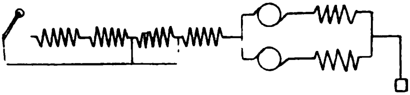
And the motors are then put in parallel at step 5. The current now flowing from the point R of the resistance divides, part passing as previously to the first motor, the other part passing to the regulating cylinder and through the rows H, I, J, F, G, to the reversing cylinder and the second motor. The following steps cut out the resistance until at full speed the motors run directly from the line in parallel.

Position 5.



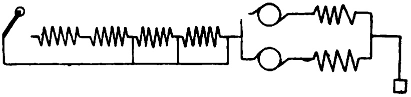
(g)

Position 6.



(h)

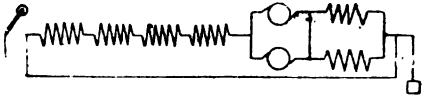
Position 7.



(i)

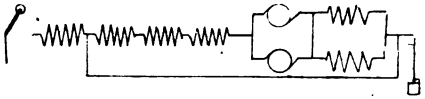
At the first step in the brake connection, the current starting from the earthed terminal passes in part to the magnets of the second motor, and in part through the rows N, M of the regulating cylinder to the magnets of the other. The inner ends of the fields are coupled by the bottom row of wipes, and the current passes to the reversing contacts and thence to the armatures, whence one portion is brought through the reversing switch directly to the point R_5 and the other part through the rows G, F, J, K, I, H. After flowing through the entire resistance it passes wholly through the rows D, F, J, I, K, L back to the starting point. This step, as before mentioned, is the only one in which the entire resistance and the upper intermediate finger are employed. The following steps cut out resistance :—

STEP 1.



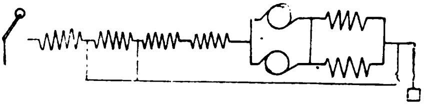
(j)

STEP 2



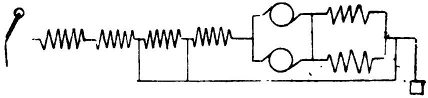
(k)

STEP 3.



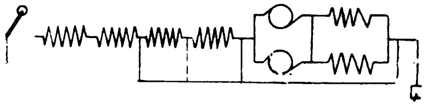
(l)

STEP 4.



(m)

STEP 5.



(n)

For reversing the brake and driving connections, the positions of the reversing cylinder are interchanged.

Figs. 187 and 188 show the general and internal construction of the Witting controller developed in fig. 186, the reversing cylinder and interlocking gear have, however, been modified and improved, the former being provided with three complete series of contacts. The first of these operates on both motors, the others on each motor singly, and thus it is only necessary in the event of one motor breaking down to turn the reversing handle round to a fresh position for it to

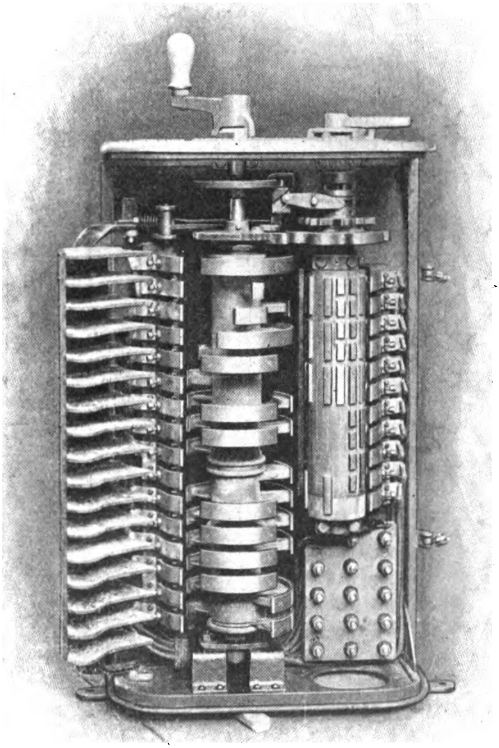


FIG. 187.

be entirely cut out. The driver has thus no need to open the controller under any circumstance.

On the left of fig. 187 are shown the fingers and the fibre plates which project inward between the wipes when swung round into position, on the right is the reversing cylinder and below it the terminal board.

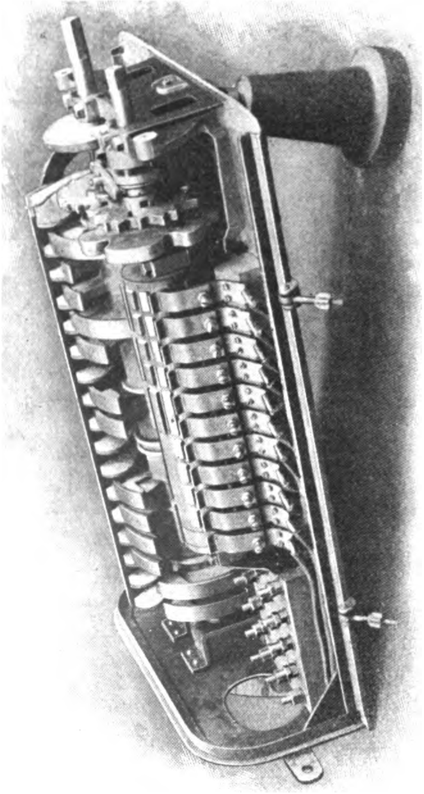


FIG. 188.

Electric travelling cranes are usually supplied with separate motors for raising, slewing, and travelling, the speed of the hauling motor most often being reduced by a worm gearing, though pinions, in many cases of leather, are sometimes used. The motor geared to the lifting drum generally runs for the sake of efficiency at a high speed, the velocity of the pitch surface of the worm on its shaft being about 20 ft. per second. The drum is fitted with a powerful brake which is automatically lifted when current is switched on to the motor. This is frequently effected by the pull of a solenoid and plunger, fig. 189, or by the torque on a solid iron armature between the poles of an electromagnet whose coils are in series with the motor armature. The brake itself is usually a band drawn down on the drum by a dead weight. It is the practice also to add an emergency brake worked by the driver's foot.

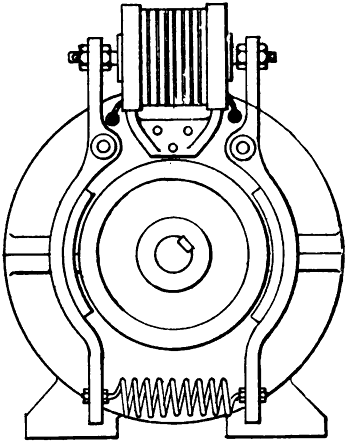


FIG. 189.

In small machines of the upright bipolar form, employed for lifts, hauling, &c., one of the magnet limbs is sometimes hinged at the bottom so that it may be driven outwards from the armature by a spring. A strap, having one end secured to the framing and the other to the magnet then acts as a brake, the pole-face being drawn up as near as possible to the armature by magnetic attraction when current is switched on, then slacks the brake. Fig 190 represents a brake of this description adapted for a motor running in either direction as employed by the Crocker-Wheeler Company.

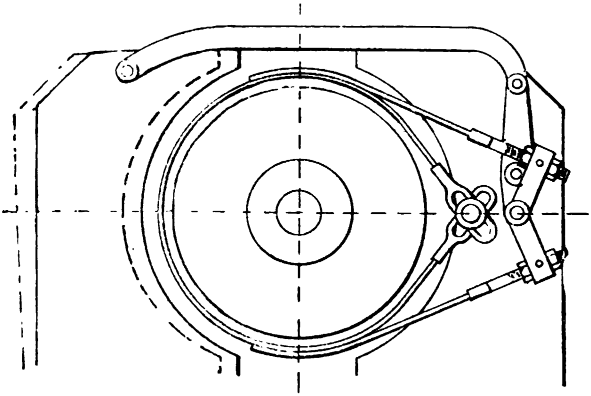


FIG. 190.

For the driving of machine tools motors may be applied in either of two ways, viz., coupled to shafting which drives off by pulleys in the ordinary way, or attached directly to individual tools. The former method is preferable where a number of small stationary tools are used, the latter for heavy tools absorbing a fair amount of power. A very useful tool to which the electric motor is applicable is a portable drill driven by a flexible spiral spring from the motor. There is little to be described in this use of motors, as the application, at present, consists in little more than bolting and gearing down the motor to an ordinary tool.

Low-pressure motors are employed in the propulsion of motor vehicles to some extent, the power being derived from accumulators. A point of importance in the design of such motors is high efficiency, increase of weight being relatively of small importance, as even the heaviest motor is light in comparison with the weight of the cells. Nevertheless, the least weight compatible with high efficiency should be aimed at.

In mining operations the main haulage and working of the pumps is often performed by large motors, while in coal cutting many advantages are derived from employing electric cutters. With regard to the motors working below the surface there is a divergence of opinion as to the desirability of enclosed or open machinery. On the one hand, should the mine be gassy, there is a probability that with an open motor any sparking might cause an explosion; on the other, it is questionable how far the ordinary enclosed motor is really gas-tight, while it certainly gets looked after less carefully than if it were open. Owing to the heavy loss caused by a breakdown, ring-wound armatures are often preferred to drums, as, should there be a short circuit, the faulty coil of a ring may be quickly cut out altogether, or, at the worst, run in its injured condition for some time. Electric cutters have the advantage over hand-wielded picks in that far more coal can be cut with fewer men, that it is removed in larger blocks with less production of small coal, and that the levels may be cut more regularly with far less arduous labour. Against these must be set the greater initial expense, and that more or less continuously created by the burning out of the armatures and repair of the comparatively complicated tools.

Most of the cutters in use fall into the two groups of wheel cutters and bar cutters. The cutting part of the first group consists of a large wheel with a cutting periphery which works into and along the seam, as in the Snell-Waterhouse tool; akin to this is the Thomson-Houston cutter, in which a cutter-bearing chain passing over two wheels saws its way through. In the second group, of which the Goolden-Atkinson cutter is an example, the active portion is, as the name implies, a bar carrying cutters. The bar mentioned cuts along the face of the level, and is driven at about 500 revolutions per minute through gear wheels by a motor. The entire carriage of the motor and bar is capable of rotation about a vertical pin borne on the truck, being rotated by a worm and wheel. The bar, itself, is about 4 ft. long and tapered, with a spiral row of holes into which the cutters are fixed. In working, the appliance is turned about the vertical axis until the distant end of the bar is against the coal, when it is started running and fed forward round the axis until the bar is at right angles to the wall. The whole truck is then hauled forward, the cutter making a horizontal cut into the solid seam. The Jeffrey cutting machine is a more elaborate contrivance. The bar is attached to a frame which is fed forward along the lower part of the truck by reduction gearing on the motor. The cutter revolves at about 200 revolutions per minute, cutting a slot 3 ft. wide in the coal to a depth of as much as 7 ft., working in at right angles to the surface.

To obtain a large starting torque the motors of these machines are usually series wound, and always, on account of their working in places liable to gas, enclosed. The gear is sometimes arranged with considerable backlash so that if the cutter becomes jammed—as wheel cutters working in soft coal are very liable to do, owing to the coal above them sinking down—a certain amount of momentum may be got up, and the wheel jerked free with a series of shocks produced by repeatedly reversing the motor, a method of procedure sufficiently indicating the tender treatment such tools receive.

CHAPTER XIV.

Motor-Generators and Dynamotors.—For the conversion of high-tension currents to a lower pressure and *vice versâ*, motor-generators having a motor and a generating armature keyed to the same shaft, or coupled shafting, and each running in its own magnetic field, may be used, or recourse may be had to a dynamotor with a single armature furnished with two distinct windings and commutators.

The motor-generator is thus, though usually possessing but a single bed and framing, electrically merely a motor coupled directly to a generator, and calls for no special description. The two armatures and their magnets are closely adjacent, and the commutators are at their outer ends, while any form of excitation may be applied to their fields. These machines are widely used for charging accumulators from high voltage circuits, in which case, like ordinary generators for the purpose, they are usually shunt wound, and they are admirably adapted for supplying the current to motors driven by the Ward-Leonard method. (Chapter XIII.)

In dynamotors the commutators of the two windings are most conveniently at each end of the armature. Since there is but a single common field to be excited the current required for this purpose is less than that necessary with a motor-generator, and the efficiency is consequently greater. Either form of machine is, of course, capable of converting from high to low pressure, or from low to high, by interchanging the brush connections; but dynamotors are at a disadvantage as compared with motor-generators, in that the ratio of the pressures is constant, while in the latter it may be varied by altering the field strengths.

Since, using the symbols previously adopted, the electromotive force induced in either winding is

$$Z\,T\,N\,10^{-8}$$

and all the quantities except T are the same for each, the electromotive force generated in each is directly proportional to its number of armature conductors. Indicating, then, the number of conductors in the motor, or primary winding by T and that of the generator, or secondary winding by T′,

$$\frac{T'}{T} = \frac{\text{E.M.F. induced in secondary}}{\text{Back E.M.F. induced in primary}}.$$

It is evident that with a perfect converter the energy put in would be equal to that given out, or putting E and C for the brush voltage and current on the primary side, and E′, C′ for the same quantities on the secondary side, the relation

$$\frac{T'}{T} = \frac{E'}{E} = \frac{C}{C'}$$

would exist. In an actual machine this condition is to a slight extent modified by the fact of the windings having resistance, so becoming

$$\frac{E'}{E} = \frac{Z\,T'\,N\,10^{-8} - C'\,R'}{Z\,T\,N\,10^{-8} + C\,R}$$

and the ratio of the pressures is thus to a small extent dependent on the current, the secondary voltage falling as the current increases. This drop cannot be obviated by any compounding of the field winding, as the increase of field strength produced would also operate on the primary, causing a fall in speed and a still further lowering of the secondary pressure.

Since the product of current and conductors in each winding is approximately the same, and the currents flow in opposite directions, there is no production of any cross flux by armature reaction.

These machines are mainly employed for converting current transmitted at high pressure into low pressure for distribution.

Boosters.—It frequently happens that it is desirable to raise the pressure of a current above that at which it is

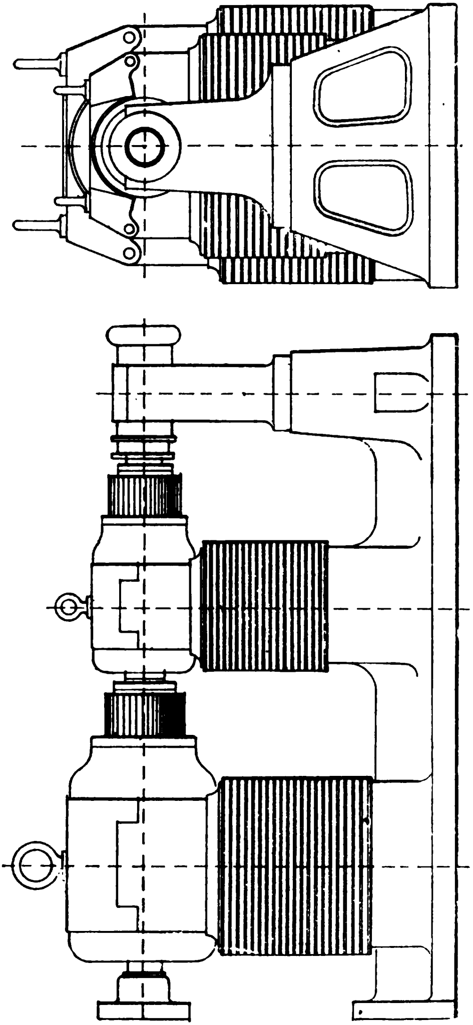


FIG. 101.

originally generated, as in a case where it is necessary to maintain the pressure at the end of a long cable equal to that on shorter circuits in the neighbourhood of the generator; or in the more common one of it being necessary to charge up secondary cells at a pressure above that of the generating plant. In this latter connection it is now standard practice to equip continuous current generating stations with boosters, which are merely shunt machines in series with the cells. Such a machine may obviously be direct-driven by an engine—as was formerly often the case—or any other motive power; but at the present time a pair are usually employed, each booster being motor-driven from one of the balancing machines, and the entire set of four machines are commonly assembled on a single bedplate, with their shafting coupled.

In a station generating at 400 to 500 volts the boosters should be capable of carrying the maximum charging current of the battery and of raising the pressure to any required extent between 20 and 100 volts.

Provision is generally made for reversing the brush connections of the boosters, so that they may be put in series with the cells when discharging.

Fig. 191 shows the half of a regulator-booster set, the boosting armature in this case being keyed on to a hollow sleeve fitting on the shaft. The magnets are fitted with a hinge, so that after swinging them back the armatures may be lifted clearly away.

Plating Machines.—For furnishing the heavy currents necessary for electro-plating and the electrolytic production of metals, generators giving out large currents at a low voltage are required. The armature winding consequently consists of relatively few bars of large sectional area, and the field winding contains a number of circuits in parallel; while, as the large current gives rise to considerable armature reaction, it is in this class of machine that the various devices already mentioned for counteracting it are most applicable. The usual pressure is about five volts, and the current is determined by the area of the electrode upon which the metal is to be deposited, suitable current densities for ordinary metals being—

Copper	3 to 30	amperes per	sq. ft.	of kathode.
Nickel	2 to 15	,,	,,	,,
Silver.........	1·5 to 4	,,	,,	,,
Gold	·7 to 1·5	,,	,,	,,

The current density is regulated to the quality of deposit required, the lower densities giving more compact and coherent deposits.

To minimise eddy currents the heavy winding should be of parallel strips bound up together or of cable; and owing to the large area of contact required between the brushes and the commutator, it is advisable to arrange the brushes at each point of collection in a double row, alternate brushes on the bar being somewhat in advance of those between them. In this way the beneficial effect of the brush resistance in preventing sparking is rendered more effective.

When current at higher pressure is available, it is not uncommon for the field of these machines to be excited separately from the higher-pressure leads; with large electro-platers, indeed, the provision of a separate exciter is not uncommon. Motor generators and dynamotors also are conveniently employed for converting down the public-supply pressure to the plater's needs; while the latter machines are sometimes belt driven, with both the sides, which are then of equal voltage, coupled in series when a greater pressure is required.

Constant-current Generators.—Although the method of distributing electrical energy, for both illuminating and motive purposes, is now to do so at constant pressure—an increase of output being an increase of current—there are circumstances in which it is more economical to transmit it at a constant current and a varying pressure. In such a case the various appliances absorbing energy must necessarily be arranged in series; and, in practice, the method is only applied to arc-lighting circuits, though now even these are usually supplied at constant pressure with the lamps in parallel.

The function of the generator on such a circuit is, therefore, to supply a current which remains constant, and must not be affected by variations in the resistance of the circuit. These variations are of two kinds—the one caused but occasionally, when resistance is switched in or out of circuit; the other, a continually occurring and rapid variation of smaller extent, caused by changes in the resistances of the individual lamps. If these last, however, pass beyond a certain limit, no machine can govern the current, and the spectacle of the whole installation of lamps jumping in unison proclaims it. Provided that generator, lamps and carbons are good, however, this display does not arise.

The current may be kept constant and unaffected by the first class of variation in resistance, by providing special regulating appliances on the generator; but the variations of the second type are of such rapid occurrence that no mechanical device can possibly act with sufficient rapidity to be effective, and the current is prevented from fluctuating on their account by means of the armature reaction, which for this purpose is made considerable in these machines. The induction in the magnetic circuit is also carried to high degrees so as to bring the iron to a well saturated state.

The various well-known machines for this constant current work differ widely in general design and in the details of their regulating gear; but a feature often found is the open-coil armature winding, which has its application only to this class of generator.

The peculiar construction of the Thomson-Houston armature has been already dealt with, but the remaining points to be described are equally characteristic.

The magnetic circuit, fig. 192, consists of cast-iron tubes, upon which the two field coils are carried, terminating internally in hemispherical pole faces. The yoke is constituted by a number of bars passing from end to end of the

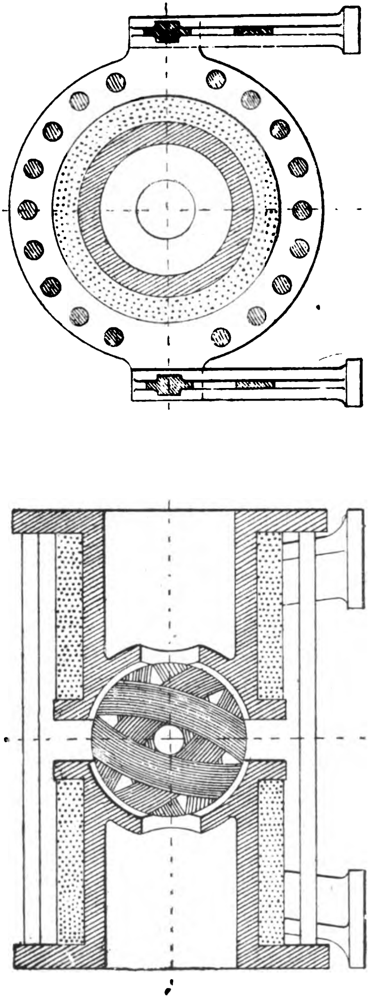


FIG. 192.

machine outside the coils, and from this design results a highly-saturated magnetic circuit.

The ends of the three armature coils are brought to a three-part commutator with air insulation between its segments, and to extinguish the sparking, which is inevitable in these high-tension machines, a special rotary blower of simple construction is provided with ducts conveying the blast to the commutator.

The regulation of the pressure is effected by placing two brushes at each of the collecting points, the angular interval between them being varied under normal conditions; and when the current for which the machine is regulated is flowing, the two brushes of a set are situated 60 deg. apart, 120 deg. thus intervening between the leading brush of one set and the trailing brush of the other, fig. 193. With this disposition the three coils are always in circuit; and, as may be seen from the figure when the diameter drawn represents the position in which the coils generate their maximum electro-motive force, the coil which is most active is always in series with a parallel pair which are less so. Should the current increase, the angular distance between a pair of brushes becomes greater than 60 deg., and then the most active coil is in series with the others for a shorter time, while the machine is also short-circuited six times in each revolution, fig. 194. On the other hand, if the current

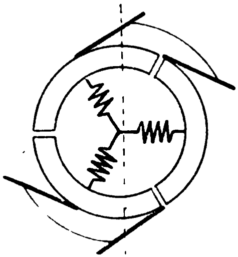


FIG. 193.

falls the brushes are brought nearer together, and the most active coil is in series with another for a longer time, while the least active coil is cut entirely out of circuit, fig. 195.

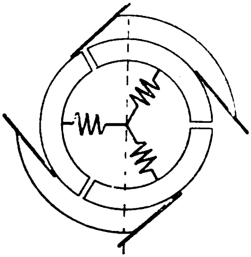


FIG. 194.

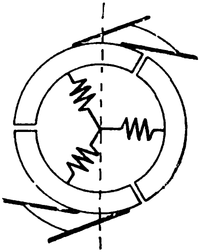


FIG. 195.

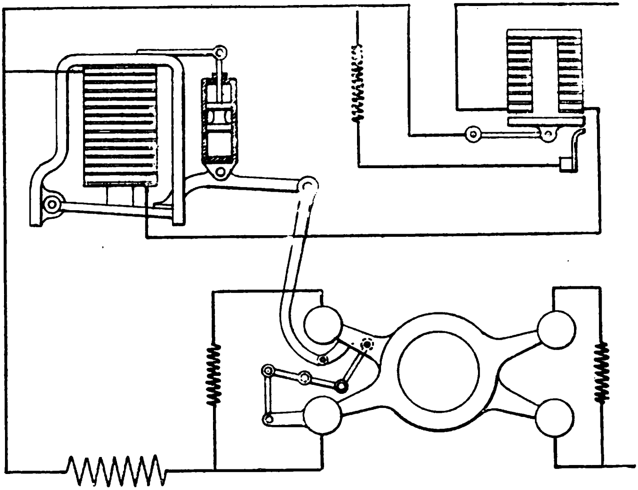


FIG. 196.

The mechanism operating the brushes, fig. 196, consists of a pair of scissor-like crossbars supporting them, and actuated themselves by levers connected to the armature of an electromagnet. The winding of this magnet is shunted by a circuit under the control of a relay, through which the whole current passes, and when running the shunt is continually being opened and closed by the vibrating armature of the relay. To prevent the inordinate sparking which would otherwise ensue with this device, a high resistance shunt is placed across the contact breaker of the relay, and to damp out the minor oscillations of the gear a dash pot is fitted to the main lever.

Upon the current increasing the relay is strengthened, and drawing up its armature opens the shunt of the main controlling magnet, which, then taking the whole current, raises the main lever, and, rotating the crossbars in opposite directions, separates the brushes. The leading brush is moved forward to give greater lead and to reduce sparking; the trailing brush is turned backwards to about thrice the extent.

This automatic adjustment of the brushes is proceeding incessantly during the running of the machine, and effectively controls the current.

The Brush machine also has an open-coil winding, but the core is a ribbon-wound disc, and the armature rotates between pole faces of similar polarity on each side, north to south, and *vice-versâ*, the flux pursuing a tangential course through the core from pole to pole, entering and leaving the armature alternately on either side at each of the four pole faces. The armature coils are laid in insulating boxes, and, projecting above the core, present the characteristically toothed appearance of the disc. To this construction is due the excellent ventilation and also the necessity of avoiding large centrifugal forces. As with all machines having disc armatures, efficient means of preventing any side-play are necessary, and for this purpose thrust collars are borne upon the shaft in the bearing next the commutator. In some instances the shaft between the armature and the commutator is hollow, and through the hole are carried the commutator connections, while the brush gear, as previously explained, consists of two sets of eight or six brushes.

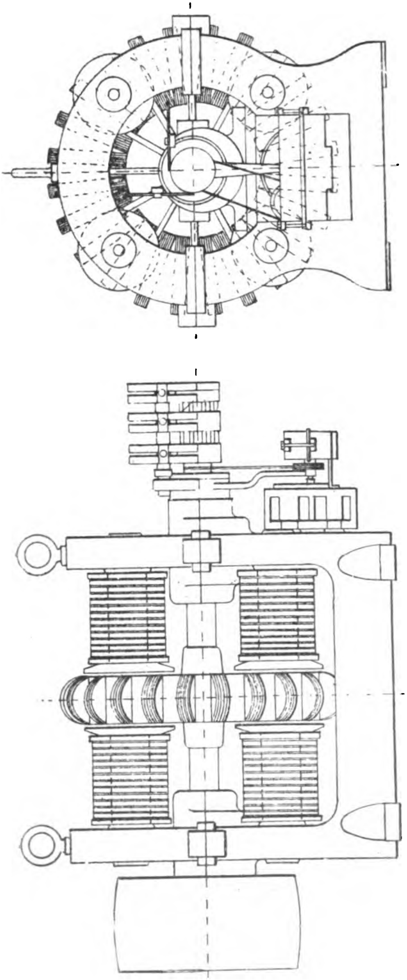


Fig. 197.

In recent forms of the machine the end rings of the framing, which serve as the magnet yokes, are divided horizontally, so that by unbolting and removing the upper half of the machine the armature and spindle may be lifted clear, as shown in fig. 197.

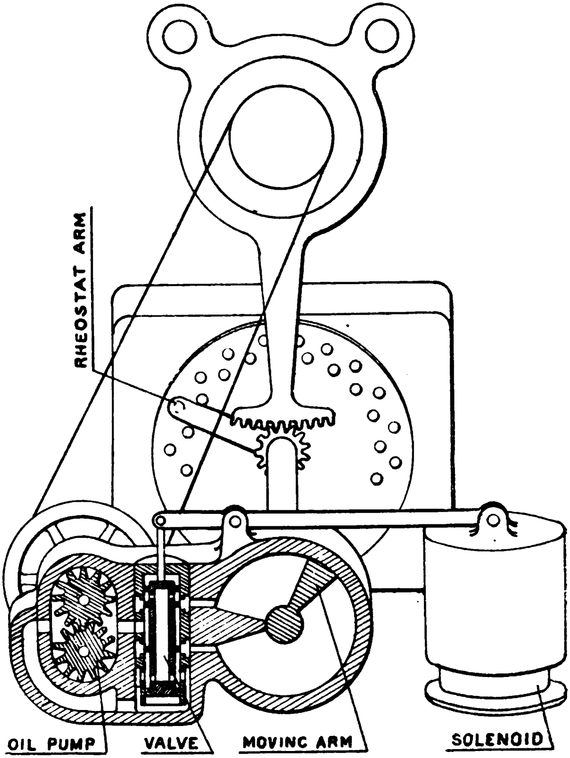


FIG. 198.

The regulation is accomplished by shunting a greater or less part of the current from the field coils through a rheostat, resistance being put in or cut out automatically by a regulator attached to the machine. The active parts of this regulator are an electro-magnet excited by the whole current, and a rotary oil pump driven by a strap from the armature shaft. Connected through a piston valve with the oil pump is a cylinder, on the axial spindle of which is borne a radial arm fitting the internal circumference of the cylinder, within which it rotates with the spindle, while a partition extending inwards from the cylinder wall divides the space inside into two regions. As is apparent from fig. 198, so long as the hollow piston valve is in its median position, the oil from the pump merely circulates through it, and the pressures exerted on each side of the arm in the cylinder are equal; but if the valve be raised or lowered the oil pressure is cut off from one side of the cylinder, and the arm and spindle are driven round. These motions are given to the valve by a lever operated by the electro-magnet in a simple manner, adjustment being made by a spiral spring attached to the lever.

The regulating spindle, passing through the flat end of the cylinder, carries at its end an arm which moves over the contacts of the rheostat, and a small pinion in addition, which gearing with a rack on an arm of the cross bar gives more or less lead to the brushes. The entire arrangement is represented diagrammatically in fig. 198.

The Excelsior Arc Machine, fig. 199, differs entirely in construction from both the preceding examples, having a closed-coil ring armature winding and polar faces embracing not only the outer circumference, but also some portion of the ends of the armature. The current is governed by a combination of the methods of shunting part of the current from the field and of shifting the position of the brushes. The current in this case is not actually shunted from the field coils, but these are wound in sections, and more or less sections are cut out by the regulator. The arm making the necessary contacts for this purpose is connected to the spindle of a small accessory motor attached to the main

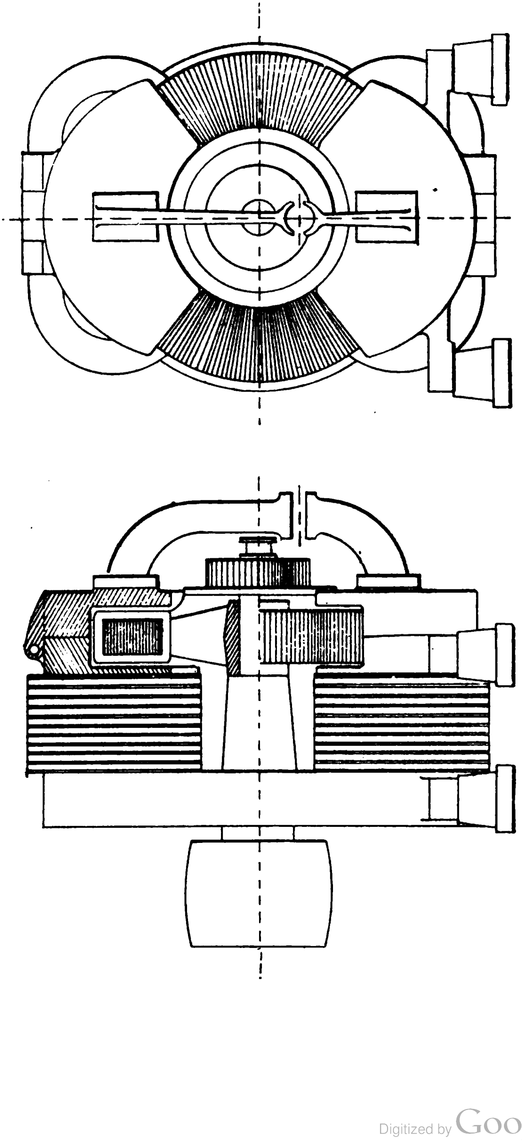


FIG. 199.

generator, and having its field drawn by an iron bracket from the main magnetic circuit.

This motor is operated by the elegant contrivance shown in fig. 200. Two levers, one of which is of insulating material and the other of iron, are, when the current is normal, held by spiral springs in such a position that the conducting lever is in contact with both of the metal contacts carried by the other. In this condition of things both brushes of the regulating motor are at the same potential, and it is, itself, of course, inactive. Should the main current rise, a magnet in circuit is strengthened and attracts the iron lever; both levers then rotating slightly, one of the contacts is broken and a part of the current passes *via* the iron lever, and the other contact to the motor which then cuts out some of the exciting coils. In the event of the main current falling, the levers are drawn back from the

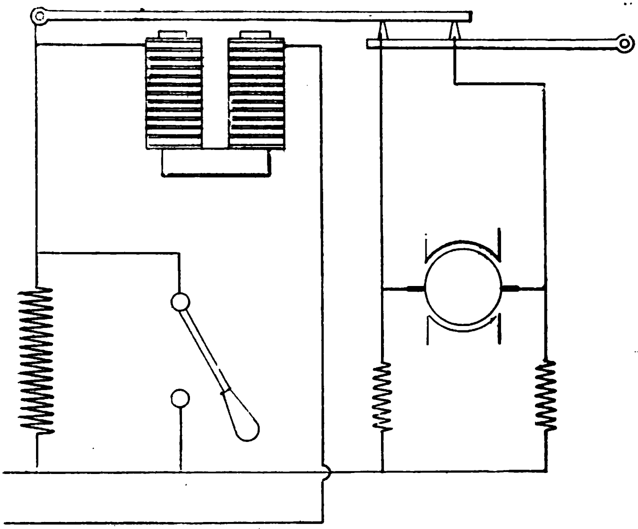


FIG. 200.

magnet and current passes through the other contact; the rotation of the motor is then in the opposite direction and the main excitation is increased.

Appropriate gearing enables the motor to give in addition an increase or reduction of lead to the generator brushes.

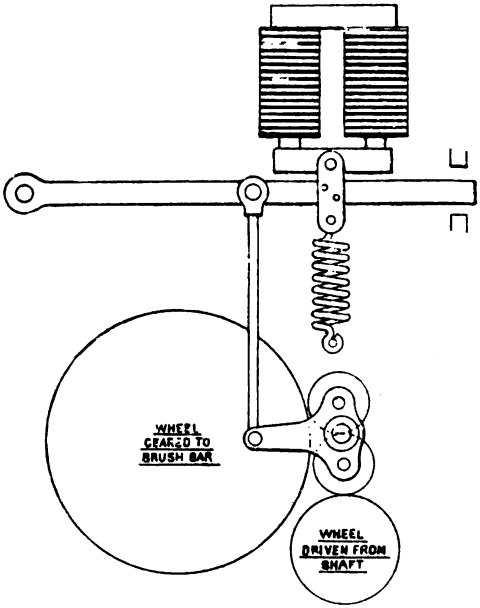


Fig 201.

Other well-known arc-lighting machines, which space does not permit detailed description of, are the Westinghouse Company's, which has a double open-coil winding with two commutators and a separately excited field. The Wood machine, which has a closed winding and regulates by shifting the brushes; and the Western, with similar winding and regulation, but, of course, a different mechanism. The gear of the Wood machine resembles that applied to the

Statter arc lighter in having the crossbar rotated one way or the other, according as one or the other of two intermediate parts, controlled by a solenoid in the main circuit, are thrown in gear with a wheel driven by the main shaft. In the Wood gear, fig. 201, the solenoid throws one or other of the two friction cones rotated by a strap-driven wheel into gear.

CHAPTER XV.

Characteristic Curves.—Of the various curves that may be set out showing the relations between any pair of the quantities, electromotive force, current, and speed for any particular machine, the characteristic, unless special qualification is made, is that which shows the relation existing between current and electromotive force at constant speed. Since generators are usually required to furnish one of these at a constant value independently of variations in the other, the importance of this particular curve, as illustrating at a glance the degree in which this condition is maintained, is evident.

Deferring until the methods of testing are discussed the consideration of its production from experiment on the machine, the predetermination of the characteristic from the drawings and the known properties of the iron or steel will here be dealt with.

As a first step, it is necessary to plot the magnetisation curve giving the connection between exciting current and armature flux. For this purpose, the number of turns in the field, the dimensions and magnetic properties of the different parts of the magnetic circuit being known, various values of armature flux are assumed and the corresponding exciting currents calculated by the method of Chapter IX. If the current required to drive the flux through the different parts of the circuit be separately calculated, a curve may be set out for each, and the final curve for the whole magnetic circuit obtained by addition of their abscissæ, as in fig. 203. It is more satisfactory, however, to add arithmetically and plot the curve directly.

As an illustration of the method of procedure the magnetic circuit shown in fig. 202 may be taken. Selecting any value—suppose 11×10^6 – for the flux per pole, the ampere turns required for the air gap are

$$\frac{10}{4\pi}\left(\frac{11 \times 10^6 \times \cdot 3}{44 \times 38}\right) = 1530.$$

Since the permeability of the air is always unity, flux and ampere turns are always directly proportional to each other, and hence if a point be set out with 11×10^6 as an ordinate and 1,530 as abscissæ, fig. 202, the straight line passing through this and the origin is the curve for the air-gap.

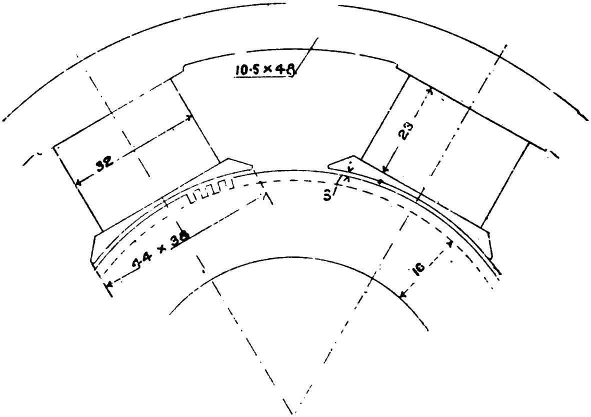


Fig. 202.

The core teeth are 1·5 centimetres deep and equal in width to the slots, while ·8 of the core length is actual iron, so that the tooth section through which the flux passes is thus—

$$38\,(22 + 1) \times \cdot 8 = 700,$$

the 1 being added to allow for fringing out of the lines.

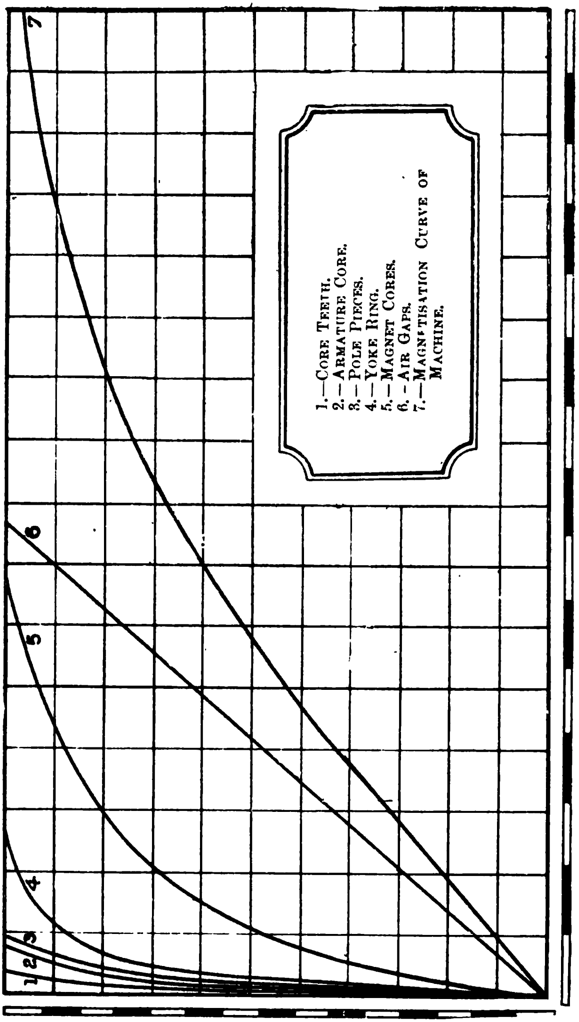

1
2
3
4
5
6
7
1.—Core Teeth.
2.—Armature Core.
3.—Pole Pieces.
4.—Yoke Ring.
5.—Magnet Cores.
6.—Air Gaps.
7.—Magnetisation Curve of Machine.

Fig. 203.

The induction is then

$$\frac{11 \times 10^6}{700} = 15700.$$

Referring to the B and H curve for the metal of the teeth the value of H corresponding to B = 15700 is found to be 36. Hence the ampere turns required are

$$1{\cdot}5 \times \frac{10}{4\pi} 36 = 43.$$

A consideration of the magnetic circuit will show the magnetising force per pole must be sufficiently great to drive the whole pole flux through the magnet core, pole piece, air-gap, and core teeth, and in addition drive one-half of it through the armature core and yoke ring half way to the next pole.

The section of the armature core, reckoning 8 of its area actual iron, is 560 square centimetres, and through this flows half of the flux, hence the induction is

$$\tfrac{1}{2} \times \frac{11 \times 10^6}{560} = 1960.$$

From the B and H curve the corresponding value of H is 3·8, and the half length of path between the poles is 25 centimetres. The necessary ampere turns are then

$$25 \times \frac{10}{4\pi} 3{\cdot}8 = 75.$$

The exciting power for the yoke ring may be calculated in exactly the same way as for the armature core.

In finding points on the curve for the cores of the magnets, leakage must be considered. The leakage co-efficient for this type of machine is about 1·25, so that for an ordinate of 11×10^6 the induction must be taken as

$$\frac{11 \times 10^6 \times 1{\cdot}25}{\pi\, 16^2} = 17100,$$

and the corresponding value of H found from the curve of B and H. Assuming other values for the pole flux the magnetisation curve for each part (including, when necessary, the pole pieces) may be set out.

From these separate curves the magnetisation curve (7) for the complete circuit is obtained either graphically or arithmetically by adding the ampere turns required by the different parts. It will be noted that the effect of the air gap is to give a more sloping character to the curve than if it were for a complete iron circuit, rendering the knee less prominent.

At constant speed the electromotive force induced in the armature is equal to the product of the flux and a constant, viz.,

$$\mathrm{T\,N}\frac{p}{n}10^{-8}$$

Hence the curve obtained may be converted into the more useful one of open-circuit volts and exciting current, by multiplying the numbers of the flux scale by

$$\mathrm{T\,N}\frac{p}{n}10^{-8}$$

and dividing those of the ampere-turn scale by the number of magnet turns in series.

The Shunt Characteristic.—In a shunt machine the exciting current is itself dependent upon the electromotive force, and so, when running on open circuit, there must be one or more values of each corresponding to any one speed. If R_s denote the shunt resistance, R_a that of the armature, C_s the shunt current, and E the electromotive force induced, then

$$C_s = \frac{E}{R_a + R_s}$$

or with sufficient accuracy, since R_a is small compared with R_s,

$$C_s = \frac{E}{R_s}$$

$$R_s = \frac{E}{C_s}$$

Hence if from the origin, fig. 204, a line be drawn at an angle θ with the current axis, θ being such that $\tan\theta = R_s$, then a perpendicular A B dropped from any point on this line represents by its length the voltage which drives a current represented by the length O B cut off the current axis through the shunt coils. The intersection of the inclined line with the curve of induced volts and exciting current evidently gives the electromotive force, and the shunt current when the machine runs on open circuit.

If the external circuit be closed so that a current flows, the terminal voltage falls to a lower value owing to drop over armature resistance and to the back turns arising from

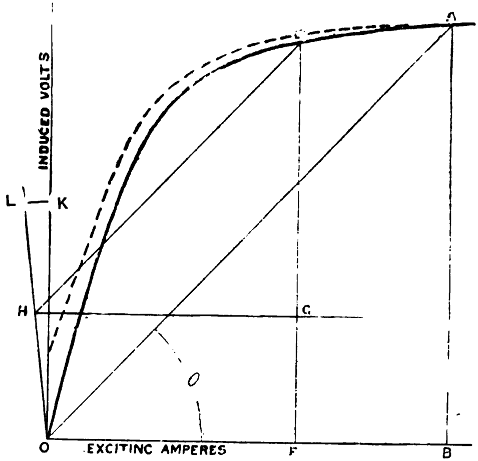


FIG. 204.

the lead which must be given to the brushes. As a result of this the exciting current itself is reduced, and consequently also the induced electromotive force. Whatever value, however, the latter may be brought down to, it is evident that the pressure driving current through the shunt coils is less by the amount $C_a R_a$. With regard to the back ampere turns of the armature, it may be assumed that the number of these is proportional to C_a, and that at full load the brush is under the pole horn; the quotient of the back ampere turns divided by the number of shunt turns in series is then the amount by which the exciting current is virtually reduced by their action. The graphical construction for finding the electromotive force and exciting current with any armature current C_a is as shown in fig. 204. From O a length OK is taken on OY, representing to the volt scale the product of armature resistance and any arbitrary value of the current, from K a distance KL is taken representing to the ampere scale the quotient of the back turns with this current divided by the number of shunt turns, and the line OL is drawn. For any current C_a a length FG equal to $C_a R_a$ is taken, and HG drawn parallel to OX, cutting OL in H, from which point HE is drawn parallel to OA. The exciting current is then given as HG, the terminal voltage as EG, and the induced voltage as EF. It will be observed that the inclined line for any current

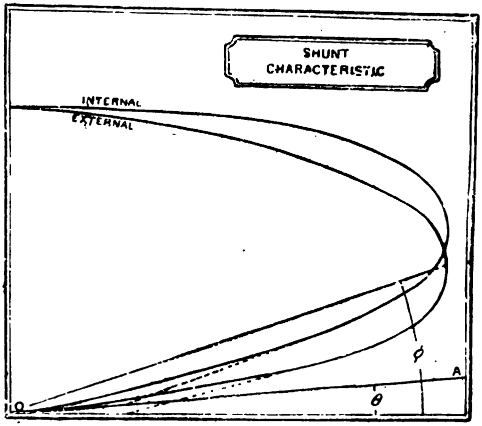


FIG. 205.

intersects the curve in two points, indicating that for any particular values of speed and current a shunt machine can run at either of two voltages. As a matter of fact, the magnetisation curve is higher for a falling than a rising flux by reason of hysteresis, and consequently, as shown by the upper dotted curve of fig. 204, the inclined lines for small currents cut it but once. By making the above construction for a sufficient number of values of C_a, and finding the corresponding voltages, the characteristic, fig. 205, may be obtained, volts being plotted as ordinates, and values of $C_a - C_s$ as abscissæ. If from O a line O A be drawn making with O X an angle θ, such that $\tan \theta = R_a$, the ordinates of this deducted from those of the internal characteristic give those of the external characteristic.

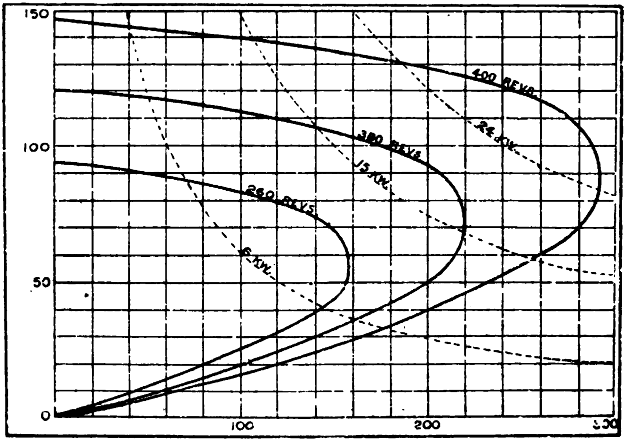


FIG. 206

It is evident from fig. 205 that if a shunt machine be run on open circuit, and the external resistance be gradually reduced, the current at first increases to a maximum—the critical current—after which, as resistance is further reduced, it falls to zero. It is thus obvious that a dead short circuit would be accompanied by no excess of current. In a well-designed machine, it is usually impossible to carry the current through the entire curve, as the armature would be burnt out by the excessive current before the maximum was reached. The critical resistance is given by the tangent of the angle ϕ between the current axis and a line joining the bend of the characteristic to the origin. If the characteristics for a number of different speeds be set out they are seen to be a number of loops within each other, fig. 206, the open circuit voltage of each being proportional to its speed.

Since the energy given out is equal to the product of electromotive force and current, the curves of equal output are a series of rectangular hyperbolas cutting the characteristics, and this is, of course, equally true for any type of characteristic, shunt or otherwise.

The Series Characteristic.—Starting as in the previous case with the magnetisation curve of the machine, it is evident that on open circuit there is no exciting current and the electromotive force is produced only on the residual flux. Converting the horizontal scale into one for exciting current and the vertical scale into one of volts as before, the magnetisation curve becomes the internal characteristic, except in so far as correction is necessary for armature back turns. To make this correction the line O A, fig. 207, is drawn from O, at the angle θ with the axis of current, θ being such that $\tan \theta = R_a + R_m$, the combined armature and series coils resistance. Let now a length O B be taken representing to scale the full load current, and an ordinate erected at B. The quotient of the full load back ampere turns divided by the number of turns in the series field coils is now set back from B giving the point D, and the line A D is drawn. The induced voltage at full load is then given by the length of the ordinate D C erected at D to cut the curve. Similarly to find the internal electromotive force corresponding to any current O B′, from the point A′ where the ordinate from B′ cuts O A a line is drawn parallel to A D cutting the axis in D′, and the ordinate at D′ gives the voltage as D′ C′. In this way the complete internal characteristic, with armature reactions taken into account, is obtained as the lower curve. From this again the external characteristic may be derived by deducting from each ordinate the length intercepted on it between the lines O A and O B. The striking difference between the series and shunt characteristics will not escape attention. Within the working range of either, however, an increase of current produces a voltage drop. In the case of constant current machines the drop of the characteristic is intentionally made large to prevent undue increase of current if the external resistance be greatly reduced.

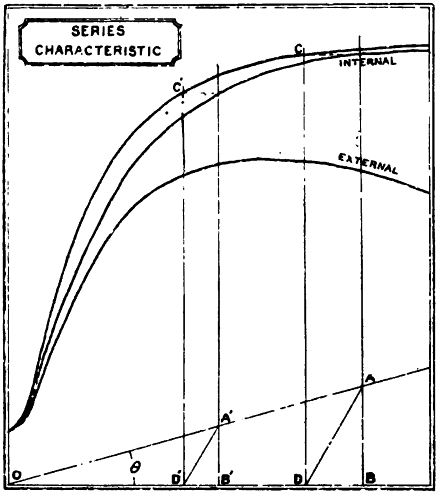


Fig. 207.

The Compound Characteristic.—It is a matter of some difficulty to plot this curve by geometrical methods, and it is certainly simpler to calculate the points on it. It is necessary, as in the previous cases, to set out the magnetisation curve for the machine showing the relation between exciting ampere turns and armature flux, then to convert the scales into those of induced electromotive force and exciting current, assuming the latter to flow in the shunt windings.

Let E_a = the voltage induced in the armature.
E_t = the terminal voltage.
C_a = the total current in amperes.
C_m = the current in the external circuit.
C_s = the shunt current.
C'_s = the total exciting current reduced to its equivalent value if all flowing in the shunt winding.
R_a = the armature resistance.
R_s = the shunt resistance.
R_m = the series coils' resistance.
N_m = number of turns in series in the series coils.
N_s number of turns in series in the shunt coils.

Then the following relations are at once evident.

$$E_t = E_a - C_a(R_a + R_m)$$
$$C_a = C_m + C_s.$$

The value of C_s depends upon whether the connections are made for a long shunt, *i.e.*, across the terminals, in which case it is given by

$$C_s = \frac{E_t}{R_s}$$

or whether it is a short shunt joined across a brush and a terminal so that

$$C_s = \frac{E_a - C_a R_a}{R_s}.$$

The back ampere turns may be considered as proportional to the current, and their value for any current C may be denoted by X_b. Then

$$C'_s = C_s + \frac{N_m}{N_s} C_m - \frac{X_b}{N_s} C_a$$

$$C'_s = \frac{E_a - C_a R_a}{R_s} + \frac{N_m}{N_s} C_m - \frac{X_b}{N_s} C_a \text{ (short shunt).}$$

The value of the right-hand side will be but slightly affected if C_m be written in place of C_a. Making this alteration, and transposing

$$C_m = \frac{C'_s R_s - E_a}{\frac{N_m}{N_s} R_s - R_a - \frac{X_b}{N_s} R_s} \quad \text{(short shunt)}$$

or substituting $R_a + R_m$ for R_a

$$C_m = \frac{C'_s R_s - E_a}{\frac{N_m}{N_s} R_s - R_a - R_m - \frac{X_b}{N_s} R_s} \quad \text{(long shunt)}$$

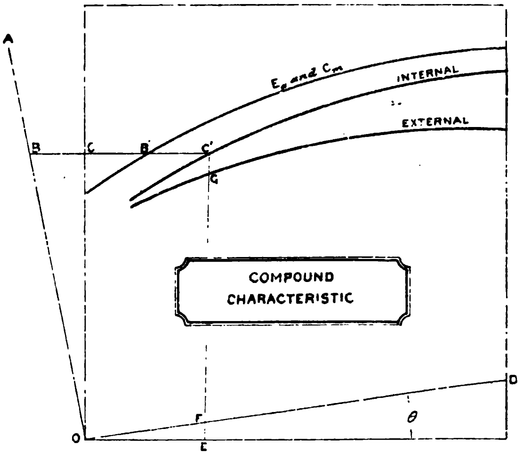


FIG. 208.

From this expression, a number of values of C_m may be calculated, E_a and C'_s being taken from the curve already constructed, and a new curve, fig. 208, set out with C_m and E_a as abscissæ and ordinates respectively. If a line O A be drawn from O, inclined to O Y at an angle whose tangent is equal to R_s, the length B C intercepted on any line drawn parallel to O X from any point on the curve is very

approximately the value of C_s for that point. Hence for every pair E_a and C_m, the corresponding C_s may be obtained, and another curve showing E_a and C_a may be drawn by adding to the abscissæ of the E_a and C_m curve the lengths cut off between O Y and O A. This is the internal characteristic.

The external characteristic may be derived from the internal curve, fig. 208, by drawing O D at such an angle θ with O X that its tangent is equal to R_a. By deducting from the ordinates the lengths cut off them by this line the required curve is obtained.

It will be evident that by changing the connections of a compound wound machine from long to short shunt, the voltage will be slightly raised.

Regulating Resistance.—The fall of voltage with increase of current in a shunt generator may be prevented, or an increase of pressure may even be produced, if an adjustable resistance be inserted in the shunt circuit, and in practice this device is commonly employed for the regulation of shunt, and often of compound, machines. The amount of this resistance may be determined in the following manner.

On the curve of induced volts and exciting ampere turns, fig. 209, let lines A B, C D, be drawn parallel to O X at heights representing to the volt scale, O A the terminal voltage required at full load plus the drop in the armature resistance, O C the terminal voltage on open circuit. Then the length A B − C D, on the scale of ampere turns, plus the armature back turns, is the number of ampere turns that must be removed from the shunt by the insertion of the resistance. Let the exciting currents corresponding to these be C_1 and C_2, then

$$R_1 = \frac{\text{full load terminal voltage}}{C_1}$$

$$R_2 = \frac{\text{open circuit voltage}}{C_2}$$

and $R_2 - R_1$ is the value of the regulating resistance when fully in. In practice this resistance is usually divided into equal steps for convenience of manufacture, and owing to the diminishing permeability of the iron as it becomes saturated, this also gives approximately equal increments of electromotive force as it is cut out of circuit.

Motor Diagrams.—The graphical method of fig. 210 brings out some useful information with respect to motors running from a constant pressure supply. The length O B is made of

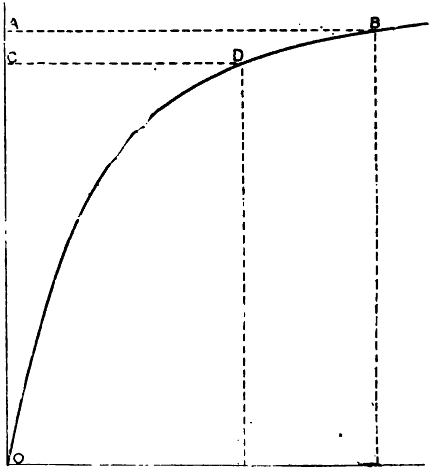


FIG. 209.

such length as to represent the current passing through the armature when at rest, *i.e.*, the quotient of supply voltage divided by resistance, O A is made to represent the supply pressure, the rectangle is completed and the diagonal drawn. The back electromotive force for any armature current O C is then given by the length C D and the resistance drop by the length D E. The energy wasted as heat is represented by the area A E D F, and the amount converted into mechanical work by the area O F D C. It is seen that when the current is nothing the back pressure must equal that of supply and that when the current is a maximum the whole energy is converted into heat. The electrical efficiency of the machine is equal to the ratio of the area O F D C to the area A E C O, and thus approaches a maximum as the back electromotive force, or, putting it another way, as the speed increases. The maximum amount of work which can be got from the motor is at that current giving the greatest value to the area O F D C; this is the case when the current and back pressure have half their maximum values, and the rate of work is then equal to

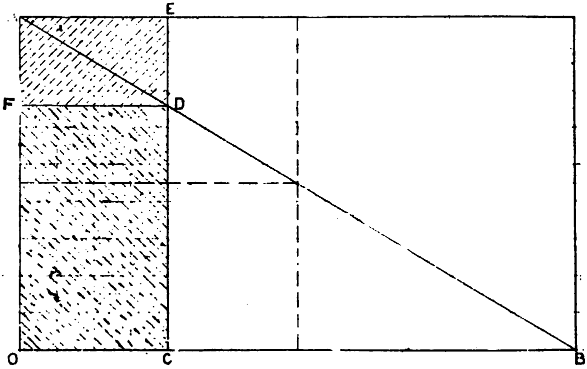


FIG. 210.

$$\frac{(\text{supply pressure})^2}{4\,(\text{resistance})}$$

The resistance referred to is for a shunt motor that of the armature, for a series or compound motor that of armature and series coils; and it should be noted that the work that may be got out of the machine is less than the area O F D C by the amount of hysteresis, eddy current and frictional losses. In fig. 211 the plotted curves show the variations in the quantities indicated in fig. 210, the current axis of both figures being similar.

If the magnetic flux of the motor be constant at all loads, its back electromotive force is proportional to the speed; and to a suitable scale the ordinates of D B, fig. 211, give the speed for any value of the current. Generally, however, the flux is affected by the current and a greater number of curves are then necessary to show the mutual relations of current, speed, and torque. In fig. 212 the curves are given for a shunt motor, of which the armature resistance is ·39 ohms, and in which, consequently, when supplied at 150 volts pressure, the maximum possible armature current is 385 amperes, though the insulation would actually, of course, be burnt out long before this maximum was reached.

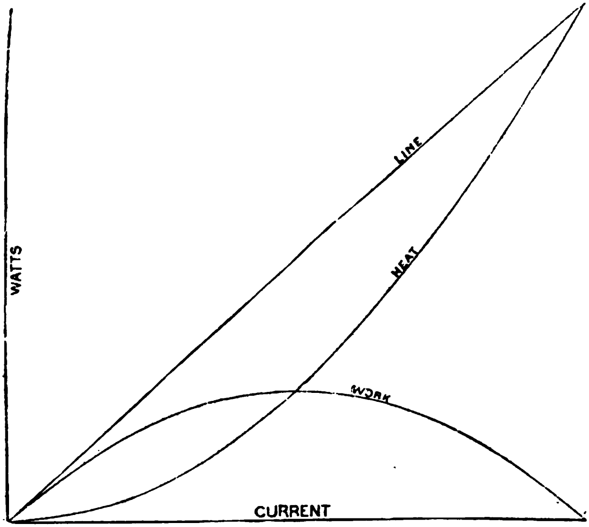


FIG. 211.

The open circuit flux is 2·5 millions C.G.S. lines and is slightly reduced by armature reaction as the current increases, while the torque being equal to

$$1{\cdot}41\,Z\,T\,C\,10^{-8}$$

is proportional to the product of flux and current, and its curve is a straight line from the origin.

The speed for any current is calculated from the relation

$$C = \frac{E - Z\,T\,N\,10^{-8}}{R},$$

whence

$$N = \frac{E - C\,R}{Z\,T}\,10^{8},$$

the speed so obtained being revolutions per second.

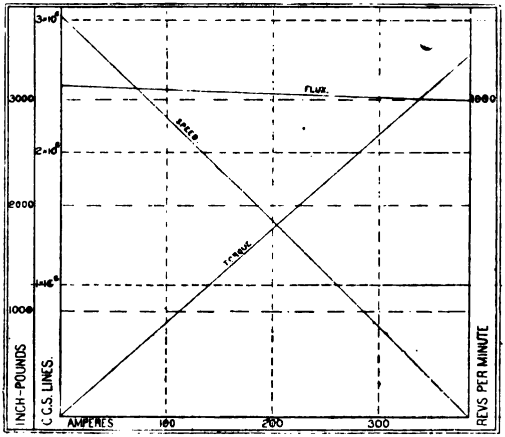


FIG. 212.

Its maximum value is that at which the back electromotive force equals the supply pressure *i.e.*, when

$$N = \frac{150}{2{\cdot}5 \times 270} \cdot 10^{2}$$

the number of external armature conductors being in this case 270.

Plotting the values of speed for various currents the curve is found to be slightly convex upwards, but for all practical purposes a straight line.

The combination of curves given in the figure shows the working under any conditions at 150 volts supply. For example, to find the current and speed when producing a torque of 250 inch-pounds, the current is seen from the torque curve to be 26 amperes, and to this current corresponds a speed of 1,666 revolutions per minute on the speed curve.

If the supply pressure be changed, the effect on the diagram is that the open circuit speed and the maximum current are altered in the same proportion, the speed curve being sensibly a straight line joining the two points. Should the flux be unaltered the torque-current curve will be unaffected, but usually, of course, the altered pressure produces a greater or less shunt current, and the exact effect of this on the flux is dependent upon the saturation of the magnets. The general effect will be that for the same

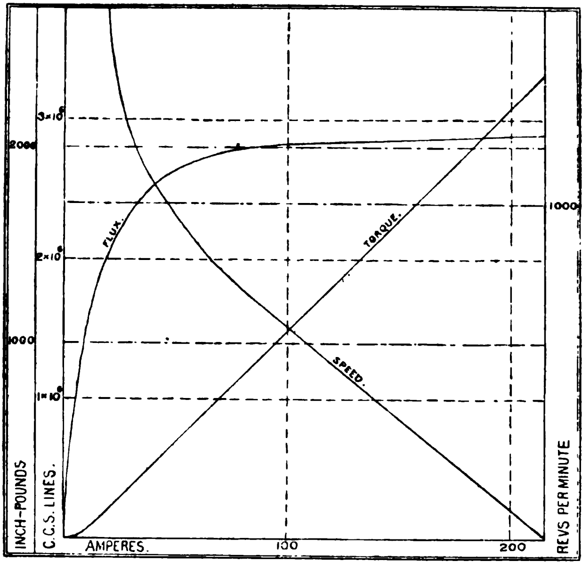


Fig. 213.

armature current an increase of supply pressure produces both greater speed and torque, the increase in the latter being less the greater the original saturation.

For the same machine when series-wound the curves are as in fig. 213. The flux, neglecting residual magnetism, is zero on open circuit and increases with the current. Selecting, as before, corresponding values of current and flux, points upon the torque curve are calculated and the curve drawn. As the combined resistance of armature and field is now ·7 ohms the maximum current is but 214·3 amperes, and the torque curve is straight except in the region of the origin, *i.e.* for small currents. The speed curve is practically a straight line for large currents with which the flux is nearly constant, but for small currents rises rapidly, approaching theoretically an infinite value as the flux vanishes.

It may also be noted, as illustrating what has been previously pointed out, viz., that the series motor has greater starting torque than the shunt motor, that the rate of increase in inch-pounds of torque per ampere is, except for very small currents, equal in the case of fig. 213 to 11 as against rather less than 9 in that of fig. 212. This difference would be still more marked if the magnets of the series machine had been larger, so permitting a lower induction to be employed with the normal working current.

The diagram of the series machine would be unaffected by an increase of supply pressure so far as flux and torque curves are concerned, but the speed curve would be raised, and the maximum current would, of course, be increased proportionally to the impressed voltage.

Both to avoid an inordinately large motor for the work, and for the sake of economy both motors would be run at a high speed and a small current, so that a narrow region on the left of figs. 212 and 213 represents the parts of the curves actually worked on.

In a similar manner to the foregoing the curves for compound wound machines may be plotted, the chief difference lying in the form of the flux-armature current curves. Those for a cumulatively compound winding are shown in fig. 214. Where for the sake of very constant speed under varying load a differential winding is adopted, certain relations must be satisfied for the given speed; and unless economy is altogether sacrificed, such a motor can only run

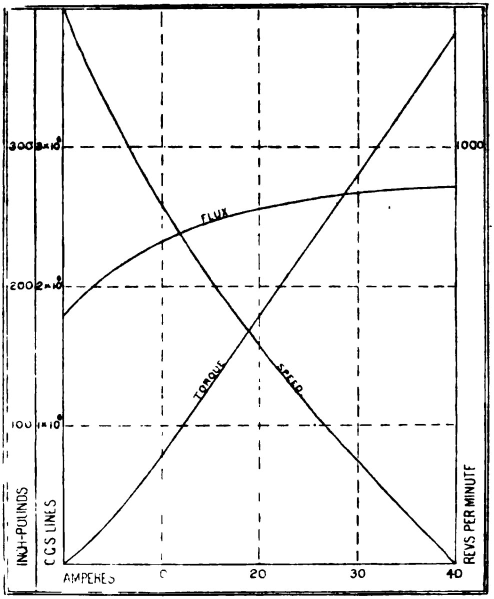


Fig. 214

at exactly the same speeds on two definite loads, with a slight increase in speed when working at intermediate rates. For each of these two loads the flux and current must have definite values, which may be determined in the following manner. The torque at each load is calculated from the expression,

$$\text{Torque (inch-pounds)} = \frac{12 \times 33.000}{2\pi} \times \text{horse power.}$$

Calling these torques 1 and 2, curves are set out as in fig. 215, with armature currents as abscissæ and values of flux as ordinates, such that the product of current and flux is equal to

$$\frac{\text{Torque}}{1{\cdot}41\,\frac{p}{n}}\,10^8$$

T denoting, as usual, the number of external armature conductors.

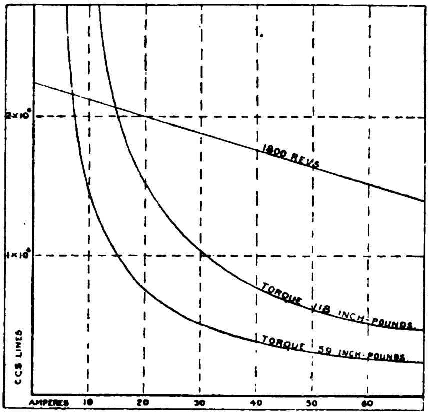


FIG. 215.

Now at any speed N current and flux are connected by the relation

$$C = \frac{E - Z\,T\,N\,10^{-8}}{R},$$

R being the resistance of armature and field. Consequently, if a distance representing $\frac{E}{R}$ amperes be set off on the current axis, and a straight line be drawn from its end, making with it an angle whose tangent is equal to

$$\frac{R\,10^8}{T\,N},$$

this will intersect the torque lines in points giving the values of flux and current for each. The required points of intersection may otherwise be found by setting off a length on the flux axis representing on the flux scale E divided by $N T 10^{-8}$, and from its extremity drawing a line inclined to the axis at an angle whose tangent is equal to

$$\frac{T N}{R 10^8}.$$

The magnetisation curve of the machine being now set out, the exciting ampere turns necessary to produce the fluxes found from fig. 215 are ascertained. The difference between these numbers of ampere turns divided by the difference of current for the two torques is the number of turns in the series winding. The shunt ampere turns must equal the number corresponding to the greater flux, as shown by the magnetisation curve, augmented by the num-

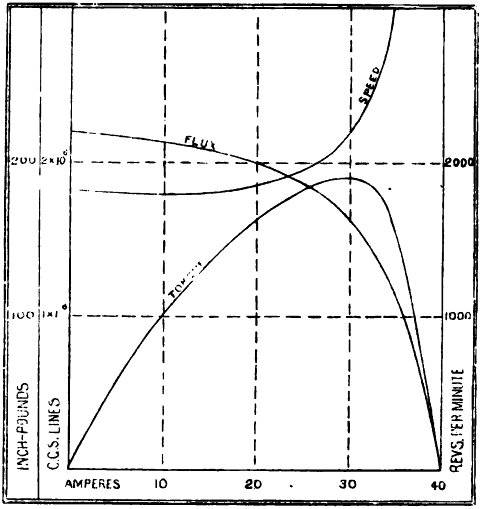


Fig. 216.

ber of ampere turns in the series coils at the smaller current. The curves for a differentially-wound motor are as shown in fig. 216.

Traction Motor Diagrams.—The speed and the current absorbed by a car on any incline, with any arrangement of motors and resistance, may readily be found by constructing a diagram based on that of fig. 181, which represents values of speed (reduced to car-miles per hour) and current for a single motor with varying load, when run at the line pressure with no resistance other than its own in circuit. In fig. 217 this curve, shown by the broken line, is firstly set out, and from it that for the two motors in parallel without resistance in may be derived.

Let E denote the line voltage, R the resistance of armature and field, M and K having the same signification as on pages 350-354. Then for any particular current C and a single motor

$$C = \frac{E - 1{\cdot}47\,K\,M}{R},$$

while for the two in parallel the speed will have some other value M′, and the combined resistance is ½ R, so that

$$C = \frac{E - 1{\cdot}47\,K'\,M'}{\frac{R}{2}}.$$

Hence, equating the right hand side and transposing,

$$M' = \frac{K}{2\,K'}\,M + \frac{E}{2{\cdot}94\,K'}.$$

The ordinates of this curve are accordingly obtained by multiplying those of the first by $\frac{K}{2\,K'}$, and adding to each the length

$$\frac{E}{2{\cdot}94\,K'}.$$

If a resistance r be inserted in series with the paralle motors, the speed takes a value M″ such that

$$C = \frac{E - 1{\cdot}47\,K'\,M''}{\frac{R}{2} + r}.$$

and so

$$M'' = \frac{E}{1{\cdot}47\,K'} - \left(\frac{E}{1{\cdot}47\,K'} - \frac{K\,M}{K'}\right)\frac{R + 2\,r}{2\,R}.$$

he ordinates of this curve are thus equal to the quantity

$$\frac{E}{1{\cdot}47\,K'}\left(\frac{R - 2r}{2\,R}\right)$$

added to

$$\frac{(R + 2\,r)\,K}{2\,R\,K'}$$

times the the ordinates of the first curve, and by taking values of r corresponding to the resistances in at the different controller notches, the curve for each may be drawn.

When the two motors are thrown in series with no external resistance in, their back electro-motive forces are added and their resistance is doubled, so that for a current C

$$C = \frac{E - 2\,(1{\cdot}47\,K\,M_s')}{2\,R}.$$

Equating the right-hand side, as before, to that of the single motor

$$M_s' = M - \frac{E}{2{\cdot}94\,K},$$

and the ordinates are less than those of the first by the length

$$\frac{E}{2{\cdot}94\,K}.$$

If resistance r be put in series,

$$C = \frac{E - 2\,(1{\cdot}47\,K\,M_s'')}{2\,R + r},$$

whence

$$M_s'' = M\left(\frac{2\,R + r}{2\,R}\right) - \frac{E\,(R + r)}{2{\cdot}94\,K\,R},$$

the ordinates of this curve being obtained by multiplying those of the first by

$$\left(\frac{2\,R + R}{2\,R}\right)$$

and deducting a length

$$\frac{E\,(R + r)}{2{\cdot}94\,K\,R}.$$

A series of curves, as in fig. 217, may thus be plotted, each corresponding to a particular notch of the controller; or,

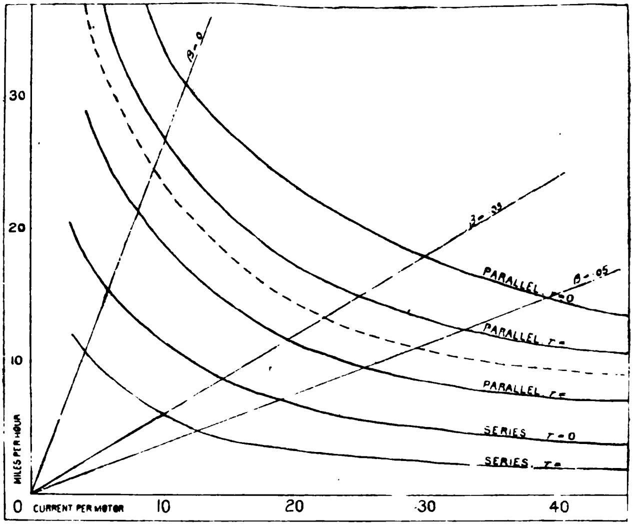


FIG. 217.

conversely, curves may be set out, and the resistances in at the notches may be determined. The speeds on various gradients may be ascertained for each notch by drawing straight lines corresponding to each gradient, as explained on page 352. Any arbitrary value of M being chosen, the total current for any gradient B is given by

$$C = \frac{4454\ W}{E\ \eta}(a + B)\ M,$$

and though, taking this expression as it stands, some error is introduced by the efficiency η varying with the load, this is easily corrected. For example, if η be taken at its value for maximum load, and the value of C be found from the equation for some smaller load at which the efficiency is η', the corrected current is

$$\frac{\eta}{\eta'}\ C.$$

Another form of traction-motor diagram, giving full information as to the running conditions of a car, is shown in fig. 218.* The internal characteristic of the motor—*i.e.*, volt and amperes—when running as a generator for some definite speed (for example, that corresponding to 12 miles per hour speed of car), is plotted out. In fig. 218 this is shown by the broken-line curve, and from it two other curves are derived—the characteristic for both motors in series by doubling the ordinates, that for both in parallel by doubling the abscissal.

From a point on the voltage scale, representing the supply pressure, a horizontal line is drawn, and below it other inclined lines—one for each controller notch. The inclination of each of these is such that the intercept on an ordinate, between itself and the horizontal line, represents on the voltage scale the C R drop due to the total resistance in circuit, with a current corresponding to the position of the ordinate.

Terminating the ampere scale at a point corresponding to the maximum current the motors are able to carry, the produced line is divided into a scale for speeds.

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* See a most instructive paper by M. B. Field, Journal of Inst. Elec. Engineers. No. 158.

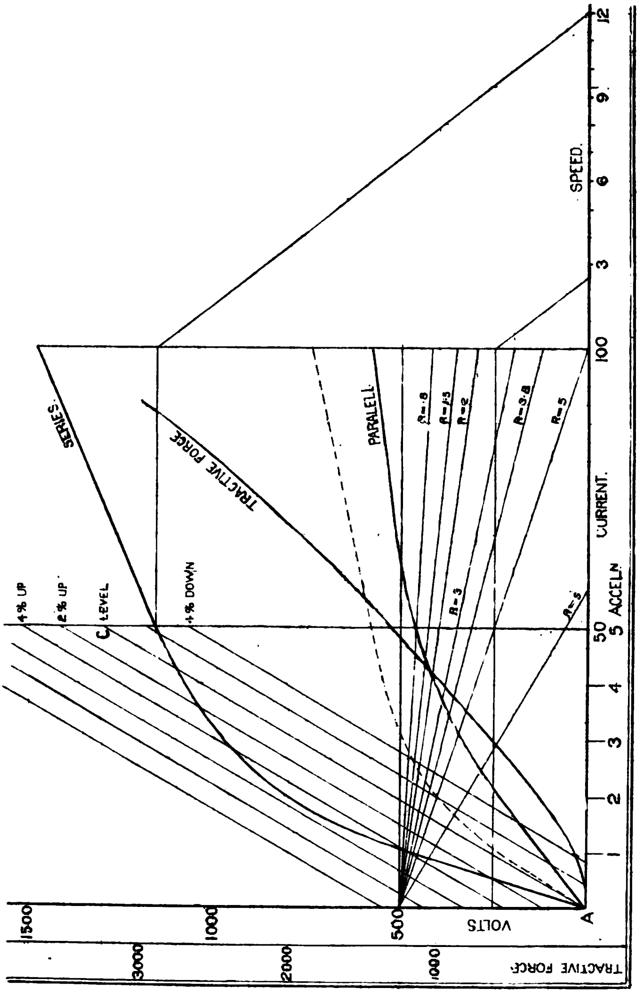
SERIES
TRACTIVE FORCE
PARALELL
R=.8
R=1.5
R=2
R=3.8
R=5
R=3
R=.5
4% UP
2% UP
C LEVEL
4% DOWN
SPEED
CURRENT
ACCELN
12
9
6
3
100
50
5
4
3
2
1
A
VOLTS
1500
1000
500
TRACTIVE FORCE
3000
2000
1000

FIG. 218.

On the same axis is also set out a curve showing the relation between current and the tractive force at the circumference of the wheels. The latter is the actual force impelling the car, and is equal to

$\left(2 \times \frac{v}{\eta} \cdot \frac{2}{12\,\mathrm{D}}\right)$ (armature torque per motor in inch-pounds).

The tractive forces necessary to maintain uniform motion on the level and on some gradients, suppose 10 per cent, are now calculated, and the distance between these numbers on the tractive force scale is divided into equal parts to form a gradient scale, which may be extended above and below. Forming a new horizontal scale of accelerations, of any convenient length, the tractive force necessary to produce any particular acceleration, suppose five miles per hour per second, on the level is calculated, and the point C corresponding to this acceleration and tractive force is joined to A. Parallel lines are then drawn from other points on the gradient scale.

To ascertain, now, the speed, etc., corresponding to any current, suppose 50 amperes, with the motors in series, it is noted that the ordinate at this current intersects the series curve at a point indicating a combined back electromotive force of 1,150 volts; through this point a horizontal line is drawn, and its point of intersection with the maximum-current ordinate is joined to the point on the speed scale agreeing with the speed for which the original characteristic has been drawn.

Through the intersection of the 50-ampere ordinate with that inclined line corresponding to the particular resistance in circuit another horizontal line is drawn, and a straight line drawn from the extremity of this parallel to that from the upper curve gives the speed 2·5 miles per hour, in fig. 218, with R = 5.

From the tractive-force curve it may now be seen that with 50 amperes the force is 1,130 pounds, and from fig. 219, which is an enlargement of part of fig. 218, it is noted that on the level the acceleration is two miles per hour per second, on a 5 per cent up gradient ·8, etc. On a 9·5 per cent gradient the acceleration is nothing, and with this incline, therefore, the car would run on this notch at a steady rate of 2·5 miles per hour, taking a current of 50 amperes. The converse, determining the current and speed for uniform motion on any gradient, is simple. From the intersection of the gradient line O Y a horizontal line is drawn cutting the tractive-effort curve; this gives the current, and the finding of the speed is then as before. For notches at which the motors are in parallel, the right-hand inclined line must be drawn from the end of a line passing through the intersection of the ordinate for the given current with the lower curve.

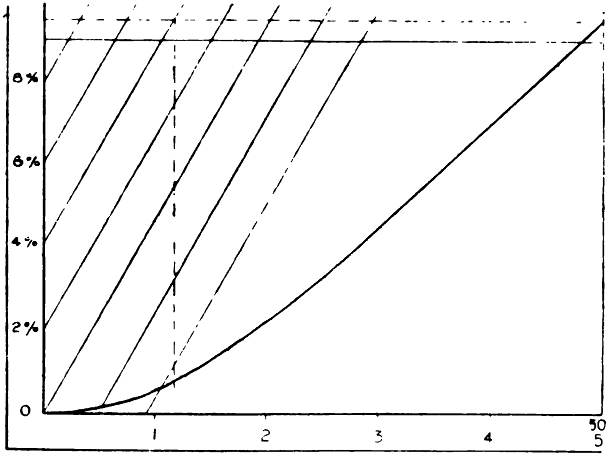

Fig. 219.

The conditions obtaining when braking may be shown by a somewhat similar diagram, fig. 220. From the characteristic (volts and amperes as generator) of the single motor the corresponding values of current and $\frac{\text{speed}}{\text{resistance}}$ are

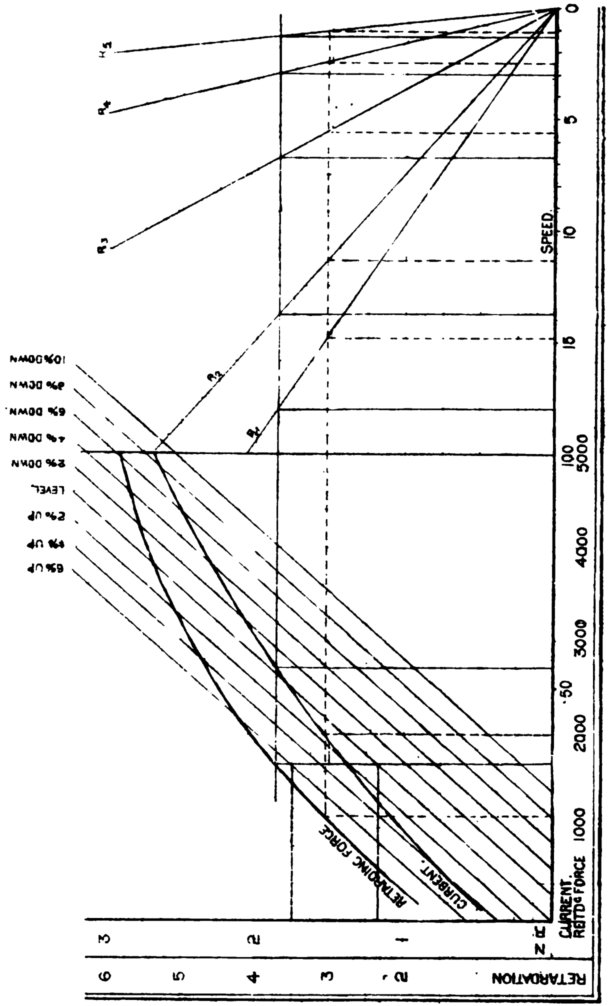
R5
R4
R3
R2
R1
0
5
10
15
SPEED.
100
5000
4000
3000
·50
2000
1000
CURRENT.
RETDG FORCE
RETARDING FORCE
CURRENT.
10% DOWN
8% DOWN
6% DOWN.
4% DOWN
2% DOWN
LEVEL
2% UP
4% UP
6% UP
3
2
1
Z
R
RETARDATION
2
3
4
5
6

FIG. 220.

readily found. In fig. 220 a curve is set out with current per motor as abscissæ and values of $\frac{\text{speed}}{\text{resistance}}$ as ordinates, and in the same straight line as the current axis a scale of speeds is set off from left to right. Straight lines are also drawn inclined to the speed axis, one for each brakeing notch, connecting values of $\frac{\text{speed}}{\text{resistance}}$ and speed for the particular resistance in circuit at the notch.

A second curve is also plotted with values of $\frac{\text{speed}}{\text{resistance}}$ as ordinates, and the corresponding retarding pulls at the circumference of the wheels as abscissæ. The latter values may be found by noting the current corresponding to the quantity $\frac{\text{speed}}{\text{resistance}}$, and finding from the characteristic the voltage with this current. E and C are then known, and the tractive force in pounds is given as

$$\frac{60 \times 33.000}{746} \times \frac{\mathrm{E\,C}\,v}{\pi\,\mathrm{D}\,\eta\,(\text{armature revs. per minute})},$$

v being the gearing ratio.

The rates at which the car would be accelerated by its own weight on various gradients if friction were absent are calculated, multiplied by $\frac{1 + a}{2}$, and the amount for each marked off from 0 along O Y. It is sufficient to do this for two gradients—for example, 5 and 10 per cent; to set off from 0 on O Y lengths, representing the numbers so obtained, and to divide their difference into five parts. By stepping off lengths equal to these above and below those marked, the retardation (negative acceleration) is obtained for the level and for down grades. From any point on the scale of retarding force, say 4,000 lb., an ordinate is raised and made equal on the retardation scale to the product of the acceleration which would be produced on the level by 4,000 lb. pull if friction were absent and $1 + a$. The extremity of this ordinate is joined to the level point on the gradient scale, and parallel lines are drawn from other points on the scale.

As an illustration of the use of the diagram, let a car be running down a 6 per cent incline at a particular instant at 13 miles per hour on the first braking notch. The current per motor is then 54 amperes. From the intersection of the horizontal line and the retarding force curve a vertical is dropped, and the point where this cuts the 6 per cent gradient line gives the retardation as 1·2 miles per hour per second. The same vertical cuts the horizontal axis (retardation = 0) at a point where the 12 per cent gradient line would be, and hence, with this notch, 18 miles per hour would be the final steady speed on that slope.

Conversely, to find the steady speed on any gradient, suppose 8 per cent down, a vertical is raised where this gradient line cuts the axis to intersect the retarding force curve, and a horizontal line is drawn where it cuts it. The current is then 40 amperes per motor, the speed on the first notch 14·8, on the second 11·3, on the third 5·6, on the fourth 2·5, and on the fifth 1·1 miles per hour.

Efficiency Curves.—The losses of energy in any dynamo-electric machine are of two kinds—(*a*) electrical and magnetic, (*b*) mechanical, on account of friction; and it is so possible to distinguish two efficiencies—(1) the electrical, or the efficiency of conversion of the electrical into mechanical energy or *vice versâ*, (2) the commercial or plant efficiency, *i.e.*, the ratio of the energy, electrical or mechanical, put into the machine to that given out. Thus in a generator the commercial efficiency is the ratio of the electrical horse power given out to the external circuit expressed as a percentage of the mechanical horse power applied to the armature shaft. The electrical efficiency is the ratio of the output to the watts generated in the armature. In the case of a motor the electrical efficiency is the ratio of the mechanical horse power transmitted to the shaft by the armature to the electrical horse power absorbed; the commercial efficiency is the horse power given out by the motor shaft expressed as a percentage of the electric horse power absorbed from the line.

American engineers usually restrict the term "efficiency" to what is here called "commercial efficiency," and designate the ratio of the energy given out by the armature to that absorbed the "economical coefficient." The ratio of the total electrical energy developed in an armature to the horse power delivered to it by the shaft, or, in a motor, the ratio of the "back watts," *i.e.*, the product of current and back electro-motive force to the horse power delivered by the armature to the shaft, is termed the "coefficient of conversion."

It will be evident that the electrical efficiency of a machine will be more or less short of 100 per cent by reason of the armature conductor and other resistances, and that if these can be calculated and deducted from the total energy the fraction remaining gives at once the efficiency. The commercial efficiency is in every case less than this, as a further mechanical loss occurs in friction in bearings, windage, hysteresis, and eddy-currents. If these, together with the electrical losses, be ascertained and deducted from the input, the remainder, expressed as a percentage, is clearly the commercial efficiency. It may also be seen that if the electrical efficiency be denoted by η, the coefficient of conversion by ζ, and the commercial efficiency by ϵ, then

$$\epsilon = \zeta \times \eta.$$

In investigating the efficiencies and losses in the various types of machine, the quantities will be indicated as follows:—

E_a = E.M.F. induced in armature.
E_t = E.M.F. at terminals.
R_a = Armature and brush resistance.
R_m = Resistance of series coils.
R_s = Resistance of shunt coils.
R = Resistance of external circuit.
C_a = Armature current.
C_s = Shunt current.
C = External current.

With a series generator

$$E_t = C\,R$$

and $$E_a = C\,(R_a + R_m + R).$$

Hence $$\eta = \frac{E_t\,C}{E_a\,C}.$$

Since the same current flows in all parts of the circuit, it is also evident that the efficiency may be written as the ratio of the external to the entire resistance in the circuit, or as

$$\eta = \frac{R}{R_a + R_m + R}$$

and from this expression it may be seen that the greater R becomes in comparison with $R_a + R_m$, the nearer does the value of η approach unity or 100 per cent. Hence, so far as this efficiency is concerned, the machine is most efficient when giving least current. When the mechanical losses are taken into consideration, however, it is seen that, as these are fairly constant at any one speed, the condition does not hold, but the maximum efficiency, with a lower value, is given at some definite value of the current.

With a shunt generator the armature current is the sum of those in the external circuit and the shunt coils, or $C_a = C + C_s$. Inasmuch as in modern machines the shunt current is but small, no considerable error will be introduced if C_a be taken as equal to C for all but the smallest loads. In that case

$$\eta = \frac{E_t\,C}{E_t\,C + E_t\,C_s}$$

from which, substituting for E_t, it may be seen that

$$\eta = \frac{1}{1 + \frac{R_a}{R} + \frac{R}{R_s}}$$

and hence, by differentiating, that η is a maximum when the internal resistance is a geometrical mean between R_a and R_s. Under this particular condition

$$\eta = \frac{1}{1 + 2\sqrt{\frac{R_a}{R_s}}}$$

For practical purposes the most favourable load is to be determined by finding the position of greatest commercial efficiency in a manner similar to that described for the series machine.

Making the same assumption for a compound as for the shunt generator, viz., that $C_a = C$, the efficiency equation is

$$\eta = \frac{E_t\,C}{C^2\,(R + R_a + R_m) + C_s{}^2\,R_s}$$

$$= \frac{1}{1 + \frac{R_a}{R} + \frac{R}{R_m} + \frac{R_m}{R}}$$

from which it may be shown that the electrical efficiency is a maximum, when

$$R^2 = R_s\,(R_a + R_m),$$

in which case

$$\eta = \frac{1}{1 + 2\sqrt{\frac{R_a + R_m}{R_s}}}.$$

The point at which the machine runs at its greatest commercial efficiency depends upon the mechanical losses as in the previous cases.

In a general way it is determined by the relative proportions of the fixed losses (hysteresis, eddy current, and friction) to the variable (resistance) loss. As the frictional loss of energy is very small in all except very small machines, these are often referred to as the "iron" and "copper" losses respectively.

The curves of fig. 221 and 222 show that to obtain the maximum efficiency at small loads the fixed losses should be small and the copper losses proportionately greater; to get it heavier loads, the iron losses should be relatively greater and the copper loss less.

The relative proportions of the various losses, as well as the efficiencies at different loads in a modern generator of fair size are clearly shown in fig. 223, which refers to an

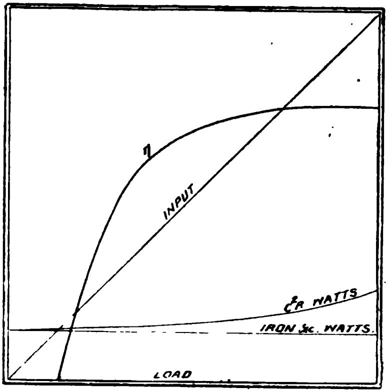

Fig. 221.

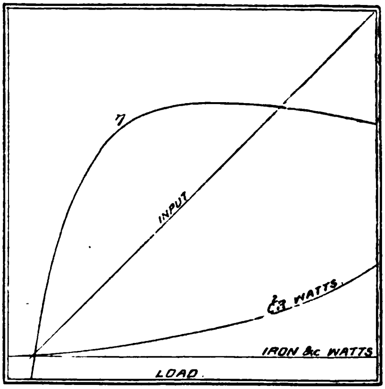

Fig. 222.

800 kilowatt machine. The ordinates of the loss curves are figured in kilowatts, and at full loads their amounts are :—

Fixed losses ...	Bearing friction and windage	·47	per cent.
	Brush friction	·71	„
	Hysteresis and eddy currents	2·70	„
Variable losses	C^2 R loss in armature and brushes	1·76	„
	C^2 R loss in shunt coils	·35	„
		6·0	„

giving a commercial efficiency of 94 per cent.

At half load :—

Fixed losses ...	Bearing friction and windage	·94	per cent.
	Brush friction	1·42	„
	Hysteresis and eddy currents	4·71	„
Variable losses	C^2 R loss in armature and brushes	·83	„
	C^2 R loss in shunt coils	·70	„
		8·6	„

the efficiency thus being 92·4 per cent.

It may be noted that the so-called "fixed" losses are not, strictly speaking, exactly so, as the eddy current loss is affected by the armature current. This variation is on account of the smallness of the total loss from this cause, negligible. The magnitude of the loss due to friction between the brushes and the commutator is also worthy of note, being in this case considerably greater than the loss by the friction in the main bearings. This arises from the fact of the machine being multipolar parallel wound and so carrying a large number of brushes; in bipolar machines the brush friction loss would be far less comparatively to the others.

In the case of motors, the expressions for the efficiency are obtained in a very similar manner to those dealt with. Thus the commercial efficiency for a motor is equal to the ratio of the mechanical horse power given out from its shaft to the electrical horse power absorbed from the line, and the difference between the two quantities is the sum of the frictional, iron, and copper losses. The remarks made as to the effect of the proportions of the iron and copper losses on the load at which a machine will run at its maximum efficiency apply equally to motors, and careful attention should be given in their design to apportioning the respective losses so as to bring the greatest efficiency, not necessarily to the motor's full load, but at its usual working load. The

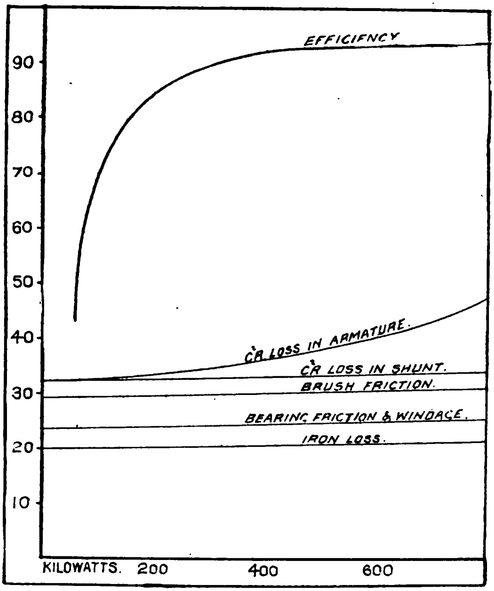

FIG. 223.

losses and efficiencies of a tramway motor—which may be regarded as typical—are shown in fig. 224.

In conclusion, it may be pointed out that the efficiency of an electrical machine, like that of steam engines and most other mechanical apparatus, increases with its size if properly designed. Thus the average efficiency of a 10 kilowatt machine is about 80, of one giving 100 kilowatts 84 per cent, and in large generators of 500 to 1,000 kilowatt capacity about 86 per cent at full load.

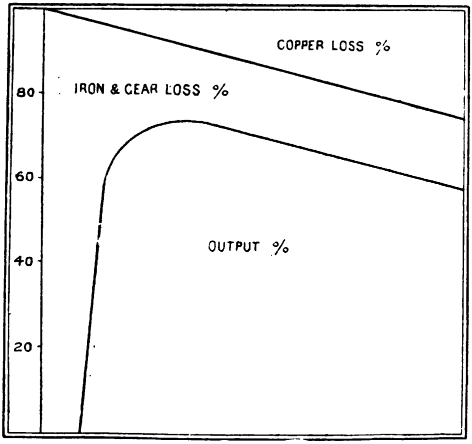

FIG. 224

CHAPTER XVI.

The Testing of Electrical Machinery.—In the foregoing chapter the methods of predicting the qualities of machines from a knowledge of the characters of the materials employed in their construction and by calculation from this and the dimensions have been briefly treated. Such predeterminations can, nevertheless, be only at the best more or less near approximations, and in every case their accuracy must depend upon a knowledge of the performance of previously made machines when actually tested.

The amount of testing through which a commercial machine is put before leaving the works varies, of course, with the character of the works and with that of the machine. Small plant is, indeed, often passed over with but sufficient trial to prove that the connections are correct and that the output is about that expected, and this is more frequently the case when of a stock pattern. For such rough testing the machine is often temporarily connected to a source of power, run at about its proper speed, and its terminal pressure noted with various currents, the latter being regulated by either a rheostat or a water resistance. On the other hand, large and heavy machines are generally put through a thorough testing both for regulation and efficiency. It is the object of the present chapter to deal with the methods of testing magnetic leakage, heating regulation, and efficiency.

The Magnetic Leakage.—This may be ascertained by passing a loop of wire tightly round the armature, and bringing the ends well twisted together to a ballistic galvanometer. If the armature be stationary, so that the coil is in such a position that the whole armature flux passes through it, and the exciting current be on, then upon switching off or reversing the exciting current the galvanometer will swing over to some position θ, owing to the current induced in the loop by the change of armature flux. A similar loop wound closely round the centre of a magnet coil, or preferably, if possible, round the iron under the centre of the coil, and similarly connected to the galvanometer, will produce a swing of greater extent θ' when the exciting current is cut off or reversed, owing to the change in the magnet flux. Since the induced currents and deflections are proportional to the fluxes,

$$\frac{\text{Armature flux}}{\text{Flux in magnet coil}} = \frac{\theta}{\theta'},$$

it is necessary that the loop should in each case have the same number of turns and the wires be of the same length. As ballistic galvanometers are of rare occurrence in commercial works, recourse may be had to a voltmeter, which will give readings proportional, roughly, to the fluxes.

With a multipolar machine the loops must be placed in corresponding positions, but in machines of moderate or large proportions the flux requires an appreciable time to vary, and unless a galvanometer with an unusually heavy needle is available the determination cannot well be made. This difficulty may be obviated to some extent by increasing or reducing the shunt current by some definite amount, instead of cutting it off or reversing, operations which would be extremely risky on account of the field coils' inductance.

The Determination of the Magnetic Density in the Air Gap.—This operation has been already discussed in pp. 101-104, and to obtain complete information it should be performed both when running on open circuit and when carrying the full load current.

The Magnetisation Curve.—It is evident that by varying the exciting current and making the previous determination for each value the magnetisation curve of the machine could be deduced; the method would entail, however, an inordinate amount of toil, and a far less troublesome means can be applied.

The machine is run at a constant speed, with its field separately excited on open circuit. The terminals being connected to a voltmeter, the exciting current is increased by steps, and the volts for each of its noted values are read. Dividing these readings by $T N \frac{p}{n} 10^{-8}$ and by 60, the flux per pole for each value of the exciting current is obtained. In practice shunt machines may be allowed to excite themselves, the shunt current being read from an inserted ammeter; but for strict accuracy the noted volts must be increased by the product of the shunt amperes and the armature resistance. Moreover, as it is generally impracticable to keep the speed constant, speed readings may be taken at the same instant as the current and volts are read; the variation can then be corrected for, by reducing or increasing the volts in the same proportion as the speeds.

Owing to hysteresis, the magnetisation curve for increasing exciting current is somewhat lower than that for falling values; it is therefore well to take two sets of readings, one with it increasing, the other while being reduced.

It will be evident that the effect of armature reaction on the curve can be investigated if an external current be allowed to flow, but the volts must in this case be increased by the internal drop due to this current.

Insulation Test.—In addition to the testing for insulation which the separate parts of machine should receive before being assembled together, the complete machine should have a rigorous testing, particularly if designed for high-pressure work, both between separate circuits, and between each of them and the framing of the machine or earth. To this end it should be subjected to pressures far exceeding any with which it is likely to be concerned in its usual working, in any case from twice to three times as great. The high-pressure supply for this test is commonly obtained from a small alternator.

Heating Test.—For this test, it is to be noted that the machine, especially if large, must be run for a considerable time before the ultimate stationary temperature is attained, the later rate of temperature rise being extremely slow. The regions usually tested for heating are the field coils, the armature, and the commutator, the usual method being to place the bulb of a sensitive mercury thermometer closely against the surface, and to protect it from cooling by holding a handful of cotton wool over it. A much more satisfactory testing of the heating of field coils may be made if a length of platinum wire be interwound at about the middle of their thickness when the copper is wound. The resistance of platinum varies largely with temperature, and if its coefficient be found before winding, its temperature, which is that of the hottest part of the coil, can be ascertained at any subsequent time by merely measuring its resistance.

The Characteristic Curve.—The usual test for obtaining this is to run the generator or the motor as a generator at a constant speed with a voltmeter connected across the terminals. An ammeter is inserted in the external circuit consisting of a water or other adjustable resistance. This resistance is then varied, and the corresponding values of the current and the volts are read. It is usually necessary, on account of its not being perfectly constant, to also read the speeds. If the speed variation is but small, it may be corrected for in a rough test by reducing or increasing the volts and amperes proportionally to the speeds—a method which is more justifiable in the case of shunt machines than of others. A more accurate, though correspondingly laborious method is as follows: Let C be the current, N, the noted speed, E_1 the voltage, and N the standard speed for which the curve is to be found.

The speed is now altered by adjusting the driving power, and the resistance also adjusted until the current is again equal to C. Let N_2, E_2, be the new speed and voltage.

Then the required voltage E corresponding at the speed N to the current C, is given by

$$E = \frac{N - N_2}{N_1 - N_2}(E_1 - E_2) + E_2.$$

Having found in this way a sufficient number of pairs of values of E and C, the external characteristic may be set out, and from it, if required, the internal may be derived by the method of page 400.

The Efficiency of the Motor.—It is frequently by running as a motor that the efficiency of a generator may be most conveniently determined. Whether this is being done, or the machine be intended for a motor, there are several methods of making the test. Two only will be dealt with.

(i.) The electrical method in which the machine is run at its normal speed with the field disconnected and separately excited. The armature current being adjusted for the correct speed, and the excitation got up to its proper working value, both armature watts and field watts are read from an ammeter and voltmeter in each circuit. The whole of the power thus absorbed is evidently that required to make up the resistance and constant losses when the machine is carrying the armature current taken in the test. By measuring the armature and field resistances while hot, the C^2 R loss can be calculated, and this deducted from the total power taken up, gives the amount of the fixed losses. For any output, therefore, the C^2 R loss may be calculated, and this deducted from the total power taken up gives the amount of the fixed losses. For any output, therefore, the C^2 R loss may be calculated and added to the fixed losses Then

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}} \quad \text{(generator)}$$

$$\eta = \frac{\text{input} - \text{losses}}{\text{input}} \quad \text{(motor.)}$$

(ii.) The brake method in which the machine running as a motor has the power input measured by the voltmeter and ammeter, and the output measured mechanically by a Prony brake. For small machines this brake may consist simply of cords to which wooden blocks are attached bearing round the pulley, each end of the cords being connected to a spring balance. For larger sizes a long bar clamped to two heavy wooden blocks, bolted round the pulley, is more suitable, the distant end of the bar being restrained from rotation by a spring balance. The difference of pulls in pounds on the balances in the first case being P, and the radial distance from the centre of the pulley to that of the cords being R, the torque is P R, and the work done at N revolutions per minute is at the rate of

$$\frac{2 \pi N P R}{33000} \text{ horse power.}$$

With the second form of brake the rate of work is

$$\frac{2\pi N P R}{33000} \text{ horse power,}$$

where P denotes the pounds pull on the spring balance, and R is the perpendicular distance from the direction of its pull on the bar to the centre of the pulley. In either case η is obtained by multiplying the horse power by 746 and dividing by the watts absorbed from the supply.

The chief practical difficulty in applying this test is the prevention of the pulley becoming too hot, on which account the readings should be taken as rapidly as possible.

Separation of Hysteresis, Frictional, and Eddy Current Losses.—The accurate testing of the frictional loss is difficult on account of its small amount, but it may be done if the machine can be driven from a motor, of which the efficiency is accurately known at all loads, by so driving it with the field unexcited and noting the power taken in by the motor. Or, if the machine be fitted with a pulley, a light strap with a spring balance at each end may be passed over the pulley and the machine pulled round; in this case the difference of readings between the spring balances multiplied by the radius of the pulley gives the frictional torque from which the loss on its account, at any speed, may be calculated. A method depending on the moment of inertia of the revolving parts has also been published by the author.

The frictional loss thus found for any speed will at any other speed vary proportionately.

If the field is now excited and the power absorbed by the motor driving the machine excited and on open circuit is measured, the work spent in supplying combined friction, hysteresis, and eddy current loss is known. Deducting from this the amount of the first, the two last losses combined are known. The hysteresis loss varies as the speed, the other as the square of the speed. Hence may be written—

$$\text{Hys. loss} \times N + \text{eddy loss} \times N^2 = \text{combined loss (i).}$$

Again running at a different speed and noting the combined loss

$$\text{Hys. loss} \times N' + \text{eddy loss} \times N'^2 = \text{combined loss (ii)}$$

and from these two equations each may be found.

The Hopkinson Efficiency Test.—When machines of large size are constructed in duplicate, the considerable cost of transmitting energy through them and then absorbing it by brakes or resistances may be avoided by adopting this very accurate method of determining the efficiency. It is based on the obvious fact that if two exactly similar machines running at the same speed have their brushes of like polarity connected, the only power required from externally is that which will just overcome their resistance to the motion and current flow.

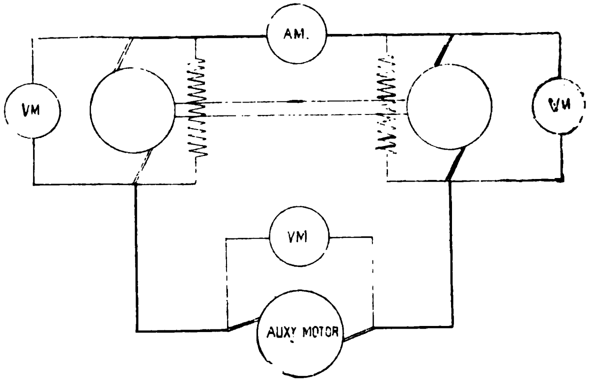

FIG. 225.

If the efficiencies of the two machines be regarded as equal, the test admits of several variations in its details, and that depending on the readings of three voltmeters is here explained.

In any case the two machines have their shafts rigidly connected in line to ensure equal speed, and an accessory generator or battery of cells is necessary for supplying the losses.

The electrical connections being made as shown in fig. 225, with an adjustable resistance in the shunt circuit of the machine which is to run as generator, current is switched on from the auxiliary generator, and at the same time the resistance is cut out from the generator under test until both the normal current and speed is obtained. The readings of the three voltmeters are then simultaneously taken.

If now C be the current circulating E_1, E_2, E_3, the voltmeter readings, the power given out by the generator is $C E_1$, and if η be its efficiency the power it absorbs is $C E_1 \frac{1}{\eta}$. The power taken in by the motor is $C E_2$, and that given out by it $C E_2 \eta$. The difference of these amounts is the power absorbed by the pair from the auxiliary source. Hence,

$$C E_1 \frac{1}{\eta} - C E_2 \eta = C E_3.$$

Hence,

$$E_2 \eta^2 + E_3 \eta = E_1,$$

a quadratic from which it is seen that

$$\eta = + \sqrt{E_1 + \tfrac{1}{4} \left(\frac{E_3}{E_2}\right)^2} - \frac{E_3}{2 E_2}.$$

In an analogous manner the efficiency may be found by employing three ammeters and making measurements of current instead of pressure.

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